ON JOINT SOURCE-CHANNEL DECODING FOR CORRELATED SOURCES

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ABSTRACT

Whenever digital communications is subject to constraints in delay and complexity, some residual redundancy remains after source coding. Shannon's separation theorem does not hold strictly and hence joint source-channel decoding is beneficial. In this paper we discuss two competing approaches exploiting residual redundancy in terms of (auto)-correlation. Both will be analyzed with respect to their impact on quantization noise and channel distortion. The first approach exploits redundancy at the receiver for *minimum mean squared error* estimation of source codec parameters. The second approach exploits correlation at the transmitter by predictive encoding. The prediction gain allows to reduce quantization noise or bitrate. The spare bitrate can be used for specific error protection and hence reduction of channel distortion. But otherwise, less redundancy is utilizable at the receiver for parameter estimation. Simulations will show that the predictive encoding scheme improves quality in a wide range of channel conditions, but in contrast to Shannon's separation theorem the utilization of correlation at the receiver side is beneficial in very bad channel conditions.

1. INTRODUCTION

In digital communications modern source encoders for analog signals such as speech, audio or video usually reveal two characteristic properties. Firstly, the codec parameters are highly sensitive to transmission errors and secondly, due to delay and complexity constraints they may exhibit residual redundancy in terms of non-uniform distributions and (auto)-correlation. Shannon has already indicated that *"any redundancy in the source will usually help if it is utilized at the receiving point."*

Recently, different techniques have been proposed how to exploit residual redundancy for *joint source-channel decoding*, e.g. [1–9]. Further on, an approach similar to the primary technique given in [1, 2] deals as reference where residual redundancy has been considered as a result of (implicit) channel encoding. Thus, correlation in source codec parameters is utilizable in order to minimize error rates in sequence or symbol detections. Further quality improvements are obtainable by *minimum mean squared error* parameter estimation within the decoding process [2, 6, 7].

Furthermore, the estimation technique can benefit from additional redundancy due to an explicit channel code [5, 6, 8, 9]. In particular, non-linear codes which are especially designed in view of the source statistics [5, 6, 8] increase robustness significantly.

In this paper we analyze, if in case of correlated sources a separate optimization of source and channel coding according to Shannon is beneficial in all channel conditions. A predictive encoding scheme is considered which allows to reduce either quantization noise or channel distortion.

The paper is structured as follows. First, the primary approach to joint source-channel coding [1, 2] utilizing (auto)-correlation in source codec parameters at the receiver side only is reviewed in Section 2. In Section 3, the alternative approach using predictive encoding will be described. Both approaches will be analyzed with respect to the parameter signal-to-noise ratio after decoding.

2. JOINT SOURCE-CHANNEL CODING

According to [1, 2] the block diagram of the primary approach to *joint source-channel decoding* is depicted in Figure 1.

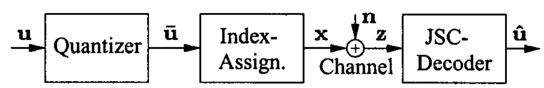

Figure 1: Transmission chain (JSC: Joint Source-Channel)

At time instant τ a signal source encoder provides a set of N parameters u_κ with $\kappa = 1, 2, \dots N$. The parameters u_κ are discrete in time and continuous in magnitude and a whole set of parameters is denoted by $\mathbf{u} = (u_1, \dots u_N)^T$. For example, the set $\mathbf{u}$ might be given by consecutive samples of a speech signal or by a frame of speech parameters determined by modern speech encoders. In order to allow digital communication the vector $\mathbf{u}$ has to be quantized to $\bar{\mathbf{u}} = (\bar{u}_1, \dots \bar{u}_N)^T \in \mathbb{U}$. Thereby, $\mathbb{U}$ represents the complete set of quantizer reproduction levels.

