Iterative Source-Channel Decoding with Code Rates near $\mathbf{r = 1}$

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***Abstract*— We propose a specific channel encoder to support error concealment of mutually independent source codec parameters. To introduce artificial dependencies a recursive non-systematic convolutional (RNSC) code is applied which exhibits a code rate near r = 1. The channel encoder may be considered as a smearing filter. A TURBO-like exploitation of artificial dependencies and of residual source redundancy according to the *iterative source-channel decoding* (ISCD) algorithm permits step-wise quality improvements. Previously known error concealment techniques are outperformed in the most interesting range of channel conditions.**

I. Introduction

In digital speech, audio, and video communications, frequently, parameter-based source encoding schemes are applied. A common feature is that the source encoder extracts frames of source codec parameters such as predictor coefficients, gain factors, etc.. Within a frame (some of) these codec parameters are statistically independent from each other.

However, the individual source codec parameters themselves normally exhibit considerable residual redundancy by their non-uniform probability distribution, their (auto)-correlation, or any other possibly non-linear dependency in time. These measurable terms of residual redundancy mark *a priori* knowledge which can be utilized at the receiver to enhance the error robustness, e.g., by error concealment using *softbit source decoding* (SBSD) [1]. Error concealment is needed to cope with annoying artifacts in the reconstructed signal if residual bit errors remain after channel decoding.

Softbit source decoding combines channel reliability information (e.g., provided by soft-output channel decoders) with the *a priori* knowledge about the individual source parameters. Based on the therewith available *a posteriori* information, SBSD determines per frame individually for each parameter the optimal estimate. As optimality criterion the *minimum mean squared error* (MMSE) of the estimated parameter is often used.

If the different parameters within a frame are mutually independent, the estimate of a specific parameter is based on past and present observations for this parameter only. In that case, the other observed parameters of the frame cannot improve the estimation process. Thus, the capabilities of *conventional softbit source decoding* [1] remain limited. Recent research interests focus on how to extend these capabilities. Such extended capabilities are of particular interest if no additional bit rate is available for channel coding with code rates $r < 1$.

In this paper we propose a specific channel encoder with a code rate very close to $r = 1$ which introduces artificial dependencies on bit-level to the statistically independent source codec parameters. These artificial dependencies and the residual redundancy are exploited by a TURBO-like algorithm [2], [3] which is known as *iterative source-channel decoding* (ISCD) [4], [5]. Besides the remarkably high code rate of $r = 1$ also the use of a *recursive non-systematic convolutional* (RNSC) code marks a key innovation if compared to formerly known ISCD schemes [4], [5].

II. Iterative Source-Channel Decoding

A. Proposed Channel Encoder – Smearing Filter

Fig. 1 a) depicts the transmitter of the proposed scheme. At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ of M source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \dots M$ denoting the position in the frame. The single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}_\kappa^{(i)}$ $(i = 1, \dots 2^{K_\kappa})$. To each quantizer reproduction level $\bar{u}_\kappa^{(i)}$ selected at time instant τ a unique bit pattern $\mathbf{x}_{\kappa,\tau}$ of K_κ bits is assigned. The frame of bit patterns at time instant τ is denoted by $\underline{\mathbf{x}}_\tau$.

