TURBO ERROR CONCEALMENT OF MUTUALLY INDEPENDENT SOURCE CODEC PARAMETERS

M. Adrat and P. Vary

Institute of Communication Systems and Data Processing (ivd) Aachen University (RWTH), Germany {adrat,vary}@ind.rwth-aachen.de

ABSTRACT

Modern error concealment techniques like *softbit source decoding* (SBSD) by parameter estimation utilize natural residual source redundancy of codec parameters as *a priori* knowledge. But, the capabilities of conventional SBSD remain limited if applied to sets of mutually independent source codec parameters.

This contribution deals with an advancement of conventional SBSD. At the transmitter, a smearing filter with a code rate near r = 1 is proposed introducing artificial channel coding redundancy on bit-level to the mutually independent codec parameters. At the receiver, a TURBO-like evaluation of this artificial channel coding redundancy and of the natural residual source redundancy according to the iterative source-channel decoding (ISCD) algorithm permits step-wise quality improvements by several iterations.

In this paper, we extensively discuss the key innovations of this TURBO *error concealment* technique over formerly known ISCD schemes. In addition, the convergence is analyzed using EXIT-charts. Finally, the remarkable performance gains over conventional SBSD are demonstrated by simulation.

1. INTRODUCTION

In digital communications, modern source compression systems for speech, audio, and video transmission are frequently based on models where-from sets of characteristic source codec parameters are extracted. For instance, such a set of codec parameters might comprise predictor coefficients, gain factors, etc. Usually, due to delay and complexity constraints in the encoding process the individual source codec parameters exhibit considerable residual redundancy by their non-uniform probability distribution, their (auto)-correlation, or any other possibly non-linear dependency in time. In addition, some of the source codec parameters of a single set reveal mutual dependencies, while some other parameters of the set can be considered as statistically independent from each other. All the measurable terms of residual source redundancy mark a priori knowledge on parameter-level which can be utilized at the receiver to enhance the error robustness, e.g., by source controlled channel decoding [1-3] or by error concealment techniques like softbit source decoding (SBSD) [4, 5].

The latter approach, error concealment by *softbit source decoding*, can cope with annoying artifacts in the reconstructed signal if residual bit errors remain after channel decoding. *Softbit source decoding* combines channel reliability information (e.g. provided by soft-output channel decoders) with the *a priori* knowledge about the individual source codec parameters. The combination yields *a posteriori* information which is exploited to determine individually the optimal estimate for each source codec parameter of a set. As optimality criterion the **minimum mean squared error** (**MMSE**) between the originally extracted source codec parameter and the corresponding estimate is often used.

The gratification of the optimality criterion depends in particular on the accuracy of the *a posteriori* information. This accuracy will be maximal if the *a posteriori* information is based on as much observations as available. Thus, it will be highest if all observed codec parameters of the present, past, and (some possibly available) future sets are taken into account. But, if some of the source codec parameters of a set are mutually independent, the *a posteriori* information of a specific codec parameter cannot exploit all given observations. Only those observed codec parameters of a set which exhibit mutual dependencies to the specific codec parameter under consideration can contribute to the estimation process. Consequently, the capabilities of *conventional* SBSD [4] by parameter estimation remain limited.

Hence, the primary objective of our work is to extend the capabilities of conventional SBSD to applications where no additional bit rate is available for channel encoding with code rates r < 1. This topic has been a matter of research interest since the invention of SBSD. For this purpose we propose a modulo-2 smearing *filter* of rate (close to) r = 1 and an iterative decoding algorithm. This specific channel encoder introduces artificial dependencies on bit-level to the statistically independent source codec parameters. These dependencies are evaluated iteratively together with the natural residual source redundancy. The TURBO-like decoding algorithm [6,7] resembles iterative source-channel decoding (ISCD) [8–13] to a great extend. But, besides the remarkably high code rate (r = 1 instead of, e.g., r = 1/2 in [13]) there are some additional major points which also mark key innovations if compared to formerly known ISCD schemes. Thus, it is our secondary objective to point out these key innovations as well.

2. ITERATIVE SOURCE-CHANNEL DECODING

2.1. Transmitter with Proposed Smearing Filter

Fig. 1 a) depicts the transmitter of an ISCD scheme.

