SOFTBIT-SOURCE DECODING BASED ON THE TURBO-PRINCIPLE

M. Adrat, J.-M. Picard, and P. Vary

Institute of Communication Systems and Data Processing (IND)
Aachen, University of Technology
{adrat,vary}@ind.rwth-aachen.de

ABSTRACT

This contribution deals with a robust decoding approach of source codec parameters called Softbit-Source Decoding. It can be considered as an error concealment technique which estimates codec parameters at the receiver utilizing residual redundancy remaining after source coding and channel decoder reliability information.

In this paper we will apply the TURBO-principle to Softbit-Source Decoding. We derive a formula that shows how different terms of residual redundancy can be transformed into independent *extrinsic* information utilizable in iterative processes. After introducing a new, iterative approach to Softbit-Source Decoding, we will analyze the influences of different bit mappings and of the number of iterations. Finally, the new approach will be compared to conventional techniques with respect to performance and complexity.

1. INTRODUCTION

Transmission bandwidth is a limited resource in digital mobile communication. Therefore, low to medium bit rate source encoders based on models are used. Characteristic parameters are extracted from the signal, but unfortunately they are partly highly sensitive against transmission errors. Some bit errors in the received parameters can result in extremely annoying artifacts.

In general, source codecs reduce redundancy, but due to delay and complexity constraints source parameters can exhibit considerable redundancy, either in terms of a non-uniform distribution or in terms of correlation, i.e., auto-correlation in time and cross-correlation between adjacent parameters. Measurements of the statistical properties of frame-oriented compression algorithms have shown that, for example, subsets of *line-spectrum frequencies* determined in modern speech codecs as the GSM-AMR codec (*adaptive multi-rate*) or scale factors of audio transform codecs like in *digital audio broadcasting* (DAB) exhibit significant auto- and cross-correlation. As already indicated by Shannon this residual redundancy can be exploited at the receiver to enhance the disturbed signal.

Recently, we have proposed an approach to error concealment by Softbit-Source Decoding [1, 2]. Reliability information gained from a soft-output channel decoder is combined with *a-priori* knowledge about statistical properties of the source to estimate the codec parameters. But an optimal utilization of all given terms of residual redundancy [3] requires high computational efforts.

The performance bounds might already be reached with less complexity by a suboptimal iterative approach. Iterative algorithms are known from the so-called TURBO technique originally introduced to channel decoding [4, 5] and recently applied to joint source-channel decoding [6, 7]. Two independent channel decoders are able to benefit from each other when *extrinsic* information extracted from the one decoder serves as additional *a-priori* knowledge in the other decoder.

In this paper we adopt the TURBO-principle to Softbit-Source Decoding. Therefore, both techniques will be briefly reviewed first. In Section 3 we will introduce the new approach and afterwards, in Sections 4 and 5, we will analyze the performance of the iterative procedure and compare it with conventional techniques.

2. SOFTBIT-SOURCE DECODING AND THE TURBO-PRINCIPLE

For further considerations the transmission model depicted in Figure 1 is assumed [7].

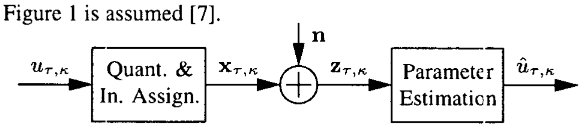

Figure 1: Transmission model

At time instant τ, a source encoder determines a set of N parameters $u_{\tau,\kappa}$, with $\kappa = 1, 2, \ldots, N$ denoting the position within the set. Each value $u_{\tau,\kappa}$, which is continuous in magnitude but discrete in time, is individually quantized by R_κ reproduction levels $\bar{u}_\kappa^{(i)}$ with $i = 1, 2, \ldots, R_\kappa$. The reproduction levels are invariant with respect to τ and a whole set is given by $\mathbb{U}_\kappa = \{\bar{u}_\kappa^{(1)}, \bar{u}_\kappa^{(2)}, \ldots, \bar{u}_\kappa^{(R_\kappa)}\}$. To each $\bar{u}_\kappa^{(i)}$ a unique bit pattern $\mathbf{x}_\kappa = \{x_\kappa(\lambda)\}^{\forall\lambda}$ with $\lambda \epsilon [1, w_\kappa]$ is assigned where the length w_κ of $\mathbf{x}_\kappa$ is usuallygiven by $w_\kappa = \log_2(R_\kappa)$. Corresponding to a set of quantizer reproduction levels $\mathbb{U}_\kappa$, the complete set of possible bit patterns is given by $\mathbb{X}_\kappa$. The bit pattern assigned to $u_{\tau,\kappa}$, i.e. $\bar{u}_\kappa^{(i_\tau)}$, is denoted by $\mathbf{x}_{\tau,\kappa}$, i.e. $u_{\tau,\kappa} \to \mathbf{x}_{\tau,\kappa}$. In order to simplify notation throughout this paper, time sequences of parameters are denoted by $\mathbf{x}_{1,\kappa}^\tau = \mathbf{x}_{\tau,\kappa}, \mathbf{x}_{\tau-1,\kappa} \ldots \mathbf{x}_{1,\kappa}$.