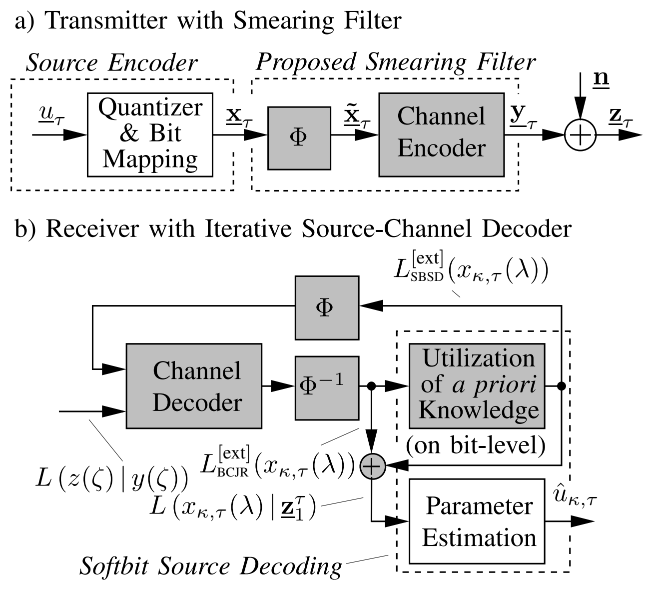

Fig. 1. Transmission scheme – proposed elements marked gray

A bit interleaver Φ scrambles the incoming frame $\underline{\mathbf{x}}_\tau$ of bit patterns. The interleaver marks a key element because a proper design of Φ enables independent reliability gains from the constituent TURBO-like decoders. This independence is also essential for the capabilities of an ISCD scheme. As the reliability gain of SBSD mainly results from the residual redundancy of the source parameters $u_{\kappa,\tau}$, independence is ensured if channel encoding is performed across uncorrelated bit patterns $\mathbf{x}_{\kappa,\tau}$.

To simplify matters, in the sequel channel encoding is restricted to a single frame $\underline{\tilde{\mathbf{x}}}_\tau$ of interleaved data bits. The formalism discussed below can easily be extended to multiples or subsets of $\underline{\tilde{\mathbf{x}}}_\tau$ [6]. For channel encoding of a single frame $\underline{\tilde{\mathbf{x}}}_\tau$ we propose terminated *recursive non-systematic convolutional* (RNSC) codes of constraint length $J+1$ and a code rate (close to) $r=1$. The code rate can be *identical* to $r=1$ if termination is realized by *truncation* or *tail biting*. The code rate is *close to* $r=1$ if for termination J tail bits are appended to the frame $\underline{\tilde{\mathbf{x}}}_\tau$. In the latter case the effective code rate approaches $r=\frac{K\cdot M}{K\cdot M+J}\approx 1$ when the source symbols $u_{\kappa,\tau}$ with $\kappa=1,\ldots M$ are encoded with $K_\kappa=K$ bits each and if the product $K\cdot M$ is significantly larger than the encoder memory J.

Thus, the RNSC code of code rate $r=1$ can be considered as a modulo-2 *smearing filter* which introduces artificial dependencies to the originally mutually independent source symbols. At the receiver these dependencies will not be exploited for error correction, but they will assist SBSD to increase the error concealing capabilities. Due to channel encoding of the non-systematic form, the data bits $x_{\kappa,\tau}(\lambda)$ of $\mathbf{x}_{\kappa,\tau}$ with $\lambda=1,\ldots K$ do not appear self-evident in the frame $\underline{\mathbf{y}}_\tau$. The channel encoded frame $\underline{\mathbf{y}}_\tau$ consists of parity check bits $y(\zeta)$ with $\zeta=1,\ldots(K\cdot M+J)$ only which are sent over an *additive white Gaussian noise* (AWGN) channel.

B. Iterative Decoding Algorithm

Fig. 1 b) shows the receiver of the proposed scheme.

At the receiver reliability information about the transmission of single data bits $x_{\kappa,\tau}(\lambda)\in\{+1,-1\}$ is generated. The reliabilities are expressed by so-called L-values [3]. The term $L\left(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_1^\tau\right)$ denotes the *a posteriori* L-value for a data bit $x_{\kappa,\tau}(\lambda)$ given the entire past of observations $\underline{\mathbf{z}}_1^\tau=\underline{\mathbf{z}}_1,\ldots\underline{\mathbf{z}}_\tau$. Every set $\underline{\mathbf{z}}_t$ represents the frame of received real-valued symbols $z(\zeta)$ with $\zeta=1,\ldots(K\cdot M+J)$ at a specific time instant $t=1,\ldots\tau$. The sign of the real-valued L-value $L\left(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_1^\tau\right)$ yields the hard decision and the magnitude $|L\left(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_1^\tau\right)|$ represents the reliability of this decision.

The *a posteriori* L-value of the proposed ISCD scheme can be separated into two additive terms[1], if a memoryless transmission channel is assumed,

$$L\left(x_{\kappa,\tau}(\lambda)\,|\,\underline{\mathbf{z}}_1^\tau\right)=L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))+L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))\,. \quad (1)$$

[1]In contrast to many other publications to the TURBO-principle, e.g. [3], the separation of the *a posteriori* L-value (1) does not reveal *intrinsic* information. The reasons are: Firstly, no channel-related L-value is given for data bits due to the *non-systematic* RNSC code. Secondly, the bitwise *a priori* knowledge will be evaluated by SBSD.

