ANALYSIS OF EXTRINSIC INFORMATION FROM SOFTBIT-SOURCE DECODING APPLICABLE TO ITERATIVE SOURCE-CHANNEL DECODING

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ABSTRACT

This contribution deals with Softbit-Source Decoding applied to an iterative source-channel decoding approach. Iterative decoding approaches have been introduced to increase the robustness of digital mobile communication systems.

Softbit-Source Decoding can be considered as an error concealment technique. In order to cope with transmission errors residual redundancy of source codec parameters is utilized. The impact of the source statistics on single data bits can be quantified in terms of so-called *extrinsic* information.

As a novelty, we will carefully analyze the *extrinsic* information of Softbit-Source Decoding. We will show, that the *extrinsic* information due to the source statistics is much less sensitive to an update in the decoder's soft-input as usually known from iterative channel decoding techniques. This might explain why in iterative decoding processes the performance improvements due to Softbit-Source Decoding are already gained with a small number of iterations. In most applications of iterative source-channel decoding no significant gain will be obtainable by more than two iterations.

1. INTRODUCTION

Iterative processes are known from the TURBO-principle, which was originally introduced as a decoding technique for concatenated channel codes [1, 2]. So-called *extrinsic* information can be extracted from the soft-output of the one decoder and serve as additional *a-priori* knowledge for the other decoder. While decoding the independent components iteratively, an update of the *extrinsic* information after each iteration allows to successively increase quality, e.g., in terms of decreasing bit error rates. In contrast to conventional decoding techniques, the TURBO-principle permits to approach Shannon's performance bounds [3] with reasonable computational complexity.

Shannon also indicated [3] that residual redundancy in parameters determined by source encoders can be utilized at the receiver side of a possibly noisy transmission system to enhance the output signal. In order to exploit a non-uniform distribution or the correlation properties of source codec parameters, we recently have proposed *Softbit-Source Decoding* by parameter estimation [4–9].

The performance of Softbit-Source Decoding can be further improved if channel coding algorithms add explicit redundancy at the transmitter side. At the receiver the additional information can be exploited in different ways, e.g., either by *joint* source-channel decoding [10–13] or by *iterative* source-channel decoding [14–19].

According to the TURBO-principle, all approaches to iterative source-channel decoding are based on the exchange of *extrinsic* information between a channel decoder and a Softbit-Source Decoder. But, first simulations have shown [14–18] that the performance improvements seem to be limited to mainly two iterations.

In this paper we will focus on the exchange of *extrinsic* information. In particular, we will carefully analyze the formula which we have introduced recently [18, 20] and which quantifies the *extrinsic* value of Softbit-Source Decoding (SBSD). On the one hand, we will be able to show that the *extrinsic* information due to the source statistics is always less sensitive to an update in the decoders input as, e.g., the famous BOXPLUS-operation [2] introduced to iterative channel decoding techniques. On the other hand, we will be able to show that the *extrinsic* information is limited to low values even under noiseless channel conditions.

The paper is structured as follows. First, we briefly review a transmission system with iterative source-channel decoding. Furthermore, the *extrinsic* information of both, channel decoding and Softbit-Source Decoding will be specified [2, 18, 20]. Afterwards, the limitations in performance improvements are demonstrated by simulation in Section 3. Finally, the analysis of Softbit-Source Decoding's *extrinsic* information is performed in Section 4.

2. ITERATIVE SOURCE-CHANNEL DECODER

A possible application for iterative source-channel decoding [18] is depicted in Fig. 1. The transmission chain consists of encoding and decoding blocks for both, the source codec as well as the channel codec.

At time instant τ, a source encoder determines a set of N uncorrelated parameters u_κ, with $\kappa = 1, 2, \ldots, N$ denoting the position within the set. Each value u_κ, which is continuous in magnitude but discrete in time, is individually quantized by R_κ reproduction levels $\bar{u}_\kappa^{(i)}$ with $i = 1, 2, \ldots, R_\kappa$. The reproduction

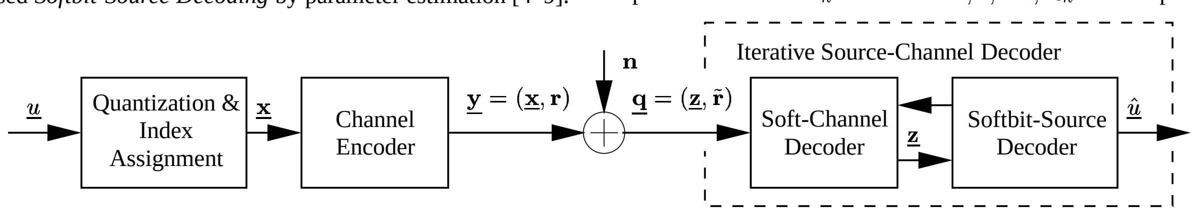

Figure 1: Transmission chain with iterative source-channel decoder

levels are invariant with respect to τ and the whole set is given by $\mathbb{U}_\kappa = \{\bar{u}_\kappa^{(1)}, \bar{u}_\kappa^{(2)}, \ldots, \bar{u}_\kappa^{(R_\kappa)}\}$. To each $\bar{u}_\kappa^{(i)}$ a unique bit pattern $\mathbf{x}_\kappa = \{x_\kappa(\lambda)\}^{\forall\lambda}$ with $\lambda \,\epsilon\, \{1, 2 \ldots w_\kappa\}$ is assigned where $x_\kappa(\lambda) \,\varepsilon\, \{+1, -1\}$ represents a single bit. The length w_κ of $\mathbf{x}_\kappa$ is usually[1] given by $w_\kappa = \log_2(R_\kappa)$. Corresponding to a set of quantizer reproduction levels $\mathbb{U}_\kappa$, the complete set of possible bit patterns is given by $\mathbb{X}_\kappa$. The bit pattern assigned to u_κ, i.e., $\bar{u}_\kappa^{(i)}$, is denoted by $\mathbf{x}_\kappa$, i.e., $\bar{u}_\kappa^{(i)} \to \mathbf{x}_\kappa$. Furthermore, sets of N parameters are denoted by $\underline{u} = (u_1, u_2 \ldots u_N)$ or $\underline{\mathbf{x}} = (\mathbf{x}_1, \mathbf{x}_2 \ldots \mathbf{x}_N)$, respectively.