At time instant τ, a set of bit patterns $\mathbf{x}_{\tau,\kappa}$ is transmitted over a channel with additive noise $\mathbf{n}$ and a possibly disturbed bit vector $\mathbf{z}_{\tau,\kappa}$ is received.

2.1. Review of Softbit-Source Decoding

Softbit-Source Decoding (SBSD) by parameter estimation determines an estimate $\hat{u}_{\tau,\kappa}$ of $u_{\tau,\kappa}$. Usually, the *minimum mean squared error* (MMSE) is used as optimality criterion individually for each parameter $E\{(u_{\tau,\kappa} - \hat{u}_{\tau,\kappa})^2\}$. $E\{\cdot\}$ denotes the expected value. With regard to the introduced notations the well known MMSE estimation rule can be written as

$$\hat{u}_{\tau,\kappa} = \sum_{\bar{u}_\kappa^{(i)} \epsilon \mathbb{U}_\kappa} \bar{u}_\kappa^{(i)} \cdot P(\bar{u}_\kappa^{(i)} | \mathbf{z}_{1,N}^\tau \ldots \mathbf{z}_{1,\kappa}^\tau \ldots \mathbf{z}_{1,1}^\tau) \quad . \quad (1)$$

$P(\bar{u}_\kappa^{(i)} | \mathbf{z}_{1,N}^\tau \ldots)$ is the *a-posteriori* probability for reproduction level $\bar{u}_\kappa^{(i)}$ at position κ when the complete set of entire history

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$\mathbf{z}_{1,\kappa}^{\tau}$ with $\kappa = 1, 2, \ldots N$ is given. Due to the fixed index assignment, $\bar{u}_\kappa^{(i_\tau)} \rightarrow \mathbf{x}_{\tau,\kappa}$, $P(\bar{u}_\kappa^{(i)}|\mathbf{z}_{1,N}^{\tau}\ldots)$ is equal to the i th of R_κ possible conditional probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$. Hence, the problem of estimating the optimal parameter value $\hat{u}_{\tau,\kappa}$ can be reduced to the problem of determining the *a-posteriori* probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$.

Recently, several solutions for the determination of $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ have been introduced, exploiting different terms of redundancy [1–3]. If, for example, the non-uniform distribution $P(\bar{u}_\kappa^{(i)})$, $P(\mathbf{x}_{\tau,\kappa})$ respectively, of the desired parameter $\bar{u}_\kappa^{(i)}$ and no correlation shall be utilized, $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ can easily be determined using Bayes' theorem $\forall \mathbf{x}_{\tau,\kappa} \epsilon \mathbb{X}_\kappa$

$$P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{\tau,\kappa}) = C \cdot p(\mathbf{z}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa}) \cdot P(\mathbf{x}_{\tau,\kappa}) \quad . \qquad (2)$$

C denotes a constant term which ensures that the sum over *a-posteriori* probabilities $\sum_{\mathbf{x}_{\tau,\kappa}\epsilon\mathbb{X}_\kappa} P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{\tau,\kappa})$ at position κ equals one. The probability density function $p(\mathbf{z}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa})$ is a channel-dependent term for $\mathbf{x}_{\tau,\kappa}$, and the last factor in Eq. (2) represents *a-priori* knowledge resulting from the non-uniform distribution. Note that thereby, the *a-posteriori* probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ are reduced to $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{\tau,\kappa})$, because only the received bit pattern $\mathbf{z}_{\tau,\kappa}$ at time instant τ of the desired parameter $\bar{u}_\kappa^{(i_\tau)}$ has been taken into account.

If on the basis of a first-order Markov model time-correlation between consecutive parameter indizes $\mathbf{x}_{\tau-1,\kappa}$ and $\mathbf{x}_{\tau,\kappa}$ shall be utilized, then parameter *a-priori* knowledge has to be considered in form of transition probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$ [2]. This form enables to approximate the parameter *a-posteriori* probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ by $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,\kappa}^{\tau})$, as the entire history of the desired parameter will be considered. However, in this simplified estimation rule most of the given parameters $\mathbf{z}_{1,N}^{\tau}, \ldots \mathbf{z}_{1,1}^{\tau}$ are still unused as κ is fixed, i.e, the cross-correlation between parameters is not considered. Therefore, some performance might be lost. In order to exploit all available information, correlation properties between adjacent parameter indizes $\mathbf{x}_{\tau,\kappa-1}$ and $\mathbf{x}_{\tau,\kappa}$ have to be utilized, too. The optimal solution was derived in [3], but the computational complexity needed for the determination of all $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots\mathbf{z}_{1,1}^{\tau})$ is extremely high. Hence, to cope with possibly given complexity constraints we will propose a new low complexity determination rule for $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ considering all given parameters $\mathbf{z}_{1,N}^{\tau}, \ldots \mathbf{z}_{1,1}^{\tau}$. This rule will be derived from the TURBO-principle.