Both terms mark so-called *extrinsic* information [3], [4] and result from the evaluation of the mutual dependencies between the data bits $x_{\kappa,\tau}(\lambda)$. In iterative TURBO-processes the (de-)interleaved *extrinsic* output of the one decoder serves as (additional) soft-input value for the other constituent decoder and vice versa. An iterative refinement of both *extrinsic* L-values usually enables step-wise reliability improvements.

Concepts how to determine bitwise *extrinsic* L-values $L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ from the artificial dependencies explicitly introduced by RNSC channel encoding are well known in literature. The most famous example is denoted as BCJR-decoder whose detailed rules are discussed, e.g., in [2], [3]. Notice, with respect to the proposed smearing filter, channel-related knowledge $L\left(z(\zeta)\,|\,y(\zeta)\right)$ is solely available for parity check bits due to the *non-systematic* RNSC channel code. Moreover, the constituent decoder provides additional (interleaved) soft-input information $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ for the data bits only. The latter one is initialized with zero in the first iteration step.

In contrast to the determination of $L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$, the technique how to combine these $L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ with the *a priori* knowledge about source codec parameters is not widely common so far. The algorithm how to compute the *extrinsic* L-value $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ of SBSD has been derived in [4], [6], [5]. It shall briefly be reviewed next:

- Firstly, merge all soft-inputs $L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ contributing to the *extrinsic* L-value of SBSD. For this, use a clear separation into present and past information [6]:
  Extract the desired data bit $x_{\kappa,\tau}(\lambda)$ from the present bit pattern $\mathbf{x}_{\kappa,\tau}=\{x_{\kappa,\tau}(\lambda),\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}\}$ and determine for all 2^{K-1} permutations of the pattern $\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}=(x_{\kappa,\tau}(1),\ldots x_{\kappa,\tau}(\lambda-1),x_{\kappa,\tau}(\lambda+1),\ldots x_{\kappa,\tau}(K))$ the term

$$\Theta(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]})=\exp\sum_{\breve{\lambda}=1,\ldots K\,,\,\breve{\lambda}\neq\lambda}\frac{x_{\kappa,\tau}(\breve{\lambda})}{2}\cdot L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\breve{\lambda})). \quad (2)$$

  Accordingly the soft-inputs related to past patterns $\mathbf{x}_{\kappa,\breve{\tau}}$ with $\breve{\tau}=1,\ldots\tau-1$ are individually mapped into terms

$$\Theta(\mathbf{x}_{\kappa,\breve{\tau}})=\exp\sum_{\breve{\lambda}=1,\ldots K}\frac{x_{\kappa,\breve{\tau}}(\breve{\lambda})}{2}\cdot L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\breve{\tau}}(\breve{\lambda})). \quad (3)$$

  For past patterns the summation has to be executed over all $\breve{\lambda}=1,\ldots K$.
- Secondly, if the codec parameters exhibit a 1st order Markov property, exploit the *a priori* knowledge in terms of $P(\mathbf{x}_{\kappa,\tau-1}\,|\,\mathbf{x}_{\kappa,\tau-2})$ by using an efficient recursive formula. For the past elements it reads

$$\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})=\Theta(\mathbf{x}_{\kappa,\tau-1})\cdot\sum_{\mathbf{x}_{\kappa,\tau-2}}P(\mathbf{x}_{\kappa,\tau-1}\,|\,\mathbf{x}_{\kappa,\tau-2})\cdot\alpha_{\tau-2}(\mathbf{x}_{\kappa,\tau-2}) \quad (4)$$

  with the probability of occurrence $\alpha_0(\mathbf{x}_{\kappa,0})=P(\mathbf{x}_{\kappa,0})$ as initialization. The summation in (4) is realized over all 2^K permutations of $\mathbf{x}_{\kappa,\tau-2}$. To measure $\alpha_\tau(\mathbf{x}_{\kappa,\tau})$ for the present time τ the first factor $\Theta(\mathbf{x}_{\kappa,\tau})$ of (4) has to be replaced by the reduced knowledge $\Theta(\mathbf{x}^{[\text{ext}]}_{\kappa,\tau})$ according to Eq. (2).