A source encoder extracts a set \underline{u}_{τ} of M source codec parameters $u_{\kappa,\tau}$ from a short time segment of the input source signal. The time index τ labels the time segment and $\kappa = 1, \ldots M$ denotes the position of the parameter in the set. Each value $u_{\kappa,\tau}$ is scalarly quantized to one of $2^{K_{\kappa}}$ quantizer reproduction levels $\overline{u}_{\kappa}^{(i)}$, $i \in [1, 2^{K_{\kappa}}]$. The sizes $2^{K_{\kappa}}$ of the individual quantizer codebooks \mathbb{U}_{κ} , $\kappa = 1, \ldots M$, and the corresponding quantizer reproduction levels $\overline{u}_{\kappa}^{(i)}$ itself are invariant with respect to τ but depend on the index κ . This dependence might be due to the specific dynamic range and the specific probability density of every parameter $u_{\kappa,\tau}$. To each quantizer reproduction level $\overline{u}_{\kappa}^{(i)}$ selected at time instant τ a unique

bit pattern $\mathbf{x}_{\kappa,\tau}$ of K_{κ} data bits $x_{\kappa,\tau}(\lambda)$ with $\lambda = 1, \ldots, K_{\kappa}$ is assigned. The complete set of M bit patterns at time instant τ is denoted by $\underline{\mathbf{x}}_{\tau}$.

A bit interleaver Φ scrambles the incoming frame $\underline{\mathbf{x}}_{\tau}$ of data bits to $\underline{\tilde{\mathbf{x}}}_{\tau}$ in a deterministic manner. In today's communication systems, usually the data bits are rearranged according to their individual importance for the subjective speech quality. The reordering performs some kind of classification making a subsequent *unequal error protection* possible¹. However, the interleaver plays another key role in iterative source-channel decoding schemes. If the interleaver Φ is appropriately designed, it allows independent reliability gains from the constituent decoders which are essential for the capabilities of ISCD. As the reliability gain of SBSD mainly results from the residual redundancy of the source codec replacementsparameters $u_{\kappa,\tau}$, independence is ensured if channel encoding is performed accoss (more or less) independent bit patterns $\mathbf{x}_{\kappa,\tau}$.

Notice, bit interleaving as well as channel encoding need not to be limited to the present set $\underline{\mathbf{x}}_{\tau}$ of bit patterns resp. $\underline{\tilde{\mathbf{x}}}_{\tau}$. Both routines can also be realized for a time sequence of T + 1 consecutive sets, e.g. $\underline{\mathbf{x}}_{\Lambda-T}, \dots, \underline{\mathbf{x}}_{\Lambda}$, if a delay of (in maximum) Λ time instants is tolerable in a practical application. To simplify matters, in the following we use the short hand notations $\underline{\mathbf{x}}_{\Lambda-T}^{\Lambda}$ resp. $\underline{\tilde{\mathbf{x}}}_{\Lambda-T}^{\Lambda}$ to denote such time sequences.

a) Transmitter with Proposed Smearing Filter







Figure 1: Transmission scheme¹ – proposed elements marked gray (notation refers to the case where no delay is accepted, T + 1 = 1, $\Lambda = \tau$)

For channel encoding of a time sequence $\underline{\tilde{\mathbf{x}}}_{\Lambda-\mathrm{T}}^{\Lambda}$ we propose terminated **recursive non-systematic convolutional (RNSC)** codes of constraint length J + 1 and a code rate (close to) r = 1. The code rate can be *identical* to r = 1 if termination is realized by *truncation* or *tail biting*. The code rate is *close to* r = 1 if for termination J tail bits are appended to the sequence $\underline{\tilde{\mathbf{x}}}_{\Lambda-\mathrm{T}}^{\Lambda}$. In the latter case the effective code rate approaches $r = \frac{(\mathrm{T}+1)\cdot K\cdot M}{(\mathrm{T}+1)\cdot K\cdot M+J} \approx 1$ when the source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \ldots, M, \tau =$ $\Lambda - \mathrm{T}, \ldots \Lambda$ are encoded with $K_{\kappa} = K$ bits each and if the product of $(\mathrm{T}+1) \cdot K \cdot M$ is significantly larger than the encoder memory J.

Thus, due to the remarkably high code rate (near) r = 1 the RNSC code can be considered as a modulo-2 *smearing filter*. This filter introduces artificial dependencies on bit-level to the originally mutually independent source codec parameters. At the receiver these dependencies will not (only) be exploited for error correction, but they will assist SBSD to increase the error concealing capabilities. Because of channel encoding of the *non-systematic* form, the data bits $x_{\kappa,\tau}(\lambda)$ of $\mathbf{x}_{\kappa,\tau}$ with $\lambda = 1, \ldots K$ do not appear self-evident in the code word $\underline{\mathbf{y}}_{\Lambda-\mathrm{T}}^{\Lambda}$. The code word $\underline{\mathbf{y}}_{\Lambda-\mathrm{T}}^{\Lambda}$ consists of parity check bits $y(\mu)$ with $\mu = 1, \ldots ((\mathrm{T}+1) \cdot K \cdot M + J)$ only which are transmitted over a memoryless **additive white Gaussian noise (AWGN)** channel.

2.2. Receiver with TURBO Error Concealment

Fig. 1 b) shows the receiver of an iterative source-channel decoding (ISCD) scheme for TURBO *error concealment*.