2.2. Review of the TURBO-Principle

The TURBO-principle is originally known as a decoding technique for concatenated channel codes [4,5]. The so called *extrinsic* information can be extracted from the soft-output of the one decoder and serve as additional *a-priori* knowledge for the other decoder. While decoding the independent processes iteratively, an update of the *extrinsic* information after each iteration allows to successively increase quality, e.g., in terms of decreasing bit error rates. In contrast to conventional decoding techniques, the TURBO-principle permits to approach Shannon's performance bounds quite close with reasonable computational complexity.

The *extrinsic* information is the key parameter in an iterative decoding process. In special cases it can be extracted from the decoder's soft-output as the following simple example for binary data should clarify:

If the bit pattern $\mathbf{x}_{\tau,\kappa}$ is channel encoded with an even single parity check code, the parity bit $q_{\tau,\kappa}$ is generated by $q_{\tau,\kappa} = \{x_{\tau,\kappa}(1) \oplus x_{\tau,\kappa}(2) \oplus \ldots x_{\tau,\kappa}(w_\kappa)\}$. $\oplus$ denotes the binary *exclusive-or* operation. From this follows immediately that each information bit $x_{\tau,\kappa}(m)$ can also be determined from the parity bit $q_{\tau,\kappa}$ and all the other information bits of $\mathbf{x}_{\tau,\kappa}$

$$\begin{aligned} x_{\tau,\kappa}(m) = \{&x_{\tau,\kappa}(1) \oplus \ldots x_{\tau,\kappa}(m-1) \oplus \\ &x_{\tau,\kappa}(m+1) \oplus \ldots x_{\tau,\kappa}(w_\kappa) \oplus q_{\tau,\kappa}\} \quad . \end{aligned} \qquad (3)$$

Thereby, information for bit $x_{\tau,\kappa}(m)$ is available twice, directly and indirectly according to Eq. (3), which is a special form of diversity. At the receiver side of a (possibly noisy) transmission system this redundancy allows to enhance the *a-posteriori* probabilities for bit $x_{\tau,\kappa}(m)$. These probabilities are not only conditioned on $z_{\tau,\kappa}(m)$, but also on all the other bits of $\mathbf{z}_{\tau,\kappa}$ as well as the received value $\tilde{q}_{\tau,\kappa}$ for bit $q_{\tau,\kappa}$. Neglecting bit correlation, the *a-posteriori* probability is given by $P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}, \tilde{q}_{\tau,\kappa})$, which can easily be re-written as

$$\begin{aligned} P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}, \tilde{q}_{\tau,\kappa}) &= C \cdot p(z_{\tau,\kappa}(m)|x_{\tau,\kappa}(m)) \cdot \\ &P(x_{\tau,\kappa}(m)) \cdot p(\{z_{\tau,\kappa}(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{q}_{\tau,\kappa}|x_{\tau,\kappa}(m)) \end{aligned} \qquad (4)$$

when a memoryless channel is assumed. Again, C denotes a constant term which ensures that the sum of *a-posteriori* probabilities $P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}, \tilde{q}_{\tau,\kappa})$ equals one. In addition to the channel related term $p(z_{\tau,\kappa}(m)|x_{\tau,\kappa}(m))$ and the *a-priori* term $P(x_{\tau,\kappa}(m))$, there exists an *extrinsic* knowledge $p(\{z_{\tau,\kappa}(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{q}_{\tau,\kappa}|x_{\tau,\kappa}(m))$ for bit $x_{\tau,\kappa}(m)$. This term results from the special diversity according to Eq. (3) and therefore diversity is required to enable iterative processes.

3. ITERATIVE SOFTBIT-SOURCE DECODING

In the transmission model depicted in Fig. 1, sets of correlated and scalar-quantized parameters $\bar{u}_\kappa^{(i)} \rightarrow \mathbf{x}_{\tau,\kappa}$ have been assumed. In contrast to the explanations in Subsec. 2.2, diversity for single bits $x_{\tau,\kappa}(m)$ has not been introduced in terms of explicit redundancy, such as parity bits, but implicit redundancy is available due to the statistical properties of the source codec parameters. A non-uniform parameter distribution or correlation properties can be interpreted as diversity for single bits $x_{\tau,\kappa}(m)$ as well. Hence, an *extrinsic* value for single bits $x_{\tau,\kappa}(m)$ might be available whenever parameter *a-posteriori* probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,\kappa}^{N}\ldots)$ are determined using Softbit-Source Decoding (SBSD).

Therefore, we will quantify the *extrinsic* value(s) of SBSD next. This will be done for a single kind of redundancy at first, e.g., according to Eq. (2) for the probabilities of occurrence $P(\mathbf{x}_{\tau,\kappa})$ [7]. Afterwards the *extrinsic* values due to the auto-correlation and cross-correlation properties will be considered, too.