Afterwards, a unique bit expression $\mathbf{x} = (x_1, \dots x_K) \in \mathbb{X}$ is assigned to the quantized set $\bar{\mathbf{u}}$ whereby a bipolar representation is assumed, i.e., $x_i \in \{-1, 1\}$ and $\mathbb{X} = \{-1, 1\}^K$. K denotes the number of bits needed to describe all possible 2^K transmission symbols $\mathbf{x} \in \mathbb{X}$. In general, in order to guarantee a unique index assignment $\bar{\mathbf{u}} \rightarrow \mathbf{x}$ the amount of possible bit expressions $|\mathbb{X}| = 2^K$ must be equal to or greater than the amount of used quantizer reproduction levels $|\mathbb{U}|$, i.e., $|\mathbb{X}| \geq |\mathbb{U}|$. In case of $|\mathbb{X}| > |\mathbb{U}|$, the index assignment might increase the redundancy in the codewords $\mathbf{x}$. The additional redundancy can be exploited at the receiver side to improve the overall quality [5, 8, 9]. Note, that thereby the number of possible quantizer reproduction levels $|\mathbb{U}|$ needs not to be a power of two [4, 5].

The quantizer in Figure 1 performs source coding and the index assignment block might carry out some channel encoding [5, 6, 8].

The codeword $\mathbf{x}$ is sent over a binary symmetric channel (BSC) with additive noise $\mathbf{n} = (n_1, \dots n_K) \in \mathbb{R}^K$ and a possibly disturbed real-valued set $\mathbf{z} = \mathbf{x} + \mathbf{n}, \mathbf{z} \in \mathbb{R}^K$, is received.

The task of the *joint source-channel decoder* is to exploit the expression $\mathbf{z}$ as well as any possibly given *a-priori* information in order to determine an optimal estimate $\hat{\mathbf{u}}$ for $\mathbf{u}$. Further on, we optimize the parameter *signal-to-noise ratio* (SNR), i.e., the term

$$\mathrm{SNR} = \frac{\mathrm{E}[||\mathbf{u}||^2]}{\mathrm{E}[||\mathbf{u} - \hat{\mathbf{u}}(\mathbf{z})||^2]} \tag{1}$$

shall be maximized. Thereby, $\mathrm{E}[\cdot]$ denotes the expected value and $||\cdot||$ the Euclidian norm. Furthermore, to indicate that the estimation of $\hat{\mathbf{u}}$ is mainly based on the received codeword $\mathbf{z}$ the notation

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$\hat{\mathbf{u}}(\mathbf{z})$ has been introduced. The numerator of Eq. (1) can be considered as fixed in case of a stationary process $\mathbf{u}$. Therefore, the maximization of Eq. (1) corresponds to the minimization of the denominator $\mathrm{E}\left[\|\mathbf{u}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$.

If a quantizer is used which is optimal in the minimum mean squared error (MMSE) sense [10, 11], the term $\mathrm{E}\left[\|\mathbf{u}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$ can be separated into two individual contributions [12, 13]. The first one results from quantization and the second one is caused by channel noise. As the considerations in Section 3 are based on this matter of fact the derivation shall be briefly reviewed next.

2.1. Separation of Quantization Noise and Channel Distortion

The expected value of the overall distortion can be re-written as

$$\mathrm{E}\left[\|\mathbf{u}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right] = \mathrm{E}\left[\|\mathbf{u}\|^2\right] - 2\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\right] + \mathrm{E}\left[\|\hat{\mathbf{u}}(\mathbf{z})\|^2\right], \tag{2}$$

where $\mathrm{E}\left[\|\mathbf{u}\|^2\right]$ specifies the signal power of the unquantized set of parameters $\mathbf{u}$ and $\mathrm{E}\left[\|\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$ the power of the estimates $\hat{\mathbf{u}}(\mathbf{z})$ respectively. The center term $\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\right]$ of Eq. (2) can be evaluated by