At the receiver reliability information about the transmission of single parity check bits $y(\mu)$ with $\mu = 1, ..., ((T + 1) \cdot K \cdot M + J)$ is processed to determine reliability information about the data bits $x_{\kappa,\tau}(\lambda)$ with $\kappa = 1, ..., M, \tau = \Lambda - T, ..., \Lambda$, and $\lambda = 1, ..., K$. Such reliability information can either be evaluated in terms of probabilities or in the so-called *log-likelihood* algebra. In *log-likelihood* algebra, reliabilities in terms of probabilities are transformed into *log-likelihood* ratios, or short *L*-values, e.g., [7]. If the transformation is restricted to a binary random variable of bipolar form, the *a posteriori log-likelihood* ratio of, e.g. the data bit $x_{\kappa,\tau}(\lambda) \in \{+1, -1\}$, is defined as [7]

$$L(x_{\kappa,\tau}(\lambda) | \underline{\mathbf{z}}_{1}^{\Lambda}) = \log \frac{P(x_{\kappa,\tau}(\lambda) = +1 | \underline{\mathbf{z}}_{1}^{\Lambda})}{P(x_{\kappa,\tau}(\lambda) = -1 | \underline{\mathbf{z}}_{1}^{\Lambda})} \quad .$$
(1)

The term $L(x_{\kappa,\tau}(\lambda) | \underline{z}_1^{\Lambda})$ represents a conditional *L*-value for $x_{\kappa,\tau}(\lambda)$ given the entire past of observations $\underline{z}_1^{\Lambda}$ (and parts of the future up to a maximum look-ahead of Λ). "log" denotes the natural logarithm and "+1" signifies the "null" element of the binary *modulo-2* addition \oplus in GF(2). The sign of the real-valued *log-likelihood* ratio $L(x_{\kappa,\tau}(\lambda) | \underline{z}_1^{\Lambda})$ yields the hard decision and the magnitude $|L(x_{\kappa,\tau}(\lambda) | \underline{z}_1^{\Lambda})|$ represents the reliability of this decision.

The *a posteriori L*-value for the specific iterative source-channel decoding (ISCD) scheme described in Section 2.1 can be separated into two additive terms, if a memoryless transmission channel is assumed

$$L(x_{\kappa,\tau}(\lambda) | \underline{\mathbf{z}}_{1}^{\Lambda}) = L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) + L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda)) \quad .$$
 (2)

Both terms mark so-called *extrinsic* information [7, 8] and result from the evaluation of the mutual dependencies between the data bits $x_{\kappa,\tau}(\lambda)$ resp. parity check bits $y(\mu)$. Notice, in contrast to all formerly known publications on iterative source-channel decoding, e.g. [8–13], the separation of the *a posteriori L*-value (2) does not reveal *intrinsic* information. The reasons are:

• Firstly, no channel-related *L*-value is given for data bits due to the *non-systematic* RNSC code. The channel-related *L*-value [7]

$$L(z(\mu) | y(\mu)) = 4 \cdot E_s / N_0 \cdot z(\mu) \quad . \tag{3}$$

is given for the parity check bits $y(\mu)$ with $\mu = 1, ... ((T + 1) \cdot K \cdot M + J)$ only. E_s denotes the energy spent to transmit a single BPSK modulated parity check bit and $\sigma_n^2 = N_0/2$ the double sided power spectral density of the AWGN.

¹Note, the interleaver Φ may not be mistaken for an interleaver breaking up burst errors on the transmission link. Such an interleaver is typically located <u>after</u> channel encoding. This kind of interleaver is not needed here, because memoryless transmission channels are considered only.

• Secondly, the bit-wise *a priori* knowledge is part of the parameter-level *a priori* knowledge being utilized by *soft-bit source decoding*. Thus, $L(x_{\kappa,\tau}(\lambda))$ is included in $L_{\text{sBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ (see also Section 3).

However, two independent *extrinsic* L-values as given in (2) are essential for an iterative TURBO-process. In iterative TURBO-processes the (de-)interleaved *extrinsic* output of the one decoder serves as (additional) soft-input value for the other constituent decoder and vice versa. An iterative refinement of both *extrinsic* L-values usually enables step-wise reliability improvements.

In literature, concepts how to determine bit-wise *extrinsic* L-values $L_{\text{BCIR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ from the artificial dependencies explicitly introduced by RNSC channel encoding are well known. The most famous example is denoted as *symbol-by-symbol maximum a posteriori* (MAP) or BCJR-channel decoder whose detailed rules are discussed, e.g., in [6, 7, 14]. Notice, while channel-related knowledge $L(z(\mu) | y(\mu))$ is solely available for parity check bits $y(\mu)$ with $\mu = 1, \ldots ((T + 1) \cdot K \cdot M + J)$, additional (interleaved) soft-input information $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ is only given for the data bits $x_{\kappa,\tau}(\lambda)$ (see Fig. 1 b.)). The latter one is usually initialized with zero in the first iteration step [11].