In order to simplify considerations throughout this paper, the channel encoded set of bit patterns $\underline{\mathbf{y}}$ is created by a systematic code, i.e., the information bits $\underline{\mathbf{x}}$ are present in the corresponding codeword $\underline{\mathbf{y}} = (\underline{\mathbf{x}}, \mathbf{r})$. Thereby, $\mathbf{r}$ represents the vector of parity bits $r(\lambda)$.

After transmission of the channel encoded set $\underline{\mathbf{y}}$ over a channel with additive noise $\mathbf{n}$, a possibly disturbed set of real valued patterns $\underline{\mathbf{q}} = \underline{\mathbf{y}} + \underline{\mathbf{n}}$ is received with $\underline{\mathbf{q}} = (\underline{\mathbf{z}}, \tilde{\mathbf{r}})$. $\underline{\mathbf{z}}$ and $\tilde{\mathbf{r}}$ denote the received sets for the information bits $\underline{\mathbf{x}}$ and parity bits $\mathbf{r}$, respectively.

The task of the iterative source-channel decoder is to determine the *optimal* approximation $\underline{\hat{u}}$ for $\underline{u}$. Usually, the *minimum mean square error* (MMSE) is used as optimality criterion individually for each parameter $E\{(u_\kappa - \hat{u}_\kappa)^2\}$. $E\{\cdot\}$ denotes the expected value.

With regard to the introduced notations, the well known MMSE estimation rule can be written as

$$\hat{u}_\kappa = \sum_{i=1}^{R_\kappa} \bar{u}_\kappa^{(i)} \cdot P(\bar{u}_\kappa^{(i)}|\underline{\mathbf{q}}) \quad . \tag{1}$$

$P(\bar{u}_\kappa^{(i)}|\underline{\mathbf{q}})$ is the *a-posteriori* probability for reproduction level $\bar{u}_\kappa^{(i)}$ when the complete set of received bit patterns $\underline{\mathbf{q}} = (\underline{\mathbf{z}}, \tilde{\mathbf{r}})$ is given. Due to the fixed index assignment $\bar{u}_\kappa^{(i)} \to \mathbf{x}_\kappa$, $P(\bar{u}_\kappa^{(i)}|\underline{\mathbf{q}})$ is equal to the conditional probability $P(\mathbf{x}_\kappa|\underline{\mathbf{q}})$. Hence, the problem of estimating the optimal parameter value $\hat{u}_\kappa$ can be reduced to the problem of determining the conditional probabilities $P(\mathbf{x}_\kappa|\underline{\mathbf{q}})$, $P(\mathbf{x}_\kappa|\underline{\mathbf{z}}, \tilde{\mathbf{r}})$ respectively.

In most applications for iterative source-channel decoding [14–19] such a problem is usually solved on bit level by approximating the parameter *a-posteriori* probabilities $P(\mathbf{x}_\kappa|\underline{\mathbf{q}})$ by *a-posteriori* probabilities for single bits $P(x_\kappa(m)|\underline{\mathbf{q}})$, i.e.,

$$P(\mathbf{x}_\kappa|\underline{\mathbf{q}}) \approx \prod_{m=1}^{w_\kappa} P(x_\kappa(m)|\underline{\mathbf{q}}) \quad . \tag{2}$$

If a memoryless transmission channel as well as independence of source and channel decoding are assumed, the bitwise probabilities $P(x_\kappa(m)|\underline{\mathbf{q}})$, $P(x_\kappa(m)|\underline{\mathbf{z}}, \tilde{\mathbf{r}})$ respectively, can be determined by

$$\begin{aligned} P(x_\kappa(m)|\underline{\mathbf{z}}, \tilde{\mathbf{r}}) = C \cdot p(z_\kappa(m)|x_\kappa(m)) \cdot P(x_\kappa(m)) \\ \cdot\, p_{ext}^{CD}(x_\kappa(m)) \cdot p_{ext}^{SBSD}(x_\kappa(m)) \quad . \end{aligned} \tag{3}$$

C denotes a constant term which ensures that the sum over conditional probabilities $\sum_{x_\kappa(m)=+1,-1} P(x_\kappa(m)|\underline{\mathbf{z}}, \tilde{\mathbf{r}})$ equals one. The probability density function (pdf) $p(z_\kappa(m)|x_\kappa(m))$ is a channel-dependent term and $P(x_\kappa(m))$ represents bitwise *a-priori* knowledge resulting from the distribution of bit $x_\kappa(m)$. In the second line of Eq. (3) there are two additional knowledges for the desired bit $x_\kappa(m)$, the so-called *extrinsic* information. The first term $p_{ext}^{CD}(x_\kappa(m))$ can be provided by a soft-channel decoder and the second term $p_{ext}^{SBSD}(x_\kappa(m))$ by Softbit-Source Decoding. Both terms are required to enable an iterative process. The prefix "soft" indicates that both decoders accept soft-inputs, e.g., in terms of channel transition pdfs $p(z_\kappa(m)|x_\kappa(m))$ and *a-priori* values $P(x_\kappa(m))$, and provide soft-outputs, e.g., in terms of *a-posteriori* probabilities $P(x_\kappa(m)|\underline{\mathbf{z}}, \tilde{\mathbf{r}})$. The soft-channel decoder enhances the bitwise *a-posteriori* probabilities $P(x_\kappa(m)|\underline{\mathbf{q}})$ by exploiting explicit redundancy in terms of parity bits and the Softbit-Source Decoder by utilizing the implicit redundancy, e.g., in terms of a non-uniform parameter distribution $P(\mathbf{x}_\kappa)$.