$$\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\right] = \sum_{\bar{\mathbf{u}}} \int_{\mathbf{u}} \int_{\mathbf{z}} \hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\, p(\mathbf{u},\bar{\mathbf{u}},\mathbf{z})\, d\mathbf{u}\, d\mathbf{z} \quad , \tag{3}$$

i.e., by determining the integrals over the value continuous components $\mathbf{u} \in \mathbb{R}^N$ and $\mathbf{z} \in \mathbb{R}^K$ as well as by summation over the discrete component $\bar{\mathbf{u}} \in \mathbb{U}$. Applying the chain rule of probability to the probability density function (pdf) $p(\mathbf{u},\bar{\mathbf{u}},\mathbf{z})$ and taking into account that the unquantized set $\mathbf{u}$ is independent from the received expression $\mathbf{z}$ when the quantized set $\bar{\mathbf{u}}$ is known,

$$p(\mathbf{u},\bar{\mathbf{u}},\mathbf{z}) = p(\mathbf{u}\,|\,\bar{\mathbf{u}},\mathbf{z})\, p(\bar{\mathbf{u}},\mathbf{z}) = p(\mathbf{u}\,|\,\bar{\mathbf{u}})\, p(\bar{\mathbf{u}},\mathbf{z}) \quad , \tag{4}$$

allows to re-arrange Eq. (3) as

$$\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\right] = \sum_{\bar{\mathbf{u}}} \int_{\mathbf{z}} \hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\, p(\bar{\mathbf{u}},\mathbf{z}) d\mathbf{z} \int_{\mathbf{u}} \mathbf{u}\, p(\mathbf{u}\,|\,\bar{\mathbf{u}}) d\mathbf{u}. \tag{5}$$

Furthermore, the right integral can be substituted by the quantized parameter set $\bar{\mathbf{u}}$, when a quantizer is used which is optimal in the MMSE sense [10, 11]. In this case the quantizer reproduction level represents the centroid of the quantization cell,

$$\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\mathbf{u}\right] = \sum_{\bar{\mathbf{u}}} \int_{\mathbf{z}} \hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\bar{\mathbf{u}}\, p(\bar{\mathbf{u}},\mathbf{z}) d\mathbf{z} = \mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\bar{\mathbf{u}}\right]. \tag{6}$$

Inserting the relation of Eq. (6) into Eq. (2) and adding and subtracting the term $\mathrm{E}\left[\|\bar{\mathbf{u}}\|^2\right]$ results in

$$\begin{aligned}\mathrm{E}\left[\|\mathbf{u}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right] &= \mathrm{E}\left[\|\mathbf{u}\|^2\right] - 2\mathrm{E}\left[\hat{\mathbf{u}}(\mathbf{z})^{\mathrm{T}}\bar{\mathbf{u}}\right] + \mathrm{E}\left[\|\hat{\mathbf{u}}(\mathbf{z})\|^2\right] \\ &= \mathrm{E}\left[\|\mathbf{u}\|^2\right] - \mathrm{E}\left[\|\bar{\mathbf{u}}\|^2\right] + \mathrm{E}\left[\|\bar{\mathbf{u}}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]\end{aligned} \tag{7}$$

Furthermore, the optimal quantizer [10, 11] allows to combine the first two terms in Eq. (7), i.e.,

$$\mathrm{E}\left[\|\mathbf{u}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right] = \mathrm{E}\left[\|\mathbf{u}-\bar{\mathbf{u}}\|^2\right] + \mathrm{E}\left[\|\bar{\mathbf{u}}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right] \tag{8}$$

Hence, the overall distortion can be separated into two independent contributions, one from quantization $\mathrm{E}\left[\|\mathbf{u}-\bar{\mathbf{u}}\|^2\right]$ and one from channel noise $\mathrm{E}\left[\|\bar{\mathbf{u}}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$. Due to the assumed optimality of the quantizer, $\mathrm{E}\left[\|\mathbf{u}-\bar{\mathbf{u}}\|^2\right]$ already achieves the absolute minimum value. Therefore, in order to minimize the overall distortion, in addition an optimal receiver is required which minimizes $\mathrm{E}\left[\|\bar{\mathbf{u}}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$.