The technique how to combine *a priori* knowledge about source parameters with these $L_{\text{BCIR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ is not widely common so far. The algorithm how to compute the *extrinsic L*-value $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ of SBSD has been derived in [8–13]. It shall briefly be reviewed next:

Merge the bit-wise soft-inputs L^[ext]_{BCIR}(x_{κ,τ}(λ)) of single data bits x_{κ,τ}(λ) to parameter-oriented soft-input information Θ(**x**_{κ,τ}) about bit patterns **x**_{κ,τ}. For this purpose, determine for all 2^K possible permutations of each bit pattern **x**_{k,t} at a specific time instant t = Λ - T,...Λ and position k = 1,...M (excluding the index pair (k, t) = (κ, τ) of the desired extrinsic L-value L^[ext]_{SBD}(x_{κ,τ}(λ)); see below) the term

$$\Theta(\mathbf{x}_{k,t}) = \exp\sum_{\breve{\lambda}=1,\dots K} \frac{x_{k,t}(\lambda)}{2} \cdot L_{\text{BCJR}}^{[\text{ext}]}(x_{k,t}(\breve{\lambda})).$$
(4)

The summation runs over the bit index $\lambda = 1, \ldots K$.

In case of the index pair $(k, t) = (\kappa, \tau)$ the bit index $\check{\lambda} = \lambda$ of the desired extrinsic *L*-value $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ has to be excluded from the summation. Thus, in this case the terms $\Theta(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]})$ have to be computed for all 2^{K-1} possible permutations of bit pattern $\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}$ by summation over all $\check{\lambda} = 1, \ldots K$, $\check{\lambda} \neq \lambda$. For convenience, in the following that specific part of the pattern $\mathbf{x}_{\kappa,\tau}$ without $x_{\kappa,\tau}(\lambda)$ will be denoted as $\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}$. Thus, $\mathbf{x}_{\kappa,\tau}$ can also be expressed as $(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}, \mathbf{x}_{\kappa,\tau}(\lambda))$.

2. Combine this parameter-oriented soft-input information $\Theta(\mathbf{x}_{\kappa,\tau})$ with the *a priori* knowledge about the source codec parameters. If the parameters $u_{\kappa,\tau}$ resp. the corresponding bit patterns $\mathbf{x}_{\kappa,\tau}$ exhibit a 1st order Markov property $P(\mathbf{x}_{\kappa,\tau} | \mathbf{x}_{\kappa,\tau-1})$ in time, past and (possibly given) future bit patterns $\mathbf{x}_{\kappa,t}$ with $t = \Lambda - T, \dots \Lambda, t \neq \tau$, can efficiently be

evaluated by a *forward-backward* algorithm. Both recursive formulas are determinable by

$$\alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1}) = \sum_{\mathbf{x}_{\kappa,\tau-2}} P(\mathbf{x}_{\kappa,\tau-1} | \mathbf{x}_{\kappa,\tau-2}) \cdot (5)$$

$$\beta_{\tau}(\mathbf{x}_{\kappa,\tau}) = \sum_{\mathbf{x}_{\kappa,\tau+1}} P(\mathbf{x}_{\kappa,\tau+1} | \mathbf{x}_{\kappa,\tau}) \cdot (6)$$

$$\Theta(\mathbf{x}_{\kappa,\tau+1}) \cdot \beta_{\tau+1}(\mathbf{x}_{\kappa,\tau+1}).$$

The summation of the *forward* recursion (5) resp. *backward* recursion (6) is realized over all 2^{K} permutations of $\mathbf{x}_{\kappa,\tau-2}$ resp. $\mathbf{x}_{\kappa,\tau+1}$.

For initialization serve $\alpha_0(\mathbf{x}_{\kappa,0}) = P(\mathbf{x}_{\kappa,0})$ and $\beta_{\Lambda}(\mathbf{x}_{\kappa,\Lambda}) = 1$. With respect to the defined size of the interleaver Φ , throughout the refinement of bit-wise *log-likelihood* values T + 1 consecutive bit patterns $\mathbf{x}_{\kappa,\tau}$ with $\tau = \Lambda - T, \dots \Lambda$ are regarded in common. In consequence, the forward recursion needs not to be recalculated from the very beginning $\alpha_0(\mathbf{x}_{\kappa,0})$ in each iteration. All terms $\mathbf{x}_{\kappa,1}^{\Lambda-T-1}$, which are scheduled before the first interleaved bit pattern $\mathbf{x}_{\kappa,\Lambda-T}$, will not be updated during the iterative feedback of *extrinsic* information and can be measured once in advance.