[1] If R_κ is not a power of 2, w_κ might be given by the integral number $w_\kappa = \lceil \log_2(R_\kappa) \rceil$. In this case *Source Optimized Channel Codes* [9, 21] can benefit from the redundancy in the bit mapping, $\bar{u}_\kappa^{(i)} \to \mathbf{x}_\kappa$.

The additional knowledges $p_{ext}^{CD}(x_\kappa(m))$ and $p_{ext}^{SBSD}(x_\kappa(m))$ shall be specified next.

2.1. The *Extrinsic* Information of Soft-Channel Decoding

The *extrinsic* information is the key parameter in an iterative decoding process. In special cases it can be extracted from the decoder's soft-output as the following simple example for binary data should clarify.

Example: If the bit pattern $\mathbf{x}_\kappa$ is channel encoded with an even single parity check code, the parity bit r_κ is generated by $r_\kappa = \{x_\kappa(1) \oplus x_\kappa(2) \oplus \ldots x_\kappa(w_\kappa)\}$. $\oplus$ denotes the binary *exclusive-or* operation. From this follows immediately that each information bit $x_\kappa(m)$ can also be determined from the parity bit r_κ and all the other information bits of $\mathbf{x}_\kappa$

$$\begin{aligned} x_\kappa(m) = \{x_\kappa(1) \oplus \ldots x_\kappa(m-1) \oplus \\ x_\kappa(m+1) \oplus \ldots x_\kappa(w_\kappa) \oplus r_\kappa\} \quad . \end{aligned} \tag{4}$$

Thereby, information for bit $x_\kappa(m)$ is available twice, directly and indirectly according to Eq. (4). At the receiver side of a (possibly noisy) transmission system this redundancy allows to enhance the *a-posteriori* probabilities for bits $x_\kappa(m)$, which are provided by the soft-channel decoder. These probabilities are not only conditioned on $z_\kappa(m)$, but also on all the other bits of $\mathbf{z}_\kappa$ as well as the received value $\tilde{r}_\kappa$ for bit r_κ. Neglecting bit correlation, the *a-posteriori* probability is given by $P(x_\kappa(m)|\mathbf{z}_\kappa, \tilde{r}_\kappa)$, which can easily be re-written as

$$\begin{aligned} P(x_\kappa(m)|\mathbf{z}_\kappa, \tilde{r}_\kappa) = C \cdot p(z_\kappa(m)|x_\kappa(m)) \cdot P(x_\kappa(m)) \\ \cdot\, p(\{z_\kappa(\lambda)\}_{\lambda \neq m}^{\forall\lambda}, \tilde{r}_\kappa|x_\kappa(m)) \end{aligned} \tag{5}$$

when the chain rule for probability is applied. Furthermore, a memoryless transmission channel is assumed. Again, C denotes a constant term which ensures that the sum of *a-posteriori* probabilities $P(x_\kappa(m)|\mathbf{z}_\kappa, \tilde{r}_\kappa)$ equals one. In addition to the channel related term $p(z_\kappa(m)|x_\kappa(m))$ and the *a-priori* term $P(x_\kappa(m))$, there exists another term for bit $x_\kappa(m)$. A comparison with Eq. (3) reveals that the *extrinsic* knowledge $p_{ext}^{CD}(x_\kappa(m))$ due to soft-channel decoding is given by $p(\{z_\kappa(\lambda)\}_{\lambda \neq m}^{\forall\lambda}, \tilde{r}_\kappa|x_\kappa(m))$. This term results from the special diversity according to Eq. (4).

Note, to simplify notation throughout this example, channel encoding had been performed over a single source codec parameter u_κ, i.e., a single bit pattern $\mathbf{x}_\kappa$ respectively. But, in order to enable iterative source-channel decoding, independence of the source and channel decoders has to be ensured. Independence means, that *extrinsic* information for the desired bit $x_\kappa(m)$ is gained from different received values $z_\kappa(\lambda)$, $\tilde{\mathbf{r}}$ with $\kappa = 1, \ldots N$, $\lambda = 1, \ldots w_\kappa$. Therefore, in an application for iterative source-channel decoding the parity check code will be determined over the index of position κ, i.e., across different bit patterns $\mathbf{x}_\kappa$.

2.2. The *Extrinsic* Information of Softbit-Source Decoding

In Subsec. 2.1, redundancy for single bits $x_\kappa(m)$ is introduced explicitly, i.e., in terms of parity bits. In contrast to this, implicit redundancy might be available due to the statistical properties of the source codec parameters $\bar{u}_\kappa^{(i)}$, and $\mathbf{x}_\kappa$ respectively. A non-uniform distribution or correlation can be exploited to enhance the robustness of the overall transmission system depicted in Fig. 1 as well.

Recently, Softbit-Source Decoding (SBSD) has been proposed to utilize different terms of residual redundancy in the source codec parameters [4–9]. Channel information for single bits $p(z_\kappa(m)|x_\kappa(m))$ is combined with parameter *a-priori* knowledge to estimate parameter *a-posteriori* probabilities. In order to simplify the following basic considerations, correlation properties between consecutive or adjacent parameters are neglected. Only redundancy due to a non-uniform distribution $P(\mathbf{x}_\kappa)$ of source codec parameter u_κ will be taken into account. If, for example, the non-uniform distribution $P(\mathbf{x}_\kappa)$ shall be utilized, the parameter *a-posteriori* probabilities $P(\mathbf{x}_\kappa|\mathbf{z}_\kappa)$ can be determined using Bayes' Theorem (mixed form) $\forall\, \mathbf{x}_\kappa \,\epsilon\, \mathbb{X}_\kappa$

$$P(\mathbf{x}_\kappa|\mathbf{z}_\kappa) = C \cdot \prod_{m=1}^{w_\kappa} p(z_\kappa(m)|x_\kappa(m)) \cdot P(\mathbf{x}_\kappa) \quad . \qquad (6)$$

In order to quantify the reliability gain due to SBSD for single bits $x_\kappa(m)$ with $m = 1, 2 \ldots w_\kappa$, the bitwise *a-posteriori* probabilities $P(x_\kappa(m)|\mathbf{z}_\kappa)$ can easily be obtained by summation when the marginal distribution[2] of parameter *a-posteriori* probabilities $P(\mathbf{x}_\kappa|\mathbf{z}_\kappa)$ is determined over all $\mathbf{x}_\kappa \,\epsilon\, \mathbb{X}_\kappa$ with a given $x_\kappa(m)$.