2.2. Optimal Receiver – Determination of an MMSE Estimate

From estimation theory it is well known [14], that $\mathrm{E}\left[\|\bar{\mathbf{u}}-\hat{\mathbf{u}}(\mathbf{z})\|^2\right]$ will be minimized (MMSE) when the estimate $\hat{\mathbf{u}}(\mathbf{z})$ is the conditional mean of $\bar{\mathbf{u}}$ given the observation $\mathbf{z}$, i.e,

$$\hat{\mathbf{u}}(\mathbf{z}) = \mathrm{E}[\bar{\mathbf{u}}\,|\,\mathbf{z}] = \sum_{\bar{\mathbf{u}}} \bar{\mathbf{u}}\, P(\bar{\mathbf{u}}\,|\,\mathbf{z}) \quad . \tag{9}$$

Hence, the estimate $\hat{\mathbf{u}}(\mathbf{z})$ is given by a weighted sum over all quantizer reproduction levels $\bar{\mathbf{u}}$ where the *a-posteriori* probabilities $P(\bar{\mathbf{u}}\,|\,\mathbf{z})$ represent the weights. Due to the unambiguous index assignment, $\bar{\mathbf{u}} \rightarrow \mathbf{x}$, the $P(\bar{\mathbf{u}}\,|\,\mathbf{z})$ are equal to $P(\mathbf{x}\,|\,\mathbf{z})$. The problem of estimating $\hat{\mathbf{u}}(\mathbf{z})$ can be reduced to the problem of determining the *a-posteriori* probabilities $P(\mathbf{x}\,|\,\mathbf{z})$. Thereby, the $P(\mathbf{x}\,|\,\mathbf{z})$ comprise channel related knowledge as well as information about residual redundancy in the source codec parameters. A common approach how to utilize residual redundancy in terms of a non-uniform distribution as well as in terms of correlation can be found, e.g., in [1, 2, 6, 7]. Further on, this approach will be referred to as primary technique. A brief introduction will be given next.

3. UTILIZATION OF RESIDUAL REDUNDANCY OF CORRELATED SOURCE CODEC PARAMETERS

The (auto)-correlation property exhibited by source codec parameters can be utilized in different ways. In the primary solution, for example, the residual redundancy is exclusively exploited at the receiver side of a transmission system by MMSE estimation of source codec parameters. In an alternative approach correlation will be eliminated by predictive encoding at the transmitter and re-inserted at the receiver.

3.1. Primary Approach to Joint Source-Channel Decoding

The primary solution has been introduced, e.g., in [1, 2, 6, 7]. According to Figure 1 the joint source-channel decoder determines an estimate $\hat{\mathbf{u}}(\mathbf{z})$ for $\bar{\mathbf{u}}$ given the observation $\mathbf{z}$. As indicated in Eq. (9) the estimation is based on the determination of *a-posteriori* probabilities $P(\bar{\mathbf{u}}\,|\,\mathbf{z})$, or $P(\mathbf{x}\,|\,\mathbf{z})$ respectively. Due to the dependencies of consecutive source codec parameters, further on the pattern $\mathbf{z}$ shall not be considered as received combination for the currently sent bit expression $\mathbf{x}$ only, but it also may include some parts of the history or the entire past. For simplification the time instant will be denoted by an additional index τ, and the entire history by $\mathbf{z}_1^\tau = \mathbf{z}_\tau, \mathbf{z}_{\tau-1} \ldots \mathbf{z}_1$.