3.1. The *extrinsic* value due to a non-uniform distribution

Softbit-Source Decoding mainly depends on the determination of *a-posteriori* probabilities $P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots)$ for bit patterns $\mathbf{x}_{\tau,\kappa}$. In order to quantify the gain due to Softbit-Source Decoding for single bits $x_{\tau,\kappa}(m)$, the probabilities $P(x_{\tau,\kappa}(m)|\mathbf{z}_{1,N}^{\tau}\ldots)$ with $m = 1, 2, \ldots w_\kappa$ can easily be obtained by

$$P(x_{\tau,\kappa}(m)|\mathbf{z}_{1,N}^{\tau}\ldots) = \sum_{\forall(\mathbf{x}_{\tau,\kappa}|x_{\tau,\kappa}(m))} P(\mathbf{x}_{\tau,\kappa}|\mathbf{z}_{1,N}^{\tau}\ldots), \qquad (5)$$

i.e., the marginal distribution of parameter *a-posteriori* probabilities has to be determined over all $\mathbf{x}_{\tau,\kappa}\epsilon\mathbb{X}_\kappa$ with a given $x_{\tau,\kappa}(m)$.

If a non-uniform distribution serves as parameter *a-priori* knowledge only, i.e correlation properties between consecutive or adjacent parameters are neglected, then parameter *a-posteriori* probabilities are given by Eq. (2). Hence, Eq. (5) can be re-written as

$$P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}) = C \cdot \sum_{\forall(\mathbf{x}_{\tau,\kappa}|x_{\tau,\kappa}(m))} p(\mathbf{z}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa}) P(\mathbf{x}_{\tau,\kappa}) \,. \quad (6)$$

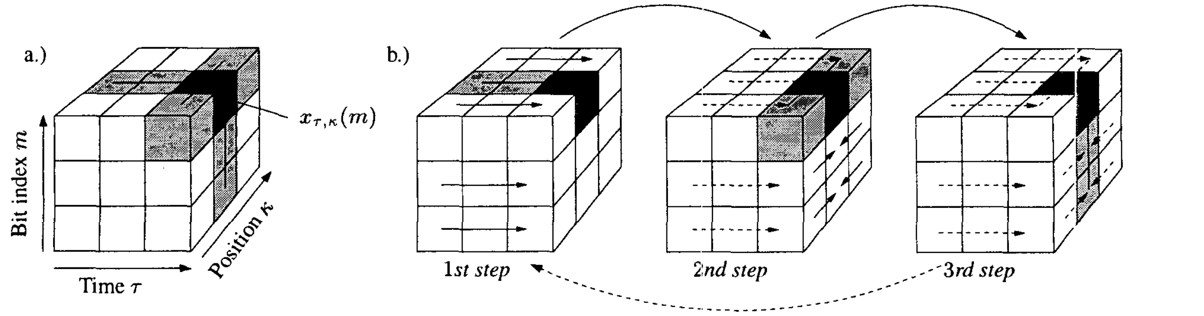


Figure 2: a.) Illustration of the conditions on combination b.) Iterative decoding approach using three steps per iteration

In case of a memoryless transmission the channel dependent parameter term $p(\mathbf{z}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa})$ is given by a product of terms for single bits

$$P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}) = C \cdot \sum_{\forall(\mathbf{x}_{\tau,\kappa}|x_{\tau,\kappa}(m))} P(\mathbf{x}_{\tau,\kappa}) \prod_{\lambda=1}^{w_\kappa} p(z_{\tau,\kappa}(\lambda)|x_{\tau,\kappa}(\lambda)) \,. \quad (7)$$

Furthermore, the *a-priori* information of parameter $\mathbf{x}_{\tau,\kappa}$ is a joint probability $P(\mathbf{x}_{\tau,\kappa} = \{x_{\tau,\kappa}(\lambda)\}^{\forall\lambda})$ of single bits $x_{\tau,\kappa}(\lambda)$ with $\lambda = 1, 2, \ldots w_\kappa$. Applying the chain rule allows to extract the probability for $P(x_{\tau,\kappa}(m))$

$$P(\mathbf{x}_{\tau,\kappa}) = P(\{x_{\tau,\kappa}(\lambda)\}_{\lambda\neq m}^{\forall\lambda}|x_{\tau,\kappa}(m)) \cdot P(x_{\tau,\kappa}(m)). \quad (8)$$

As the sum of Eq. (7) has to be determined for any fixed combination τ, κ and m, the bitwise *a-priori* knowledge $P(x_{\tau,\kappa}(m))$ as well as the channel dependent term $p(z_{\tau,\kappa}(m)|x_{\tau,\kappa}(m))$ are constant. Hence, both can be factored

$$P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa}) = C \cdot p(z_{\tau,\kappa}(m)|x_{\tau,\kappa}(m)) \cdot P(x_{\tau,\kappa}(m)) \cdot \sum_{\forall(\mathbf{x}_{\tau,\kappa}|x_{\tau,\kappa}(m))} \frac{P(\mathbf{x}_{\tau,\kappa})}{P(x_{\tau,\kappa}(m))} \prod_{\substack{\lambda=1\\ \lambda\neq m}}^{w_\kappa} p(z_{\tau,\kappa}(\lambda)|x_{\tau,\kappa}(\lambda)) \,. \quad (9)$$

The bitwise *a-posteriori* knowledge of Softbit-Source Decoding can be separated into three product terms: a channel dependent term, an *a-priori* term and an *extrinsic* value for bit $x_{\tau,\kappa}(m)$. The *extrinsic* value resulting from Softbit-Source Decoding consists of channel information as well as joint *a-priori* knowledge for the bits $\mathbf{x}_{\tau,\kappa}$ representing parameter $u_{\tau,\kappa}$ excluding bit $x_{\tau,\kappa}(m)$ itself[1]. A comparison of Eq. (9) and Eq. (4) yields that the implicit redundancy utilized by Softbit-Source Decoding might act like the explicit redundancy given in terms of parity bits.