3. Finally, the intermediate results of (4), (5) and (6) have to be combined as shown in (7) (see bottom of page). With respect to the 2^K permutations of $\mathbf{x}_{\kappa,\tau}$, the set of *backward* recursions $\beta_{\tau}(\mathbf{x}_{\kappa,\tau})$ of (6) as well as the set of parameter *a priori* knowledge values $P(\mathbf{x}_{\kappa,\tau} | \mathbf{x}_{\kappa,\tau-1})$ are separated into two subsets of equal size. In the numerator only these $\beta_{\tau}(\mathbf{x}_{\kappa,\tau}^{[ext]}, \mathbf{x}_{\kappa,\tau}(\lambda))$ resp. $P(\mathbf{x}_{\kappa,\tau}^{[ext]}, \mathbf{x}_{\kappa,\tau}(\lambda) | \mathbf{x}_{\kappa,\tau-1})$ are considered where the desired data bit takes the value $\mathbf{x}_{\kappa,\tau}(\lambda) = +1$, and in the denominator $\mathbf{x}_{\kappa,\tau}(\lambda) = -1$ respectively. The inner summation of (7) has to be evaluated for all 2^K

permutations of $\mathbf{x}_{\kappa,\tau-1}$ and the outer summation for the 2^{K-1} permutations of $\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}$.

After several iterative refinements of $L_{\text{BCIR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ and $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ the *a posteriori L*-values of (2) are utilized for estimation of parameters $\hat{u}_{\kappa,\tau}$. For this purpose, at first parameter-oriented *a posteriori* knowledge is determined and secondly combined with quantizer reproduction levels to provide the parameter estimates $\hat{u}_{\kappa,\tau}$. Parameter-oriented *a posteriori* knowledge like $P(\mathbf{x}_{\kappa,\tau} \mid \mathbf{Z}_1^{\Lambda})$ can easily be measured either from the bit-wise *a posteriori L*-values of (2) or from the intermediate results of (7), e.g. by

$$P(\mathbf{x}_{\kappa,\tau} | \underline{\mathbf{z}}_{1}^{\Lambda}) = C \cdot \beta_{\tau}(\mathbf{x}_{\kappa,\tau}) \cdot \Theta(\mathbf{x}_{\kappa,\tau})$$
$$\sum_{\mathbf{x}_{\kappa,\tau-1}} P(\mathbf{x}_{\kappa,\tau} | \mathbf{x}_{\kappa,\tau-1}) \cdot \alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1}).$$
(8)

The term C denotes a constant factor which ensures that the *to-tal probability theorem* is fulfilled. Thus, if the *minimum mean squared error* (MMSE) serves as fidelity criterion, the individual estimates are given by [4]

$$\hat{u}_{\kappa,\tau} = \sum_{\bar{u}_{\kappa}^{(i)} \in \mathbb{U}_{\kappa}} \bar{u}_{\kappa}^{(i)} \cdot P(\mathbf{x}_{\kappa,\tau} = i \mid \underline{\mathbf{z}}_{1}^{\Lambda}) \quad . \tag{9}$$

Determination rule for the *extrinsic* L-value of softbit source decoding (including bit-wise a priori knowledge $L(x_{\kappa,\tau}(\lambda))$):

$$L_{\text{SBSD}}^{[\text{ext}]}(\boldsymbol{x}_{\kappa,\tau}(\lambda)) = \log \frac{\sum_{\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}} \beta_{\tau}(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}, \boldsymbol{x}_{\kappa,\tau}(\lambda) = +1) \cdot \Theta(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]})}{\sum_{\mathbf{x}_{\kappa,\tau}} P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}, \boldsymbol{x}_{\kappa,\tau}(\lambda) = +1 | \mathbf{x}_{\kappa,\tau-1}) \cdot \alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1})} \sum_{\mathbf{x}_{\kappa,\tau-1}} P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]}, \boldsymbol{x}_{\kappa,\tau}(\lambda) = +1 | \mathbf{x}_{\kappa,\tau-1}) \cdot \alpha_{\tau-1}(\mathbf{x}_{\kappa,\tau-1}) + \alpha_{\tau-1}(\mathbf{x}_{$$

If a delay is acceptable, i.e. T + 1 > 1, Eq. (9) performs interpolation of source codec parameters due to the look-ahead of $\Lambda - \tau$ parameters. Otherwise, if T + 1 = 1 and $\Lambda = \tau$, Eq. (9) performs parameter extrapolation.

3. KEY INNOVATIONS FOR ISCD SCHEMES

The primary objective of the modulo-2 smearing filter and TURBO error concealment is to improve the capabilities of conventional SBSD. Such a scheme is of particular relevance for those systems where no bit rate is available for channel coding with code rates r < 1. However, besides the high code rate near r = 1, the specific scheme described in Section 2 also comprises some additional key innovations which can directly be applied to the formerly known ISCD approaches with, e.g., r = 1/2 [10, 13].