The application of Bayes' Theorem allows to determine the *a-posteriori* probabilities $\forall\, \mathbf{x}_\tau \in \mathbb{X}$ recursively,

$$P(\mathbf{x}_\tau\,|\,\mathbf{z}_1^\tau) = C \cdot p(\mathbf{z}_\tau\,|\,\mathbf{x}_\tau) \sum_{\mathbf{x}_{\tau-1}} P(\mathbf{x}_\tau\,|\,\mathbf{x}_{\tau-1})\, P(\mathbf{x}_{\tau-1}\,|\,\mathbf{z}_1^{\tau-1}), \tag{10}$$

when a memoryless transmission channel as well as a first order Markov property $P(\mathbf{x}_\tau\,|\,\mathbf{x}_{\tau-1})$ in the source codec parameters is assumed [2, 6, 7]. Thereby, C specifies a constant term which guarantees that the sum over all *a-posteriori* probabilities $P(\mathbf{x}_\tau\,|\,\mathbf{z}_1^\tau)$ takes on the value one. The pdf $p(\mathbf{z}_\tau\,|\,\mathbf{x}_\tau)$ represents the channel related reliability information and $P(\mathbf{x}_\tau\,|\,\mathbf{x}_{\tau-1})$ the statistical properties of the source. Further on, the approach given by Eqs. (9) and (10) will be referred to as "AK1-Estimator" (1st order *a-priori* knowledge) and the estimate is denoted by $\hat{\mathbf{u}}_{\mathbf{AK1}}$. Note, if the source codec parameters are uncorrelated and if a non-uniform distribution is utilizable only ("AK0-Estimator", 0th order *a-priori* knowledge, $\hat{\mathbf{u}}_{\mathbf{AK0}}$), Eq. (10) simplifies to

$$P(\mathbf{x}_\tau\,|\,\mathbf{z}_\tau) = C \cdot p(\mathbf{z}_\tau\,|\,\mathbf{x}_\tau) P(\mathbf{x}_\tau) \quad . \tag{11}$$

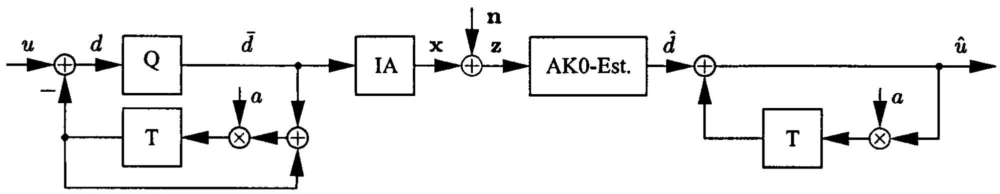

Figure 2: Predictive encoding scheme for joint source-channel decoding
(Q: quantizer, IA: index assignment, T: 1 sample time delay, AK0-Est.: MMSE-Estimator utilizing 0th order *a-priori* knowledge)

3.2. Predictive Encoding Scheme For JSC-Decoding

In the alternative approach to joint source-channel (JSC) coding the correlation in consecutive source codec parameters shall be reduced or even eliminated by predictive encoding. To simplify notation throughout this section we restrict the following basic considerations to a single component u_κ of the parameter set $\mathbf{u}$. Furthermore, without loss in generality the index of position κ can be neglected, i.e., $u_\kappa \to u$. The block diagram for the predictive encoding scheme is depicted in Figure 2.

At first, the correlation in u is reduced or even eliminated by encoding with a 1st order linear predictor. The residual signal is quantized to $\bar{d} \in \mathbb{D}$ within the loop. If the source codec parameter u can be modelled as a 1st order Gauss-Markov process with zero mean and variance σ_u^2, then the prediction residual d is Gaussian distributed as well, but with the reduced variance

$$\sigma_d^2 = (1-a^2)\sigma_u^2 \quad . \tag{12}$$

Thereby, the term $(1-a^2)^{-1}$ represent the prediction gain. If a minimal prediction error variance σ_d^2 is desired, the filter coefficient a must correspond to the correlation factor ρ of the Gauss-Markov source. The prediction gain can be exploited either for quantization with higher resolution, i.e., for reductions of quantization noise $\mathrm{E}\big[|d-\bar{d}|^2\big]$, or for bitrate reduction. The spare bitrate can be used for stronger error protection, e.g., in terms of a more robust index assignment ($|\mathbb{X}| \geq |\mathbb{U}| \geq |\mathbb{D}|$), i.e., for reductions of channel noise $\mathrm{E}\Big[|\bar{d}-\hat{d}(\mathbf{z})|^2\Big]$.