Hence, in [18, 20] we have shown that under these constraints Eq. (6) can be transformed into

$$P(x_\kappa(m)|\mathbf{z}_\kappa) = C \cdot p(z_\kappa(m)|x_\kappa(m)) \cdot P(x_\kappa(m))$$
$$\cdot \sum_{\forall(\mathbf{x}_\kappa|x_\kappa(m))} P(\{x_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}|x_\kappa(m)) \prod_{\substack{\lambda=1\\ \lambda\neq m}}^{w_\kappa} p(z_\kappa(\lambda)|x_\kappa(\lambda)) \qquad (7)$$

$\forall(\mathbf{x}_\kappa|x_\kappa(m))$ means: summation over all $\mathbf{x}_\kappa$ with fixed but arbitrary $x_\kappa(m)$

when a memoryless channel is assumed. Again, in addition to the channel related term $p(z_\kappa(m)|x_\kappa(m))$ and the *a-priori* term $P(x_\kappa(m))$, there exists an *extrinsic* information for bit $x_\kappa(m)$. A comparison with Eq. (3) yields that

$$p_{ext}^{SBSD}(x_\kappa(m)) = \sum_{\forall(\mathbf{x}_\kappa|x_\kappa(m))} \frac{P(\mathbf{x}_\kappa)}{P(x_\kappa(m))} \prod_{\substack{\lambda=1\\ \lambda\neq m}}^{w_\kappa} p(z_\kappa(\lambda)|x_\kappa(\lambda)) \quad . \qquad (8)$$

The *extrinsic* information resulting from Softbit-Source Decoding consists of channel information as well as joint *a-priori* knowledge for the bits representing parameter $\mathbf{x}_\kappa$ excluding the desired bit $x_\kappa(m)$ itself[3]. A comparison of Eq. (5) and Eq. (7) indicates that the implicit redundancy utilized by Softbit-Source Decoding might act like the explicit redundancy given in terms of parity bits.

3. AN EXAMPLE APPLICATION TO ITERATIVE SOURCE-CHANNEL DECODING

Before the sensitivity of the different *extrinsic* information given by Eq. (5) and Eq. (8) will be analyzed, an example application shall point out the behavior of both if applied to iterative source-channel decoding. In order to enhance the visibility in the simulation results, the Softbit-Source Decoder will consider residual redundancy in terms of time correlation as well. As shown in [18], the explanations given in Subsec. 2.2 can be extended to this case as well.

[2]If Softbit-Source Decoding is the final step in an iterative decoding process, the marginal distribution and the approximation given in Eq. (2) can cancel out. Thereby, the overall performance might increase.

[3]Note: $P(\{x_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}|x_\kappa(m)) = P(\mathbf{x}_\kappa)/P(x_\kappa(m))$

Therefore, for simulation each parameter u_κ is modelled individually by a Gauss-Markov process of the order one. For this, white Gaussian noise is processed by a first-order recursive filter and afterwards normalized to $\sigma_{u_\kappa}^2 = 1$ [8]. The filter coefficient allows to adjust auto-correlation properties, e.g., $\rho = 0.8$. Different parameters $u_\kappa, u_{\hat{\kappa}}$ $(\kappa \neq \hat{\kappa})$ within one set $\underline{u}$ are statistically independent. The parameters u_κ are individually quantized by a 16-level Lloyd-Max quantizer using $w_\kappa = 4$ bits. Furthermore, a set of parameters $\underline{u}$ consists of $N = 4$ entries u_κ with $\kappa = 1, \ldots N$. *Folded binary* is applied for index assignment as discussed in [18].

Channel encoding is performed by a simple single parity check code over the index of position κ. For each bit index m, $m = 1, \ldots w_\kappa$ an even parity bit $r(m)$ is generated by $r(m) = \{x_1(m) \oplus x_2(m) \oplus \ldots x_N(m)\}$. As transmission channel serves an AWGN channel with known E_b/N_0.

At the receiver side decoding will be performed iteratively in two independent steps. In each iteration the *extrinsic* information of the one decoder will be used as additional *a-priori* knowledge by the other one. For the first iteration all *extrinsic* values of Softbit-Source Decoding are initialized with $p_{ext}^{SBSD}(x_\kappa(m)) = 0.5$. The first step soft-channel decoding is performed according to [2] and the second step Softbit-Source Decoding is done as discussed in [18]. The updated *extrinsic* value of Softbit-Source Decoding is fed back for different numbers of iterations.

After the last iteration the bitwise *a-posteriori* probabilities according to Eq. (3) are transformed into parameter *a-posteriori* probabilities $P(\mathbf{x}_\kappa|\underline{\mathbf{q}})$ (see Eq. (2)) and finally, $P(\mathbf{x}_\kappa|\underline{\mathbf{q}})$ is inserted in the MMSE estimation rule given by Eq. (1).

Fig. 2 depicts the parameter *signal-to-noise ratio (SNR)* for simulations with different numbers of iterations.