Non-Systematic Channel Codes

In Section 2 a recursive non-systematic convolutional (RNSC) code of code rate r = 1 serves as modulo-2 smearing filter. In general, in iterative TURBO processes the use of an RNSC code can be superior to an adequate recursive systematic convolutional (RSC) code of the same code rate r and the same constraint length J + 1 as considered, e.g., in [10, 13]. The reason shall be explained by the following example:

Let us suppose that a rate r = 1/2 RSC code with generator PSfrag replacements of clarity, before this method shall be applied to the term D denotes the one-tap delay operator and the maximum power J of D^J in the feed-forward polynomial F(D)resp. feedback polynomial G(D) determines the constraint length J + 1 of the code. There exist (less than)² $(2^{J+1})^2$ ways to design the RSC code. If an RNSC code of the same rate r and constraint length J + 1 is used, e.g., with $\mathbf{G} = (F_1(D)/G(D), F_2(D)/G(D))$, there exist (less than) $(2^{J+1})^3$ possible arrangements. We can benefit from the higher number of valid arrangements, e.g., to design the EXIT characteristic of the channel code such that it is better suited to match that one of SBSD (see the following sections).

Puncturing Pattern •

Instead of using the RNSC code of r = 1, an RSC mother code of code rate r' = 1/2 can also be used to determine the same channel encoded frames $\mathbf{y}_{_}$ when after RSC encoding all systematic data bits are eliminated by puncturing. Thus, the puncturing pattern offers an additional design option. Even higher quality gains might be available by *partial puncturing* of data bits and of parity check bits.

Number of Iterations

It turns out that the number of profitable iterations is higher if RNSC codes are used in ISCD schemes. As demonstrated in Section 5 and as confirmed by the convergence analysis in Section 4 up to 9 iterations reveal remarkable quality gains. Most previously known ISCD schemes using RSC codes provide quality gains by mainly 2 iterations only [10, 13]. A few more iterations become reasonable if the index assignment is optimized in view of ISCD [15, 16].

Implementational Aspect .

The determination rule (7) for the extrinsic L-value of SBSD includes the bit-wise a priori knowledge. This bit-wise a priori knowledge can either be expressed in terms of an L-value $L(x_{\kappa,\tau}(\lambda))$ or in terms of a probability $P(x_{\kappa,\tau}(\lambda))$. In order to extract $L(x_{\kappa,\tau}(\lambda))$ from $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ it is necessary to extract $P(x_{\kappa,\tau}(\lambda))$ from the 1st order Markov probability $P(\mathbf{x}_{\kappa,\tau} | \mathbf{x}_{\kappa,\tau-1})$ in (7). However, this would cause a loss of optimality due to the approximation

$$P(\mathbf{x}_{\kappa,\tau} | \mathbf{x}_{\kappa,\tau-1}) = P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]} | \mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda)) \cdot P(x_{\kappa,\tau}(\lambda) | \mathbf{x}_{\kappa,\tau-1})$$
(10)
$$\approx P(\mathbf{x}_{\kappa,\tau}^{[\text{ext}]} | \mathbf{x}_{\kappa,\tau-1}, x_{\kappa,\tau}(\lambda)) \cdot P(x_{\kappa,\tau}(\lambda)).$$

The approximation that the data bit $x_{\kappa,\tau}(\lambda)$ is independent from the preceding bit pattern $\mathbf{x}_{\kappa,\tau-1}$ is usually not fulfilled strictly. Therefore, here we do not extract bit-wise a priori knowledge from the extrinsic L-value of SBSD.

4. ANALYSIS OF CONVERGENCE BEHAVIOR

In order to predict the convergence behavior of iterative processes, a so-called "EXIT-chart"-analysis has been proposed in [17-19]. By using the powerful "EXIT-chart"-analysis, the mutual information measure is applied to the input/output relations of the individual constituent soft-input/soft-output decoders. On the one hand, the information exhibited by the extrinsic input log-likelihood ratio, and on the other hand, the information comprised in the extrinsic output L-values after soft-output decoding are closely related to the information content of the originally

TURBO error concealment algorithm, some notations have to be defined. Figure 2 depicts an enlargement of the two constituent decoders of TURBO error concealment shown in Figure 1 b).

Figure 2: Soft-output decoder of ISCD using L-values

With respect to the softbit source decoding part of the TURBO error concealment algorithm (see upper part of Figure 2) the mutual information measure at the input has to be evaluated between the data bit $x_{\kappa,\tau}(\lambda)$ and $L_{\text{BCJR}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$. The *mutual* information at the output has to be analyzed between $x_{\kappa,\tau}(\lambda)$ and $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$. In case of the soft-input/soft-output channel decoder it is the other way around (see lower part of Figure 2). To simplify notation we define:

 $\mathcal{I}^{[in]}$ is the *mutual information* at the input of both decoders

 $\mathcal{I}^{[out]}$ is the *mutual information* at the output of both decoders.