At the receiver side of the transmission system depicted in Figure 2 an AK0-Estimator determines an estimate $\hat{d}_{AK0}$ for $\bar{d}$ according to Eqs. (9) and (11). Thereby, the non-uniform distribution of $\bar{d}$ as well as the observation $\mathbf{z}$ are taken into account. Finally, the (auto)-correlation is re-inserted by a 1st order recursive filter.

3.3. Robustness of the Predictive Encoding Scheme

In order to determine the robustness of the joint source-channel decoder using a predictive encoding scheme the overall distortion $\mathrm{E}\big[|u-\hat{u}|^2\big]$ has to be analyzed. Corresponding to Figure 2 the unquantized parameter value u_τ at time instant τ as well as the estimate $\hat{u}_\tau(\mathbf{z})$ can be expressed as

$$u_\tau = d_\tau + \sum_{i=1}^{\tau} a^i\,\bar{d}_{\tau-i} \quad \text{and} \quad \hat{u}_\tau = \sum_{i=0}^{\tau} a^i\,\hat{d}_{\tau-i} \quad . \tag{13}$$

Inserting these relations into $\mathrm{E}\big[|\hat{u}_\tau - u_\tau|^2\big]$ and re-writing the expected value according to Eq. (2) results in

$$\begin{aligned}
\mathrm{E}\big[|\hat{u}_\tau - u_\tau|^2\big] &= \mathrm{E}\left[\left|\sum_{i=0}^{\tau} a^i\,\hat{d}_{\tau-i} - d_\tau - \sum_{i=1}^{\tau} a^i\,\bar{d}_{\tau-i}\right|^2\right] \\
&= \mathrm{E}\Big[|\hat{d}_\tau - d_\tau|^2\Big] + \mathrm{E}\left[\left|\sum_{i=1}^{\tau} a^i\,\Big[\bar{d}_{\tau-i} - \hat{d}_{\tau-i}\Big]\right|^2\right] \\
&\quad + 2\mathrm{E}\left[\Big(\hat{d}_\tau - d_\tau\Big)\sum_{i=1}^{\tau} a^i\,\Big[\bar{d}_{\tau-i} - \hat{d}_{\tau-i}\Big]\right]
\end{aligned} \tag{14}$$

The first term in the second line can be immediately separated into quantization noise and channel distortion according to Eq. (8),

$$\mathrm{E}\Big[|\hat{d}_\tau - d_\tau|^2\Big] = \mathrm{E}\big[|\bar{d}_\tau - d_\tau|^2\big] + \mathrm{E}\Big[|\hat{d}_\tau - \bar{d}_\tau|^2\Big] \quad , \tag{15}$$

Thereby, an optimal quantizer for the prediction residual d_τ has been considered [10, 11]. In addition, this quantizer allows to determine the last term in Eq. (14) according to Eqs. (3) to (6) by

$$\begin{aligned}
&\mathrm{E}\left[\Big(\hat{d}_\tau - d_\tau\Big)\sum_{i=1}^{\tau} a^i\,\Big[\bar{d}_{\tau-i} - \hat{d}_{\tau-i}\Big]\right] = \\
&\qquad \mathrm{E}\left[\Big(\hat{d}_\tau - \bar{d}_\tau\Big)\sum_{i=1}^{\tau} a^i\,\Big[\bar{d}_{\tau-i} - \hat{d}_{\tau-i}\Big]\right] \quad .
\end{aligned} \tag{16}$$