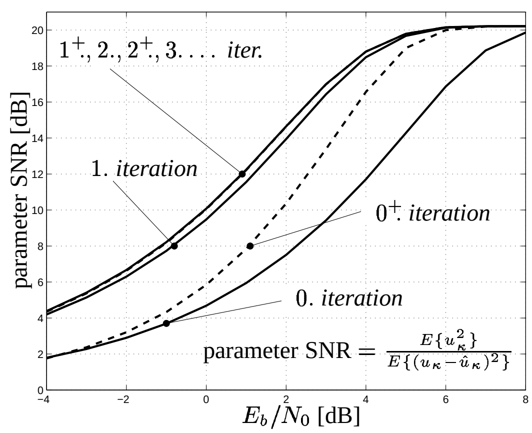

Figure 2: Simulation results for different numbers of iterations if Softbit-Source Decoding is concatenated with a single parity check code

The curve labelled "0. *iteration*" neglects channel decoding and Softbit-Source Decoding. The desired parameters $\hat{u}_\kappa$ are directly estimated by exploiting the corresponding systematic part $\mathbf{z}_\kappa$ of the received bit pattern $\mathbf{q}_\kappa$. If in the first step soft-channel decoding is carried out (see "0^+ *iteration*" in Fig. 2), the *extrinsic* value $p_{ext}^{CD}(x_\kappa(m))$ is utilizable for each bit $x_\kappa(m)$. This results in a gain of several dB for the parameter SNR for moderate E_b/N_0. Next, the "1. *iteration*" will be completed, when in addition the *extrinsic* value of Softbit-Source Decoding $p_{ext}^{SBSD}(x_\kappa(m))$ is taken into account. Due to the correlation of $\rho = 0.8$, the parameter SNR can further be improved by up to 3.7 dB. When the next iteration is started and the updated $p_{ext}^{SBSD}(x_\kappa(m))$ is used as additional *a-priori* knowledge, the channel decoder can benefit from the new information (*"1$^+$. iteration"*). A slight increase in quality of about 0.69 dB is noticable. But afterwards, when the *"2. iteration"* is completed the Softbit-Source Decoder seems not to be able to take advantage of the updated *extrinsic* value of the soft-channel decoder. There is no considerable gain. More than 2 iterations do not increase the parameter SNR any further. Similar results for different parameter settings ($w_\kappa = 3$, $N = 3$, $\rho = 0.95$) are obtained in [18].

4. ANALYSIS OF EXTRINSIC INFORMATION

Improving the Softbit-Source Decoder's soft-input in terms of additional *a-priori* knowledge, i.e., an update of the soft-channel decoder's *extrinsic* value $p_{ext}^{CD}(x_\kappa(m))$, seems not to enable the Softbit-Source Decoder to enhance the overall quality any further.

Hence, in the present section the sensitivity of SBSD's soft-output with respect to variations in the soft-input values shall be analyzed. In order to permit a fair comparison with the properties of a soft-channel decoder, common constraints will be defined first. Therefore, the sensitivity of a soft-channel decoder shall be briefly reviewed, too.

Furthermore, for the following considerations the so-called log-likelihood algebra for single bits will be used. In addition, a bipolar transmission of data bits $x_\kappa(m)$ is assumed.

If the binary random variable takes on the value $x_\kappa(m)\,\epsilon\,\{+1,-1\}$, the log-likelihood ratio [2] is defined as

$$L(x_\kappa(m)) = \log\frac{P(x_\kappa(m)=+1)}{P(x_\kappa(m)=-1)} \tag{9}$$

with log representing the natural logarithm. The sign of $L(x_\kappa(m))$ yields the hard decision and the magnitude $|L(x_\kappa(m))|$ represents the reliability of this decision.

4.1. Sensitivity of Soft-Channel Decoding

In Subsec. 2.1 the *extrinsic* information $p_{ext}^{CD}(x_\kappa(m))$ was specified which can be determined by soft-channel decoding. Hence, using the log-likelihood algebra for the *extrinsic* term, results in

$$L_{ext}^{CD}(x_\kappa(m)) = \log\frac{p(\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{r}_\kappa | x_\kappa(m)=+1)}{p(\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{r}_\kappa | x_\kappa(m)=-1)}. \tag{10}$$

Furthermore, as the received bits $\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{r}_\kappa$ represent a diversity transmission for the desired bit $x_\kappa(m)$, applying the binary *exclusive-or* operation on $\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}$ and $\tilde{r}_\kappa$ according to Eq. (4) provides the additional knowledge.

In log-likelihood algebra this corresponds to [2]

$$L_{ext}^{CD}(x_\kappa(m)) = \frac{1+\prod_{\forall\hat{q}}\tanh(L(\hat{q})/2)}{1-\prod_{\forall\hat{q}}\tanh(L(\hat{q})/2)} \quad , \tag{11}$$

when independence of the received bits $\hat{q}\,\epsilon\,\{\,\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{r}_\kappa\}$ is assumed. Thereby, in order to simplify notation $\hat{q}$ denotes one of the received bits $\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}$ or $\tilde{r}_\kappa$. $L(\hat{q})$ specifies the soft-input value for $\hat{q}$ according to Eq. (9). Using the definition for $\tanh$, Eq. (11) can be transformed into [2]

$$L_{ext}^{CD}(x_\kappa(m)) = 2\cdot\operatorname{artanh}\left(\prod_{\forall\hat{q}}\tanh(L(\hat{q})/2)\right) \quad . \tag{12}$$

Hence, if $|L(\hat{q})/2|$ with $\hat{q}\,\epsilon\,\{\,\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}, \tilde{r}_\kappa\}$ of different magnitude are assumed, the received bits $\hat{q}$ exhibit different reliabilities. Furthermore, if all except one $|L(\hat{q})/2|$ can be considered as such high, that $\tanh$ can be considered as saturated, then all except one $\tanh(L(\hat{q})/2)$ in the product of Eq. (12) can be approximated by their sign, i.e. $\tanh(L(\hat{q})/2) \approx \operatorname{sign}\{L(\hat{q})\}$. In this case a product of signs can be factorized, while the $|L(\hat{q})/2|$ with the smallest magnitude remains. However, for this term the functions $\tanh$ and artanh cancel out, which results in the famous BOXPLUS operation ($\boxplus$) for even parity check codes defined in [2],

$$L_{ext}^{CD}(x_\kappa(m)) \approx \prod_{\forall\hat{q}}\operatorname{sign}\{L(\hat{q})\}\cdot\min_{\forall\hat{q}}\{|L(\hat{q})|\} \tag{13}$$

It is obvious, that under the given assumptions the *extrinsic* information grows proportionally to the soft-input term. If the minimum input value $|L(\hat{q})|$ of the soft-channel decoder increases, the *extrinsic* information for the desired bit $x_\kappa(m)$ increases as well. The absolut value for the gradient is always one and the range of possible $L_{ext}^{CD}(x_\kappa(m))$ is $\mathbb{R}$.