3.2. The *extrinsic* values due to correlation properties

In the previous basic considerations the Markov properties in time $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$ and position $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa-1})$ have been neglected. Thereby, the *a-posteriori* probability of the desired bit $x_{\tau,\kappa}(m)$ (see Eq. (5)) has been reduced to $P(x_{\tau,\kappa}(m)|\mathbf{z}_{\tau,\kappa})$. In order to improve quality, all received values $\mathbf{z}_{1,N}^{\tau} \ldots \mathbf{z}_{1,1}^{\tau}$ have to be taken into account. In this case the optimal solution will be obtained when the marginal distribution of $P(\mathbf{x}_{1,N}^{\tau} \ldots |\mathbf{z}_{1,N}^{\tau} \ldots)$ is determined over all $\mathbf{x}_{1,N}^{\tau} \ldots \mathbf{x}_{1,1}^{\tau}$ given the desired bit $x_{\tau,\kappa}(m)$

$$P(x_{\tau,\kappa}(m)|\mathbf{z}_{1,N}^{\tau} \ldots \mathbf{z}_{1,1}^{\tau}) = \sum_{\forall(\mathbf{x}_{1,N}^{\tau} \cdots \mathbf{x}_{1,1}^{\tau}|x_{\tau,\kappa}(m))} \!\!\cdots\sum\cdots\sum P(\mathbf{x}_{1,N}^{\tau} \ldots \mathbf{x}_{1,1}^{\tau}|\mathbf{z}_{1,N}^{\tau} \ldots \mathbf{z}_{1,1}^{\tau}) \,. \quad (10)$$

Of course, this formal solution is by far too complex. Each bitwise *a-posteriori* probability $P(x_{\tau,\kappa}(m)|\mathbf{z}_{1,N}^{\tau} \ldots)$ is a function of three indizes: the time instant τ, the position within a set of parameters κ, and the bit position m. Therefore, to reduce complexity we consider (with respect to *extrinsic* information) only such values $z_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})$ which fulfill the following conditions on combination for $\tilde{\tau}, \tilde{\kappa}, \tilde{m}$

- $\tilde{m} = m$, $\tilde{\kappa} = \kappa$, and $\tilde{\tau} = \tau - T + 1, \tau - T + 2, \ldots \tau$.
- $\tilde{\tau} = \tau$, $\tilde{m} = m$, and $\tilde{\kappa} = 1, 2, \ldots N$
- $\tilde{\tau} = \tau$, $\tilde{\kappa} = \kappa$, and $\tilde{m} = 1, 2, \ldots w_\kappa$

Figure 2 a.) illustrates these conditions. In the resulting 3-dimensional space every partition represents a single bit. If $x_{\tau,\kappa}(m)$ is the desired bit (marked dark grey), then only such received bits will be considered, which are in one line with $x_{\tau,\kappa}(m)$ in any of the three dimensions (medium grey). Furthermore, in the time dimension the entire history is limited to the latest T time instants. Hence, taking the previous constraints into account and using Bayes' theorem, Eq. (10) can be reduced to

$$P(x_{\tau,\kappa}(m)|\{z_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})\}^{\forall(\tilde{\tau},\tilde{\kappa},\tilde{m})}) = C \cdot \sum_{\forall(\{x_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})\}^{\forall(\tilde{\tau},\tilde{\kappa},\tilde{m})}|x_{\tau,\kappa}(m))} \!\!\sum\sum p(\{z_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})|x_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})\}^{\forall(\tilde{\tau},\tilde{\kappa},\tilde{m})}) \cdot P(\{x_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})\}^{\forall(\tilde{\tau},\tilde{\kappa},\tilde{m})})$$

Applying the chain rule to the last term and assuming independence of the different directions τ, κ and m, allows to factorize the *a-priori* knowledge,

$$P(\{x_{\tilde{\tau},\tilde{\kappa}}(\tilde{m})\}^{\forall(\tilde{\tau},\tilde{\kappa},\tilde{m})}) \approx P(\{x_{\tilde{\tau},\kappa}(m)\}_{\tilde{\tau}\neq\tau}^{\forall\tilde{\tau}}|x_{\tau,\kappa}(m)) \cdot P(\{x_{\tau,\tilde{\kappa}}(m)\}_{\tilde{\kappa}\neq\kappa}^{\forall\tilde{\kappa}}|x_{\tau,\kappa}(m)) \cdot P(\{x_{\tau,\kappa}(\tilde{m})\}^{\forall\tilde{m}}). \quad (11)$$