Hence, it is easy to prove that inserting Eqs. (15) and (16) into Eq. (14) yields

$$\mathrm{E}\big[|\hat{u}_\tau - u_\tau|^2\big] = \mathrm{E}\Big[|\bar{d}_\tau - d_\tau|^2\Big] + \mathrm{E}\left[\left|\sum_{i=0}^{\tau} a^i\,\Big[\bar{d}_{\tau-i} - \hat{d}_{\tau-i}\Big]\right|^2\right] . \tag{17}$$

If the term $\bar{d}_{\tau-i} - \hat{d}_{\tau-i}$ can be considered as white and stationary process with zero mean, then Eq. (17) simplifies to

$$\begin{aligned}
\mathrm{E}\big[|\hat{u}_\tau - u_\tau|^2\big] &= \mathrm{E}\Big[|\bar{d}_\tau - d_\tau|^2\Big] + \sum_{i=0}^{\tau} a^{2i}\,\mathrm{E}\Big[|\bar{d}_{\tau-i} - \hat{d}_{\tau-i}|^2\Big] \\
&= \mathrm{E}\Big[|\bar{d}_\tau - d_\tau|^2\Big] + \frac{1}{1-a^2}\,\mathrm{E}\Big[|\bar{d}_\tau - \hat{d}_\tau|^2\Big] .
\end{aligned} \tag{18}$$

That is, the overall distortion in source codec parameter values $\hat{u}_\tau$ can be expressed as a linear combination of quantization noise and channel distortion in terms of prediction residuals $\bar{d}_\tau$.

3.4. Comparison of the Different Approaches

The robustness of both systems can be evaluated and compared by means of the previous relations. In order to increase comparability, the overall distortion of the predictive encoding scheme given in Eq. (18) can be re-written with respect to Eq. (12) by

$$\mathrm{E}\big[|u - \hat{u}_{AK0}|^2\big] = (1-a^2)\,\mathrm{E}\big[|u - \bar{u}|^2\big] + \mathrm{E}\big[|\bar{u} - \hat{u}_{AK0}|^2\big] . \tag{19}$$

Predictive encoding allows to reduce the quantization noise by the prediction gain. However, less residual redundancy is utilizable to improve transmission quality under noisy channel conditions as a non-uniform distribution can be exploited only. The trade-off between quantization noise and channel distortion might be adjusted by a sub-optimal filter coefficient $a \neq \rho$ used for predictive encoding. In contrast, the overall distortion of the primary system is

$$\mathrm{E}\big[|u - \hat{u}_{AK1}|^2\big] = \mathrm{E}\big[|u - \bar{u}|^2\big] + \mathrm{E}\big[|\bar{u} - \hat{u}_{AK1}|^2\big] . \tag{20}$$

In comparison to Eq. (19) the quantization noise might be higher, but channel distortion might be less as correlation is utilizable.

Of course, the impact of the different residual redundancy on the distortion due to channel noise $\mathrm{E}\big[|\bar{u} - \hat{u}|^2\big]$ remains to be evaluated.

4. SIMULATION RESULTS

For simulation the source codec parameter u_τ is modelled by a 1st order Gauss-Markov process. Therefore, white Gaussian noise with zero mean and variance $\sigma_u^2 = 1$ is processed by a 1st order recursive filter. The filter coefficient ρ allows to adjust (auto)-correlation, e.g., $\rho = 0.8$. In case of the primary approach to joint source-channel decoding the source codec parameters u_τ are quantized by an 32-level Lloyd-Max quantizer. In case of the predictive encoding scheme the encoding with $a = \rho$ eliminates (auto)-correlation. The residual signal d_τ is quantized by an 32-level Lloyd-Max quantizer as well, whereas the quantization intervalls are adapted to the reduced variance $\sigma_d^2 = 0.46$. For both approaches a *source optimized channel code* (SOCC) [5, 8] using $K = 6$ bits serves as index assignment. Thereby, the redundancy in codewords $\mathbf{x}_\tau$ is increased ($|\mathbb{X}| = 64 > |\mathbb{U}| = 32$). SOCCs are non-linear channel codes which exploit source and channel statistics. In addition, in the design of SOCCs a parameter based quality measure is considered, e.g., like the parameter SNR given in Eq. (1). The transmission channel is modelled by *additive white Gaussian noise* (AWGN) with known E_x/N_0. E_x denotes the energy per codeword $\mathbf{x}_\tau$ and $N_0/2$ the spectral density of the AWGN. At the receiver side of the transmission system parameter estimation is performed using 1st order *a-priori* knowledge for the primary approach and 0th order *a-priori* knowledge for the predictive encoding scheme.