4.2. Sensitivity of Softbit-Source Decoding

In Subsec. 2.2 the *extrinsic* information $p_{ext}^{SBSD}(x_\kappa(m))$ for Softbit-Source Decoding was determined. It has been shown that the *extrinsic* value consists of channel information $p(z_\kappa(\lambda)|x_\kappa(\lambda))$ as well as joint *a-priori* probabilities for the bits representing parameter $\mathbf{x}_\kappa$ excluding the desired bit $x_\kappa(m)$ itself.

Hence, if the log-likelihood algebra is applied to Eq. (8), the term $L_{ext}^{SBSD}(x_\kappa(m))$ is given by [18, 20]

$$L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{\displaystyle\sum_{\forall(\mathbf{x}_\kappa|x_\kappa(m)=+1)}\frac{P(\mathbf{x}_\kappa)}{P(x_\kappa(m)=+1)}\prod_{\substack{\lambda=1\\\lambda\neq m}}^{w_\kappa}p(z_\kappa(\lambda)|x_\kappa(\lambda))}{\displaystyle\sum_{\forall(\mathbf{x}_\kappa|x_\kappa(m)=-1)}\frac{P(\mathbf{x}_\kappa)}{P(x_\kappa(m)=-1)}\prod_{\substack{\lambda=1\\\lambda\neq m}}^{w_\kappa}p(z_\kappa(\lambda)|x_\kappa(\lambda))} \tag{14}$$

In order to analyze the sensitivity of Softbit-Source Decoding with respect to variations in the soft-input values, some simplifications of $L_{ext}^{SBSD}(x_\kappa(m))$ are necessary. Therefore, according to the derivation of the BOXPLUS operation it is assumed, that only one of the received bits $\{z_\kappa(\lambda)\}_{\lambda\neq m}^{\forall\lambda}$ has to be considered as unreliable. All the other bits can be considered as error free. The bit index of the unreliable received value might be λ with $\lambda\neq m$. Thereby, the influence of $z_\kappa(\lambda)$ on the desired bit $x_\kappa(m)$ will be maximized.

As a consequence of these assumptions, only the four possible bit combinations of $x_\kappa(m)$ and $x_\kappa(\lambda)$ have to be taken into account. To simplify notation further on we define

$$P(\mathbf{x}_\kappa^{(\eta|\mu)}) = P(\,\{x_\kappa(\zeta)\}_{\zeta\neq m,\lambda}^{\forall\zeta}, x_\kappa(\lambda)=\eta | x_\kappa(m)=\mu) \tag{15}$$

with $\eta,\mu\epsilon\{+1,-1\}$, $\zeta\,\epsilon\,\{1,2\ldots w_\kappa\}$. Hence, as only $z_\kappa(\lambda)$ is noisy just two elements of each sum given in Eq. (14) are unequal to zero,

$$L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{\displaystyle\sum_{\eta\epsilon\{+1,-1\}}P(\mathbf{x}_\kappa^{(\eta|+1)})\cdot p(z_\kappa(\lambda)|x_\kappa(\lambda)=\eta)}{\displaystyle\sum_{\eta\epsilon\{+1,-1\}}P(\mathbf{x}_\kappa^{(\eta|-1)})\cdot p(z_\kappa(\lambda)|x_\kappa(\lambda)=\eta)}. \tag{16}$$

If additive white Gaussian noise is assumed on the transmission channel, the channel dependent term $p(z_\kappa(\lambda)|x_\kappa(\lambda)=\eta)$ for bit $x_\kappa(\lambda)$ can be substituted by

$$\begin{aligned} p(z_\kappa(\lambda)|x_\kappa(\lambda)=\eta) &= \frac{1}{\sqrt{2\pi\sigma_n^2}}\cdot e^{-\frac{(z_\kappa(\lambda)-\eta)^2}{2\sigma_n^2}} \\ &= \frac{1}{\sqrt{2\pi\sigma_n^2}}\cdot e^{-\frac{z_\kappa^2(\lambda)+\eta^2}{2\sigma_n^2}}\cdot e^{\eta\frac{z_\kappa(\lambda)}{\sigma_n^2}} \\ &= \alpha(\lambda)\cdot e^{\eta\frac{z_\kappa(\lambda)}{\sigma_n^2}} \quad . \end{aligned} \tag{17}$$

σ_n^2 is the variance of the additive white Gaussian noise with spectral density $N_0/2$ and zero mean. The bipolar transmission of data bits $x_\kappa(m)\,\epsilon\,\{+1,-1\}$ represents a transmission with constant envelope. Hence, the energy per bit amounts $E_b = 1$.