If needed, an additional subscript "BCJR" resp. "SBSD" will be added to distinguish between BCJR-decoding and softbit source decoding. Note, the upper limit for both measures is constrained to the information content of $x_{\kappa,\tau}(\lambda)$. Thus, $\mathcal{I}^{[\mathrm{in}]}$ and $\mathcal{I}^{[\mathrm{out}]}$ are bound to values less than or equal to the entropy of $x_{\kappa,\tau}(\lambda)$.

4.1. Extrinsic Information Transfer (EXIT) Characteristics The mutual information measure $\mathcal{I}^{[out]}$ at the output of each decoder depends on the settings applied at the input. Observations obtained by simulation reveal [17-19] that the extrinsic input log-likelihood ratio can be modeled by a Gaussian distributed random variable with variance $\sigma_L^2 = 4/\sigma_n^2$ and mean

²In certain cases, F(D) and G(D) exhibit a common devisor so that the effective number of valid arrangements is slightly reduced.



Figure 3: EXIT characteristics of SBSD for various index assignments (upper part "optimized" [15, 16], lower part *natural binary*), quantizer codebook size (from left to right K = 3, 4, 5 bit/parameter) and correlation (in each subplot from bottom to top $\rho = \{0.0, 0.7, 0.8, 0.9, 0.95\}$)

 $\mu_L = \sigma_L^2/2 \cdot x_{\kappa,\tau}(\lambda)$. Since both are adjustable by a single variable σ_L^2 , for arbitrary σ_L^2 the *a priori* relation $\mathcal{I}^{[in]}$ can immediately be evaluated by numerical integration. Notice, the soft-input/soft-output channel decoder also depends on the channel-related input ratio $L(z_{\kappa,\tau}(\lambda) | x_{\kappa,\tau}(\lambda))$ which is mainly adjusted by the E_s/N_0 value (see Eq. (3)). Thus, the EXIT characteristics \mathcal{T} of soft-output decoders are (in general form) defined as [19]

$$\mathcal{I}^{[\text{ext}]} = \mathcal{T}\left(\mathcal{I}^{[\text{in}]}, E_s/N_0\right) \quad . \tag{11}$$

Of course, since *softbit source decoding* has no direct access to the channel-related *L*-values the EXIT characteristics \mathcal{T} of SBSD are (more or less) independent of E_s/N_0 . However, if defined settings for $\mathcal{I}^{[in]}$ resp. σ_L^2 (and for E_s/N_0) are adjusted, $\mathcal{I}^{[out]}$ is quantifiable by means of Monte-Carlo simulation.

The combination of EXIT characteristics of two soft-output decoders in a single diagram is referred to as EXIT-chart [19]. In this EXIT-chart representation both EXIT characteristics have to consider swapped axes, because the output of the one decoder serves as input for the other one, $\mathcal{I}^{[out]} \rightarrow \mathcal{I}^{[in]}$. The main contribution of EXIT-charts is that an analysis of the convergence behavior of an entire concatenated TURBO-scheme is realizable by solely studying the EXIT characteristics of the single components.

In literature, e.g. [17–19], the EXIT characteristics of soft-output channel decoders have already carefully been analyzed for various RSC and RNSC codes of different code rate r, constraint length J + 1, and generator polynomials **G**.

Thus, in the following we restrict ourselves to a detailed analysis of the EXIT characteristics of *softbit source decoding*. Some EXIT characteristics of SBSD have already been discussed by us in [10, 11], but for different configurations. In [10, 11] only these EXIT characteristics of SBSD are shown which are required to analyse the convergence of iterative source-channel decoding schemes with a *systematic* channel code. Due to the *systematic* form of the channel code the *softbit source decoding* part of ISCD schemes also gains access to the channel-related *L*-values.

In the following, we discuss some other EXIT characteristics of SBSD. These are in particular needed if SBSD is concatenated with a *non-systematic* channel code. Remember, in the latter case the EXIT characteristics of SBSD are independent from E_s/N_0 .

4.2. EXIT characteristics of Softbit Source Decoding

Figure 3 depicts some representative, E_s/N_0 -independent EXIT characteristics of SBSD. For simulation of the EXIT characteristics the source codec parameters $u_{\kappa,\tau}$ are modeled by a 1st order Gauss-Markov process with auto-correlation $\rho = \{0.0, 0.7, 0.8, 0.9, 0.95\}$ and quantized by a Lloyd-Max quantizer using K = 3, 4 or 5 bits/parameter. As index assignment serves either *natural binary* or that specific mapping which has particularly been *optimized*³ in view of ISCD [15, 16].

The three lower subplots show the simulation results for the *nat-ural binary* mapping and the upper three subplots for the *optimized* mapping [15, 16]. The number of bits used to encode each source

³The optimization maximizes the Euclidean distance between these quantizer reproduction levels which differ in a single bit position [15, 16].

codec parameter increases from the left subplots to the right. In each subplot every single curve represents one specific correlation ρ . The lowest curve depicts the EXIT characteristic for $\rho = 0.0$ and the highest curve is for $\rho = 0.95$.