- Finally, separate the $\alpha_\tau(\mathbf{x}_{\kappa,\tau})$ with $\mathbf{x}_{\kappa,\tau} = \{x_{\kappa,\tau}(\lambda), \mathbf{x}^{[\text{ext}]}_{\kappa,\tau}\}$ for $x_{\kappa,\tau}(\lambda) = \{+1, -1\}$ and calculate

$$L^{[\text{ext}]}_{\text{SBSD}}(x_{\kappa,\tau}(\lambda)) = \log \frac{\sum_{\mathbf{x}^{[\text{ext}]}_{\kappa,\tau}} \alpha_\tau(x_{\kappa,\tau}(\lambda) = +1, \mathbf{x}^{[\text{ext}]}_{\kappa,\tau})}{\sum_{\mathbf{x}^{[\text{ext}]}_{\kappa,\tau}} \alpha_\tau(x_{\kappa,\tau}(\lambda) = -1, \mathbf{x}^{[\text{ext}]}_{\kappa,\tau})} \quad (5)$$

In (5), the operator "log" denotes the natural logarithm. After several iterative refinements of $L^{[\text{ext}]}_{\text{BCJR}}(x_{\kappa,\tau}(\lambda))$ and $L^{[\text{ext}]}_{\text{SBSD}}(x_{\kappa,\tau}(\lambda))$ the *a posteriori* L-values of (1) are utilized for estimation of parameters $\hat{u}_{\kappa,\tau}$ [1], [4], [5], [6].

In the remainder, the specific *iterative source-channel decoding* scheme with an RNSC code of code rate near $r = 1$ will also be referred to as *TURBO error concealment*.

III. Simulation Results

The capabilities of *TURBO error concealment* shall be demonstrated by simulation. Instead of using any specific speech, audio, or video encoder, we model the M statistically independent source parameters of a frame by 1st order Gauss-Markov processes (zero mean and unit variance) with auto-correlation ρ.

First Experiment: In a first experiment we set $\rho = 0.95$. Such a value is typical, e.g., for the scale factors of audio transform codecs. For simplicity we apply interleaving Φ on the present frame $\underline{\mathbf{x}}_\tau$ only. Thus, we adjust the frame length to $M = 500$. Note, in practice a smaller frame length M might be used in conjunction with interleaving of several consecutive frames.

Each component $u_{\kappa,\tau}$ is scalarly quantized by a Lloyd-Max quantizer using $K = 3$ bit/parameter. The index assignment is realized by a *natural binary* mapping.

Taking the interleaver size of $K \cdot M = 1500$ bits into account, it is usually sufficient to apply a simple random interleaver in order to ensure independent reliability gains of both constituent decoders. The bit interleaved frame $\tilde{\underline{\mathbf{x}}}_\tau$ is channel encoded using a memory $J = 6$ terminated RNSC code with generator polynomial $\mathbf{G} = (\frac{1}{1+D+D^2+D^3+D^4+D^6})$. With respect to the $J = 6$ terminating bits, the effective code rate increases to $r = 1500/1506 \approx 0.996$.

The transmission channel is modeled by *additive white Gaussian noise* (AWGN) with known E_s/N_0 (E_s: energy per BPSK-modulated parity check bit $y(\zeta)$, $N_0/2$: spectral density resp. variance of the zero mean AWGN).

The *parameter signal-to-noise ratio* (SNR) between the originally generated parameters $u_{\kappa,\tau}$ and the reconstructed estimates $\hat{u}_{\kappa,\tau}$ is used for quality evaluation. The related simulation results are shown in Fig. 2.

The dashed curve depicts the result for *conventional* SBSD without the smearing filter according to [1]. Due to the assumed statistical independence of the original source parameters $u_{\kappa,\tau}, u_{\breve{\kappa},\tau}$ with $\kappa, \breve{\kappa} \in [1, M]$, $\kappa \neq \breve{\kappa}$, the estimation of a particular $\hat{u}_{\kappa,\tau}$ exhibiting a fixed but arbitrary position κ in the frame $\underline{\hat{u}}_\tau$ is based on past and present observations for that specific position κ only. The observations for the other $M - 1$ positions remain unexploited.