If the definition [2] for the reliabiliy value of the channel L_c is considered

$$L_c = 4\frac{E_b}{N_0} \tag{18}$$

the term $1/\sigma_n^2$ in Eq. (17) can be re-written as

$$\frac{1}{\sigma_n^2} = 2\frac{E_b}{N_0} = \frac{L_c}{2} \quad . \tag{19}$$

Thereby, Eq. (17) results in

$$p(z_\kappa(\lambda)|x_\kappa(\lambda)=\eta) = \alpha(\lambda)\cdot e^{\eta\frac{L_c\cdot z_\kappa(\lambda)}{2}} \quad . \tag{20}$$

Finally, inserting Eq. (20) in the channel dependent term of Eq. (16) allows to find a relation between the soft-input value of Softbit-Source Decoding $L_c\cdot z_\kappa(\lambda)$ and the *extrinsic* information for $x_\kappa(m)$,

$$L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{\sum\limits_{\eta\epsilon\{+1,-1\}} P(\mathbf{x}_\kappa^{(\eta|+1)})\cdot e^{\eta\frac{L_c\cdot z_\kappa(\lambda)}{2}}}{\sum\limits_{\eta\epsilon\{+1,-1\}} P(\mathbf{x}_\kappa^{(\eta|-1)})\cdot e^{\eta\frac{L_c\cdot z_\kappa(\lambda)}{2}}}. \tag{21}$$

Thereby, $\alpha(\lambda)$ cancels out. Furthermore, if applied in an iterative process the soft-input term $L_c\cdot z_\kappa(\lambda)$ might be enhanced by *extrinsic* information of another soft-output decoder.

4.2.1. Bounds for $L_{ext}^{SBSD}(x_\kappa(m))$

It is obvious, that the maximum *extrinsic* information achievable with Softbit-Source Decoding is limited by the distribution of the source codec parameters $P(\mathbf{x}_\kappa)$. In case of a noisefree transmission channel the channel related term $L_c\cdot z_\kappa(\lambda)$ given in Eq. (21) approaches plus or minus infinity, depending on the transmitted bit $x_\kappa(\lambda)$. When $\eta=+1$ is sent, one bound is given by

$$\lim_{L_c\cdot z_\kappa(\lambda)\to+\infty} L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{P(\mathbf{x}_\kappa^{(+1|+1)})}{P(\mathbf{x}_\kappa^{(+1|-1)})} \quad , \tag{22}$$

and otherwise, when $\eta=-1$ is sent, another bound is

$$\lim_{L_c\cdot z_\kappa(\lambda)\to-\infty} L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{P(\mathbf{x}_\kappa^{(-1|+1)})}{P(\mathbf{x}_\kappa^{(-1|-1)})} \quad . \tag{23}$$

Indeed taking transmission errors into account might be the much more interesting case. Hence, next let us assume a heavily disturbed transmission channel. In this case $L_c\cdot z_\kappa(\lambda)$ approaches zero and the *extrinsic* information takes on the value

$$\lim_{L_c\cdot z_\kappa(\lambda)\to 0} L_{ext}^{SBSD}(x_\kappa(m)) = \log\frac{\sum\limits_{\eta\epsilon\{+1,-1\}} P(\mathbf{x}_\kappa^{(\eta|+1)})}{\sum\limits_{\eta\epsilon\{+1,-1\}} P(\mathbf{x}_\kappa^{(\eta|-1)})}. \tag{24}$$

Usually, the values, which are achievable under noiseless or heavily disturbed channel conditions, are not symmetric with respect to $L_c\cdot z_\kappa(\lambda)=0$. Therefore, even under worst error conditions *extrinsic* information is available.

4.2.2. Monotonic behavior of $L_{ext}^{SBSD}(x_\kappa(m))$

Of course, the behavior and sensitivity of $L_{ext}^{SBSD}(x_\kappa(m))$ in between the extrem cases of Subsect. 4.2.1 have to be discussed as well. Both properties of the *extrinsic* information enable conclusions for the capability of Softbit-Source Decoding in iterative processes.

The sensivity of $L_{ext}^{SBSD}(x_\kappa(m))$ with respect to the soft-input term $L_c\cdot z_\kappa(\lambda)$ is given by the gradient. Therefore, the partial derivative of Eq. (21) for $L_c\cdot z_\kappa(\lambda)$ has to be determined. It is easy to prove that the partial derivative is given by Eq. (25).

It is obvious, that only the denominator is a function of $L_c\cdot z_\kappa(\lambda)$ and that it is always positive valued. Furthermore, both terms of the numerator are part of the denominator and therefore the denominator is still greater than the numerator. Hence, it can be stated that the bitwise *extrinsic* information of Softbit-Source Decoding is a monotonic function in $L_c\cdot z_\kappa(\lambda)$ and that the absolut value of the gradient is always less than 1.

4.2.3. Highest sensitivity of $L_{ext}^{SBSD}(x_\kappa(m))$

In order to find the soft-input value $L_c\cdot z_\kappa(\lambda)$ where the highest sensitivity of $L_{ext}^{SBSD}(x_\kappa(m))$ can be obtained, the maximum absolute value of the gradient has to be identified. Determining the maximum value of Eq. (25) is equivalent to the minimization of the denominator of Eq. (25). It is easy to prove that the denominator is minimized when

$$L_c\cdot z_\kappa(\lambda) = \frac{1}{2}\log\frac{P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(-1|+1)})}{P(\mathbf{x}_\kappa^{(+1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)})} \quad . \tag{26}$$

If this value is inserted in Eq. (25), the maximum absolute value of the gradient, i.e., the highest sensitivity of $L_{ext}^{SBSD}(x_\kappa(m))$, will be achieved

$$\left.\frac{\partial L_{ext}^{SBSD}(x_\kappa(m))}{\partial(L_c\cdot z_\kappa(\lambda))}\right|_{\max} = \frac{\sqrt{P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)})} - \sqrt{P(\mathbf{x}_\kappa^{(-1|+1)})P(\mathbf{x}_\kappa^{(+1|-1)})}}{\sqrt{P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)})} + \sqrt{P(\mathbf{x}_\kappa^{(-1|+1)})P(\mathbf{x}_\kappa^{(+1|-1)})}}. \tag{27}$$

4.3. Comparison of the sensitivities

Finally, the sensitivities of soft-channel decoding and the Softbit-Source Decoding shall be compared by simulation. In order to determine parameter *a-priori* knowledge in terms of probabilities of occurrence $P(\mathbf{x}_\kappa)$, each parameter u_κ is modelled by a white Gaussian source with variance $\sigma_{u_\kappa}^2 = 1$. The parameters are individually quantized by a 16-level Lloyd-Max quantizer using $w_\kappa = 4$ bit. For index assignment serves *Natural Binary*.