If in addition a memoryless transmission channel is considered, the determination rule given in Eq. (10) can be re-written according to Eqs. (7)-(9) as

$$\begin{aligned} P(x_{\tau,\kappa}(m)|\mathbf{z}_{1,N}^{\tau} \ldots) &\approx C \cdot p(z_{\tau,\kappa}(m)|x_{\tau,\kappa}(m)) \cdot P(x_{\tau,\kappa}(m)) \cdot \\ &\cdot \sum_{\forall(\{x_{\tilde{\tau},\kappa}(m)\}^{\forall\tilde{\tau}}|x_{\tau,\kappa}(m))} P(\{x_{\tilde{\tau},\kappa}(m)\}_{\tilde{\tau}\neq\tau}^{\forall\tilde{\tau}}|x_{\tau,\kappa}(m)) \prod_{\lambda=\tau-T+1}^{\tau-1} p(z_{\lambda,\kappa}(m)|x_{\lambda,\kappa}(m)) \\ &\cdot \sum_{\forall(\{x_{\tau,\tilde{\kappa}}(m)\}^{\forall\tilde{\kappa}}|x_{\tau,\kappa}(m))} P(\{x_{\tau,\tilde{\kappa}}(m)\}_{\tilde{\kappa}\neq\kappa}^{\forall\tilde{\kappa}}|x_{\tau,\kappa}(m)) \prod_{\substack{\lambda=1\\ \lambda\neq\kappa}}^{N} p(z_{\tau,\lambda}(m)|x_{\tau,\lambda}(m)) \\ &\cdot \sum_{\forall(\mathbf{x}_{\tau,\kappa}|x_{\tau,\kappa}(m))} \frac{P(\mathbf{x}_{\tau,\kappa})}{P(x_{\tau,\kappa}(m))} \prod_{\substack{\lambda=1\\ \lambda\neq m}}^{w_\kappa} p(z_{\tau,\kappa}(\lambda)|x_{\tau,\kappa}(\lambda)) \,. \end{aligned} \quad (12)$$

The *a-posteriori* knowledge for bit $x_{\tau,\kappa}(m)$ is enhanced by three independent *extrinsic* terms. The last line in Eq. (12) is known from Eq. (9) as *extrinsic* value due to the non-uniform distribution. The center two lines implicitly contain the *extrinsic* information due to the correlation properties (see Subsec. 5.1). Both terms combine channel information as well as *a-priori* knowledge for

[1]Note, in order to simplify notation $P(\{x_{\tau,\kappa}(\lambda)\}_{\lambda\neq m}^{\forall\lambda}|x_{\tau,\kappa}(m))$ is re-written as $P(\mathbf{x}_{\tau,\kappa})/P(x_{\tau,\kappa}(m))$ (see Eq. (8)).

the bits which are arranged in one line with $x_{\tau,\kappa}(m)$ either in time τ or in position κ.

3.3. Utilizing the *extrinsic* values iteratively

Due to the independent *extrinsic* values an iterative evaluation of Eq. (12) might be possible. Figure 2 b.) illustrates an example solution of the determination of $x_{\tau,\kappa}(m)$. In the 1*st step* the first 2 lines of Eq. (12) are considered only and *extrinsic* information for $x_{\tau,\kappa}(m)$ is calculated for all $m = 1, \ldots w_\kappa$ with $\kappa = 1, \ldots N$. Afterwards, in the 2*nd step*, the *extrinsic* value given in the 3rd line of Eq. (12) has to be determined, where the *extrinsic* value of the 1*st step* is enclosed as additional *a-priori* knowledge. The dashed arrows represent the enclosed information. In the 3*rd step* the *extrinsic* information due to the non-uniform distribution is calculated, while the results of the 1*st* and 2*nd step* serve as additional *a-priori* knowledge. After the 3*rd step* all received values $\mathbf{z}^{\tau}_{\tau-T+1,N} \cdots \mathbf{z}^{\tau}_{\tau-T+1,1}$ have once been taken into account, either directly (solid arrows) or indirectly (dashed arrows). The first iteration is finished and a second iteration might start where the 1*st step* uses the *extrinsic* information of the 2*nd* and 3*rd step* as additional *a-priori* knowledge.

4. SIMULATION RESULTS

For simulations the source model proposed in [3] is used. This model allows to adjust parameter correlation properties in time τ by the *auto correlation factor* ρ and in position κ by the *cross correlation factor* δ. In our simulations we set $N = 5$. The parameters $u_{\tau,\kappa}$ are quantized by a 32-level Lloyd-Max quantizer using 5 bits, i.e., $R_\kappa = R = 32 \forall \kappa$. The bit patterns $\mathbf{x}_{\tau,\kappa}$ are assigned according to the *natural binary* bit mapping. The time span T used in the decoder is limited to $T = 5$. As transmission channel serves an AWGN channel with known E_b/N_0.

4.1. Improvements due to the iterative approach

Fig. 3 depicts the parameter *signal-to-noise ratio* (SNR) as a function of E_b/N_0 for simulations with $\rho = 0.95$ and $\delta = 0.8$. Such auto and cross correlation factors can be found for scale factors determined in audio transform codecs as, for example, in the MPEG audio codec for *digital audio broadcasting* (DAB).