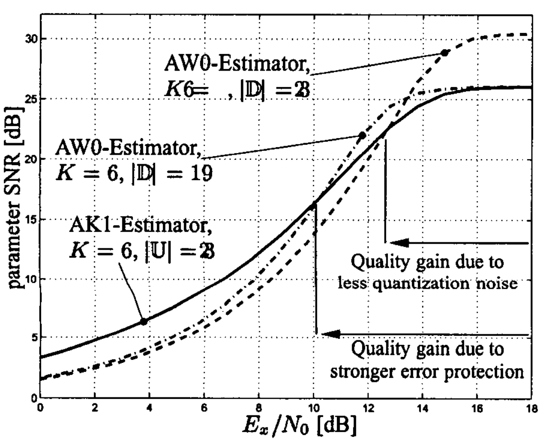

Figure 3: Simulation results with auto-correlation of $\rho = 0.8$

The considerations of the previous sections can be verified by simulation (see Figure 3). The predictive encoding scheme (dashed curve) is able to benefit from the prediction gain for $E_x/N_0 > 12.7$ dB. The prediction gain allows quality improvements with respect to quantization noise by up to $\Delta = -10 \log_{10}(1 - a^2) = 4.4$ dB. If the prediction gain is used for bitrate reductions the spare bitrate can be exploited for specific error protection. It is easy to prove that $|\mathbb{D}| = 19$ quantizer reproduction levels are sufficient to achieve a similar basic performance as the primary approach. Applying a re-designed SOCC [5, 8] by using the optimal $|\mathbb{D}| = 19$ out of $|\mathbb{X}| = 64$ possible codewords $\mathbf{x}_\tau$ provides the dashed-dotted curve in Figure 3. Now the predictive encoding scheme is superior to the primary approach in a wider range of channel conditions, i.e., down to $E_x/N_0 = 10.2$ dB. In contrast to Shannon's separation theorem the primary approach (solid curve) utilizing residual redundancy at the receiver side still outperforms the predictive encoding scheme for $E_x/N_0 < 10.2$ dB.

In addition to the better performance of the predictive encoding scheme to joint source-channel decoding in the most interesting range of channel conditions, it offers higher adaptivity and usually requires less complexity. For example, the optimal receiver of the primary approach needs 1st order *a-priori* knowledge $P(\mathbf{x}_\tau \mid \mathbf{x}_{\tau-1})$ which has to be measured once in advance and which has to be stored in tables of size $|\mathbb{X}| * |\mathbb{X}|$ for any fixed but arbitrary ρ. In contrast, the predictive encoding scheme demands a table of size $|\mathbb{X}|$ to store the 0th order *a-priori* knowledge $P(\mathbf{x}_\tau)$ and allows to easily adapt the prediction coefficient a. With respect to these savings in the optimal receiver, the additional complexity needed to perform predictive encoding can be considered as neglectable small.

5. CONCLUSION

In this contribution the utilization of residual redundancy in correlated source codec parameters by joint source-channel decoding has been discussed. Thereby, two competing approaches were analyzed with respect to their impact on a given quality measure. The signal-to-noise ratio depends on quantization noise as well as channel distortion. In this paper formulas for both approaches have been derived and thus enable a comparison. It was shown by simulation that the elimination of correlation by predictive encoding allows to increase the robustness in the most interesting range of channel conditions. In contrast to Shannon's separation theorem the primary approach is best in case of bad channel conditions.

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