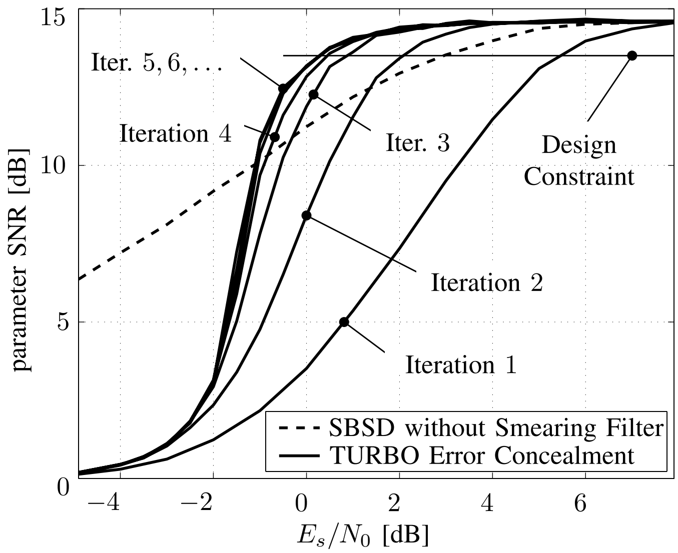

Fig. 2. Simulation Results of First Experiment

The solid curves show the simulation results if the proposed smearing filter is applied at the transmitter and if the resulting explicit artificial dependency and the implicit residual redundancy of the source parameters are evaluated iteratively at the receiver. The BCJR-decoder serves as component decoder for the RNSC code. If both decoding steps are executed only once, the performance of the proposed scheme is always inferior to the conventional approach due to channel coding of the *non-systematic* form. But, if the reliability gain of SBSD is fed back to the BCJR-decoder and if a second iteration is carried out, the reference is outperformed. Up to 5 iterations reveal significant quality gains in the most interesting range of channel conditions. The convergence after 5 iterations is confirmable with information theoretical measures like the so-called *extrinsic information transfer* (EXIT) charts [7], [6].

If in a practical application a baseline parameter SNR of 13.5 dB is accepted to provide a tolerable reconstruction quality (see "Design Constraint" in Fig. 2), the higher robustness permits to decrease the lower limit for an acceptable performance from $E_s/N_0 = 3.06$ dB (conventional SBSD) down to $E_s/N_0 = 0.27$ dB (after 5 iterations). The reference remains superior if E_s/N_0 drops below -1.01 dB.

Second Experiment: The results shall be confirmed by a second experiment using modified parameter configurations. For this purpose, auto-correlation of the source codec parameters is reduced to $\rho = 0.9$ and the resolution of the Lloyd-Max quantizer is increased to $K = 4$ bit/parameter. The index assignment is replaced by an *optimized* mapping which has explicitly been designed in view of the ISCD process [8].

A simple random interleaver design of size $K \cdot M = 2000$ is used to ensure independence of the *extrinsic* reliability gains. Channel encoding is realized by a memory $J = 5$, terminated RNSC code with $\mathbf{G} = (\frac{1}{1+D+D^2+D^3+D^5})$. The effective code rate increases to $r = 2000/2005 \approx 0.9975$.

Figure 3 shows the simulation results. Again, the *conventional* SBSD approach [1] without *smearing filter* is outperformed in the most interesting range of channel conditions. After only 2 iterations the new *TURBO error concealment* scheme becomes superior. Up to 9 iterations make further substantial quality improvements possible.