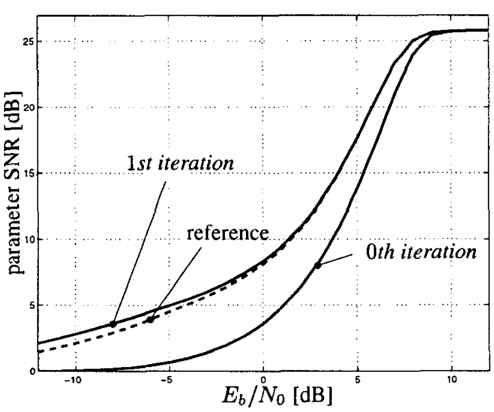


Figure 3: Simulation results

The curve labelled 0*th iteration* shows the obtained simulation result if none of the *extrinsic* terms of Eq. (12) is utilized. A stepwise addition of *extrinsic* information according to Fig. 2 b.) allows to increase the parameter SNR by up to 4.77 dB. Furthermore, in case of low channel qualities the reference system defined by Eq. (12) (see Fig. 2 a.)) can be slightly outperformed. The reason is that due to the iterative solution all partitions of the cube have been considered once, directly or indirectly. On the other hand, the reference exploits only such partitions which are in one direct line with the desired partition.

4.2. Different numbers of iterations

Usually, more than one iteration does not improve quality any further. It is a known fact [5] that in the first iteration the different *extrinsic* information can be considered as statistically independent. But afterwards, the iterations will use the same information indirectly and therefore they will become more and more correlated. This has a major impact on the parameter *a-priori* knowledge.

As Softbit-Source Decoding (SBSD) is a parameter estimation technique, a precise *a-priori* knowledge is very important. In order to separate a bitwise *a-priori* knowledge independently for the three dimensions τ, κ and m, in the derivation of Eq. (12) the overall *a-priori* information for bit $x_{\tau,\kappa}(m)$ has been approximated according to Eq. (11).

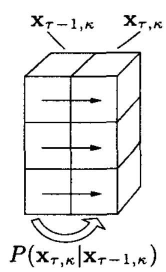
Of course, due to the assumed independence of the different directions some information might be lost. However, usually we cannot benefit from the new dependencies introduced by higher numbers of iterations, especially as these are based on channel related terms. Therefore, sometimes it might happen that quality will slightly be increased by a larger number of iterations, but in most cases the overall *a-priori* knowledge gets more imprecise. Hence, the performance of SBSD by parameter estimation will decrease.

5. COMPARISON WITH CONVENTIONAL SOFTBIT-SOURCE DECODING

Finally, the iterative approach to SBSD shall be compared with the conventional technique. The main difference between both methods is the utilization of the correlation properties. The conventional approach exploits the correlation properties explicitly in terms of parameter transition probabilities for consecutive indizes τ, i.e., $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$, or for adjacent indizes κ, i.e., $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa-1})$. The new iterative approach does it implicitly.

5.1. Impact of the implicit utilization of correlation properties

The implicit utilization of correlation properties might result in significant quality degradations. In order to simplify the analysis of the performance loss we restrict the following basic considerations to bit patterns $\mathbf{x}_{\tau-1,\kappa}$ and $\mathbf{x}_{\tau,\kappa}$ of consecutive parameters.



It is easy to prove that in the introduced iterative approach the parameter transition probability $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$ is approximated by transition probabilities on bit level

$$\tilde{P}(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa}) = \prod_{\lambda=1\ldots m} P(x_{\tau,\kappa}(\lambda)|x_{\tau-1,\kappa}(\lambda)). \tag{13}$$

Note that in the estimation rule given by Eq. (12) the probabilities $P(x_{\tau,\kappa}(\lambda)|x_{\tau-1,\kappa}(\lambda))$ are implied in the probabilities $P(\{x_{\tilde{\tau},\kappa}(m)\}^{\forall\tilde{\tau}}_{\tilde{\tau}\neq\tau}|x_{\tau,\kappa}(m))$. The impact of the approximation is illustrated in Fig. 4, where the settings of the experiment described in Sec. 4 are reused. The curve labelled *original* shows the true parameter transition probability $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$ for the example situation where $\mathbf{x}_{\tau-1,\kappa}$ represents the 15th quantizer reproduction level. If bit patterns are assigned according to the *natural binary* bit mapping the approximation given in Eq. (13) yields the dashed curve. Unnatural hops arise. In particular, in the given example situation the approximation $\tilde{P}(\mathbf{x}_{\tau,\kappa} = 10000|\mathbf{x}_{\tau-1,\kappa} = 01111)$, i.e., $\mathbf{x}_{\tau,\kappa}$ represents $i = 16$ given that $\mathbf{x}_{\tau-1,\kappa}$ represents 15, is untruly very small. The higher the *Hamming* distance between $\mathbf{x}_{\tau,\kappa}$ and $\mathbf{x}_{\tau-1,\kappa}$ is, the more unlikely is $\tilde{P}(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$.