According to the assumption given in the previous subsections, the transmission of the bits $x_\kappa(3)$ and $x_\kappa(4)$ (least significant bits) is considered to be reliable. Without loss of generality, both bits are set to $x_\kappa(3) = -1$ and $x_\kappa(4) = -1$. Hence, the impact of the noisy bit $x_\kappa(\lambda=2)$ on the desired bit $x_\kappa(m=1)$ (most

Partial derivative of $L_{ext}^{SBSD}(x_\kappa(m))$ for $L_c\cdot z_\kappa(\lambda)$:

$$\frac{\partial L_{ext}^{SBSD}(x_\kappa(m))}{\partial(L_c\cdot z_\kappa(\lambda))} = \frac{P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)}) - P(\mathbf{x}_\kappa^{(-1|+1)})P(\mathbf{x}_\kappa^{(+1|-1)})}{(P(\mathbf{x}_\kappa^{(+1|-1)})e^{+\frac{L_c z_\kappa(\lambda)}{2}} + P(\mathbf{x}_\kappa^{(-1|-1)})e^{-\frac{L_c z_\kappa(\lambda)}{2}})(P(\mathbf{x}_\kappa^{(+1|+1)})e^{+\frac{L_c z_\kappa(\lambda)}{2}} + P(\mathbf{x}_\kappa^{(-1|+1)})e^{-\frac{L_c z_\kappa(\lambda)}{2}})}$$

$$= \frac{P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)}) - P(\mathbf{x}_\kappa^{(-1|+1)})P(\mathbf{x}_\kappa^{(+1|-1)})}{P(\mathbf{x}_\kappa^{(+1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)})e^{+L_c z_\kappa(\lambda)} + P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(-1|+1)})e^{-L_c z_\kappa(\lambda)} + P(\mathbf{x}_\kappa^{(-1|-1)})P(\mathbf{x}_\kappa^{(+1|+1)}) + P(\mathbf{x}_\kappa^{(-1|+1)})P(\mathbf{x}_\kappa^{(+1|-1)})} \tag{25}$$

significant bit) for both, the soft-channel decoder as well as the Softbit-Source Decoder, is depicted in Fig. 3.

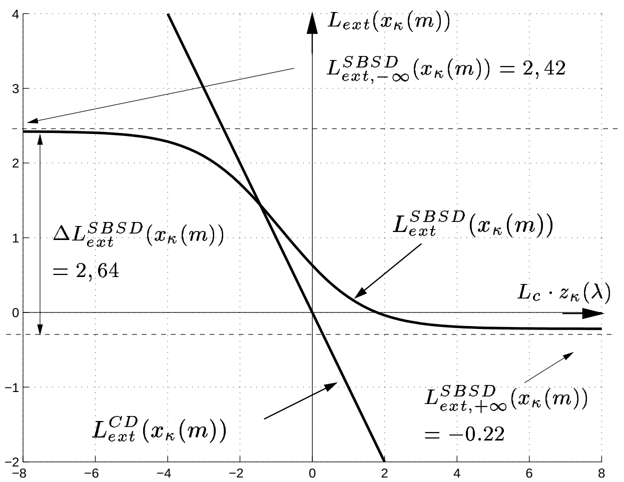

Figure 3: Impact of a single soft-input value $L_c \cdot z_\kappa(\lambda)$ on the *extrinsic* information of different soft-output decoders

It is obvious, that the soft-output of soft-channel decoding $L^{CD}_{ext}(x_\kappa(m))$ grows proportional to the soft-input term $L(\hat{q}) = L_c \cdot z_\kappa(\lambda)$. According to Eq. (13) the absolut value for the gradient amounts always 1 and the range of possible $L^{CD}_{ext}(x_\kappa(m))$ is $\mathbb{R}$. In contrast, the soft-output of Softbit-Source Decoding is bounded by $L^{SBSD}_{ext,-\infty}(x_\kappa(m)) = 2,42$ and $L^{SBSD}_{ext,+\infty}(x_\kappa(m)) = -0.22$. Between these bounds the *extrinsic* information is a monotonic function in the soft-input term $L_c \cdot z_\kappa(\lambda)$, while the maximum absolute value of the gradient is strictly limited to 0.58.

The comparison of both soft-output decoders yields that (under comparable assumptions) the *extrinsic* information $L^{SBSD}_{ext}(x_\kappa(m))$ due to the source statistics is always less sensitive to an update in the decoder's soft-input. Furthermore, if the reliability of the minimum soft-input value increases, e.g., due to additional *a-priori* knowledge, already a low number or even no iterations might be sufficient for $L^{SBSD}_{ext}(x_\kappa(m))$, to reach some kind of saturation. Therefore, no significant gain will be obtainable by more than two iterations.

5. CONCLUSIONS

In this paper we applied Softbit-Source Decoding to an iterative source-channel decoding approach. We carefully analyzed the *extrinsic* information due to Softbit-Source Decoding. We quantified bounds for the *extrinsic* soft-output value, which are in general induced by the source statistics. In addition, we have shown that between these bounds the *extrinsic* value is a monotonic function in the decoder's soft-input values. Furthermore, a formula for the highest sensitivity has been derived. It shows that Softbit-Source Decoding's *extrinsic* information is always much less sensitive to variations in the soft-input as usually known from soft-channel decoder. Therefore, in applications to iterative source-channel decoding the performance improvements due to Softbit-Source Decoding are already gained with a small number of iterations.

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