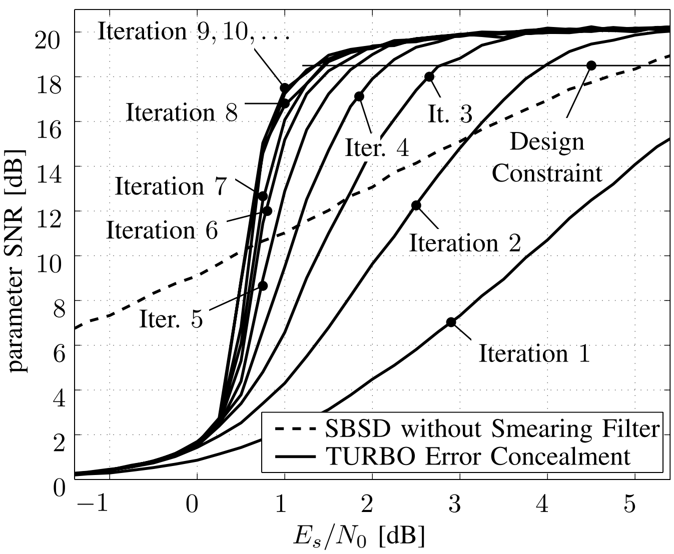

Fig. 3. Simulation Results of Second Experiment

If a baseline *parameter SNR* of 18.5 dB provides a tolerable reconstruction quality the range of acceptable performance is extended from $E_s/N_0 = 5.1$ dB (conventional SBSD) down to $E_s/N_0 = 1.35$ dB (after 9 iterations). Conventional SBSD remains best if $E_s/N_0 < 0.55$ dB.

IV. Additional Remarks

The primary objective of the proposed scheme with *smearing filter* and *TURBO error concealment* is to extend the capabilities of conventional SBSD if no bit rate is available for channel coding with code rates $r < 1$. However, it is not the high code rate $r = 1$ alone which distinguishes the proposed scheme from formerly known ISCD approaches [4], [5]. It is also the *non-systematic* form of channel encoding.

All formerly known ISCD schemes consider channel codes of the *systematic* form. For instance, it is most common to use *recursive systematic convolutional* (RSC) codes. However, first experiments have shown that the error robustness can significantly be improved if the RSC code is replaced by an appropriately designed *recursive non-systematic convolutional* (RNSC) code of the same code rate r and constraint length $J+1$. This can be explained as follows:

The matrix $\mathbf{G}$ of generator polynomials of an RSC (as used in the former approaches to ISCD) consists of a 1 due to the systematic form and of rational polynomials due to the recursive structure, e.g., if the code rate amounts to $r = 1/2$ it is $\mathbf{G} = (1,\, F(D)/H(D))$. The term D denotes the one tap delay operator and the maximum power J of D^J in $F(D)$ resp. $H(D)$ determines the constraint length $J+1$ of the code. If the constraint length is fixed, there exist (in maximum) $2^{J+1} \times 2^{J+1}$ possibilities to design the RSC code. The precise number is less because in some cases $F(D)$ and $H(D)$ exhibit a common devisor so that the effective constraint length is reduced. However, at the transmitter the RSC code generates for every data bit $x_{\kappa,\tau}(\lambda)$ two code bits $y(\zeta)$. One of these code bits is identical to the data bit. At the receiver channel-related reliability information is available for both bits and some additional information is given for the data bit due to the *extrinsic* L-value provided by the other constituent decoder.

In contrast, if an RNSC code with $\mathbf{G} = (F_1(D)/H(D),\, F_2(D)/H(D))$ of the same code rate r and constraint length $J+1$ is used, there exist (in maximum) $2^{J+1} \times 2^{J+1} \times 2^{J+1}$ possibilities to find an appropriate channel coding component for ISCD. Of course, channel-related reliability information will not be available for the data bit $x_{\kappa,\tau}(\lambda)$ anymore, but for this bit there exists still the *extrinsic* L-value provided by the other constituent decoder. Thus, the RNSC code with $\mathbf{G} = (F_1(D)/H(D),\, F_2(D)/H(D))$ can also be interpreted as an RSC code with $\mathbf{G} = (1,\, F_1(D)/H(D),\, F_2(D)/H(D))$ whose systematic data bit has been eliminated by puncturing. As some reliability information is given for all code bits of this punctured RSC the error robustness of the ISCD scheme can benefit from the lower effective code rate $r = 1/3$.

A detailed comparison of applying either RSC codes or RNSC codes to ISCD is a key topic for future research work.

V. Conclusions

In this paper, error concealment by *softbit source decoding* has been improved in the most interesting range of channel conditions. At the transmitter an RNSC channel encoder/smearing filter is used which adds artificial dependencies to frames of statistically independent source parameters. The encoding scheme exhibits a code rate (very close to) $r = 1$ and is designed to assist SBSD. At the receiver an iterative, TURBO-like utilization of this artificial dependency and of the residual redundancy of the source parameters permits stepwise improvements of robustness by several iterations.

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