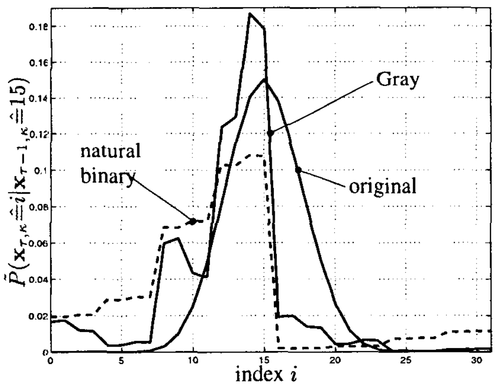


Figure 4: Different approximations of $\tilde{P}(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$

Therefore, an index assignment according to the *Gray* mapping[2] seems to be more suitable, because bit patterns of neighbouring indizes i always possess the *Hamming* distance $d = 1$.

5.2. Simulation result

To demonstrate the performance of both approximations, either with *natural binary* or with *Gray* mapping, they are compared with the optimal parameter estimator proposed in [3]. The optimal solution (*PARAM-OPT*) explicitly exploits *a-priori* knowledge due to a non-uniform distribution, correlation properties in time according to $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa})$ as well as cross-correlation properties according to $P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa-1})$. The settings of Sec. 4 are reused for this simulation.

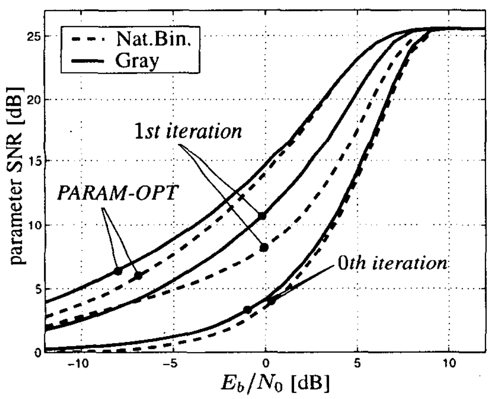


Figure 5: Simulation results

Fig. 5 depicts the simulation results. In general, the iterative solution is outperformed by the optimal parameter estimator because of the imprecise approximation of *a-priori* knowledge given in Eq. (13). Furthermore, in all simulations with *Gray* mapping, transmission errors result in minor quality degradation due to the short *Hamming* distance of neighboured symbols. The gain due to *Gray* mapping is highest for the iterative Softbit-Source Decoder, because, as described in the previous subsection, the mapping can also be exploited to approximate the parameter *a-priori* knowledge more precisely. In contrast to the solution without any *extrinsic* knowledge (*0th iteration*) significant improvements are obtainable with only 1 *iteration*.

[2] $i\epsilon[0,1\ldots31]$ is mapped into $\tilde{i}\epsilon[20,21,23,22,18,19,17,16,0,1,3,2,6,7,5,4,12,13,15,14,10,11,9,8,24,25,27,26,30,31,29,28]$

5.3. Rough estimates of the computational complexity

In addition to the simulation results the optimal solution [3] and the iterative approach to Softbit-Source Decoding shall be compared with respect to their computational efforts. In the following some rough estimates will be performed. In order to simplify the considerations we assume that the number of parameters within a set N is equal to the time span T as well as the number of bits $w = w_\kappa$ per parameter with $\kappa = 1\ldots N$, i.e., $N = T = w$.

The optimal solution [3] exploits Markov properties in time $(P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau-1,\kappa}))$ and position $(P(\mathbf{x}_{\tau,\kappa}|\mathbf{x}_{\tau,\kappa-1}))$. Hence, each $\mathbf{x}_{\tau,\kappa}$ has 2^w possible predecessors $\mathbf{x}_{\tau-1,\kappa}$ and 2^w possible neighbours $\mathbf{x}_{\tau,\kappa-1}$ and therefore the determination of 2^w possible $\mathbf{x}_{\tau,\kappa}$ at a fixed time instant τ and position κ requires at least arithmetical operations of the order $O(2^{3w})$. Data memory of the same order is necessary to store the parameter *a-priori* knowledge.

The computational efforts of the iterative approach can be estimated corresponding to Fig. 2 b.). In the *1st step* operations of the order $O(Nw2^T)$ are necessary, in the *2nd step* operations of the order $O(w2^N)$ and in the *3rd step* operations of the order $O(2^w)$. In total this results in arithmetical operations as well as data memory of the order $O(w^2 2^w)$ to determine 2^w possible $\mathbf{x}_{\tau,\kappa}$.

Hence, the iterative approach allows significant complexity reductions if $w > 2$, i.e. $O(w^2 2^w) \ll O(2^{3w})$. Note that a re-usage of intermediate results can lower complexity demands further.

6. CONCLUSION

In this paper, we applied the TURBO-principle to Softbit-Source Decoding. As a novelty, we introduced an approach which utilizes different terms of residual redundancy iteratively. The derived formula shows how, in particular, to exploit correlation properties. Their implicit utilization allows to reduce complexity demands significantly. Simultaneously, quality decreases due to a loss in parameter *a-priori* information. A suitable approximation has been found in terms of a *Gray* mapping. Furthermore, we explained why more than 1 iteration does usually not increase performance.

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