In general, for all simulation results it can be stated that the *extrinsic* information at the output of the decoder takes the value $\mathcal{I}_{\text{SBSD}}^{[\text{out}]} = 0$ bit if the input relation $\mathcal{I}_{\text{SBSD}}^{[\text{in}]} = 0$ bit (resp. if $\sigma_L^2 = 0$). If the information content $\mathcal{I}_{\text{SBSD}}^{[\text{in]}}$ at the input (resp. σ_L^2) increases, the $\mathcal{I}_{\text{SBSD}}^{[\text{out}]}$ increases as well. In addition, the more correlation is available, the higher the increase of the *extrinsic* information $\mathcal{I}_{\text{SBSD}}^{[\text{out}]}$ at the output can be. However, for none of the simulation results the EXIT characteristic is able to reach entropy, e.g., in case of *natural binary* index assignment $\mathcal{I}_{\text{SBSD}}^{[\text{out}]} < 1$ bit. Notice, in Figure 3 even the input relation $\mathcal{I}_{\text{SBSD}}^{[\text{inf}]} < 1$ bit. The reason is that we limited the design parameter σ_L^2 to a reasonable value $\sigma_L^2 \ll \infty$.

The comparison of the EXIT characteristics of the *natural binary* bit mapping (lower subplots) and the *optimized* mapping [15, 16] (upper subplots) reveals that the latter index assignment exhibits higher *mutual information* $\mathcal{I}_{SBSD}^{[out]}$ at the output for a fixed but arbitrary configuration of bit length *K* and correlation ρ . Moreover, while for *natural binary* most of the EXIT characteristics of SBSD increase approximately linear with a relatively flat slope, the corresponding curves for the *optimized* mapping [15, 16] reveal a relatively steep gradient. The slope becomes even steeper if the input relation $\mathcal{I}_{SBSD}^{[in]}$ increases.

As we have already shown in [10, 11] for every parameter configuration of correlation ρ , quantizer codebook size 2^{K} , bit mapping, and look-ahead Λ the maximum *mutual information* value $\mathcal{I}_{\text{SBSD,max}}^{[out]}$ can also be quantified by means of analytical considerations. Table 1 summarizes the upper bounds for exemplary situations.

To simplify matters, no delay for a look-ahead is accepted, i.e., T + 1 = 1 and $\Lambda = \tau$. The theoretical upper bounds confirm the corresponding simulation results of Figure 3.

However, it has to be pointed out that the bounds $\mathcal{I}_{\text{SBSD,max}}^{[out]}$ do not comprise any information about the adverse effects of the different bit mappings on instrumental quality measures like the parameter **signal-to-noise ratio** (**SNR**). Thus, a higher $\mathcal{I}_{\text{SBSD,max}}^{[out]}$ does not guarantee a higher parameter SNR value as well.

5. SIMULATION RESULTS

The error concealing capabilities of the specific approach to iterative source-channel decoding proposed in this paper and the analysis of the convergence behavior shall be demonstrated by simulation. For the sake of reproducibility, instead of using any specific speech, audio, or video encoder we model the Mstatistically independent source codec parameters of a set by Mindependent 1st order recursive filters with filter coefficient ρ (auto-correlation). For simplicity we apply interleaving Φ on the present frame $\underline{\mathbf{x}}_{\tau}$ only and adjust the frame length to M = 500. Note, in practice a smaller frame length M might be used in conjunction with interleaving of several consecutive frames.

First Experiment

In a first experiment, the source codec parameters $u_{\kappa,\tau}$ exhibit an auto-correlation of $\rho = 0.95$. Such a value is typical, e.g., for the scale factors of audio transform codecs as applied to the *digital audio broadcasting* (DAB) system. Every $u_{\kappa,\tau}$ is scalarly quantized by a Lloyd-Max quantizer using K = 3 bit/parameter. The index assignment is realized by a *natural binary* mapping.

Taking the interleaver size of $K \cdot M = 1500$ bits into account, it is usually sufficient to apply a simple random interleaver in order to ensure independent reliability gains of both decoders.

The bit-interleaved frame $\tilde{\mathbf{x}}_{\tau}$ is channel encoded using a memory J = 6, terminated RNSC code with generator polynomial $\mathbf{G} = (\frac{1}{1+D+D^2+D^3+D^4+D^6})$. With respect to the J = 6 terminating bits, the effective code rate amounts to $r = 1500/1506 \approx 0.996$. As transmission channel serves AWGN with known E_s/N_0 .

The *parameter SNR*, e.g. [4], between the originally generated source codec parameter $u_{\kappa,\tau}$ and the reconstructed estimates $\hat{u}_{\kappa,\tau}$ is used for quality evaluation. The related simulation results are shown in the left part of Figure 4.

The dashed curve depicts the result for *conventional* SBSD without the *smearing filter* according to [4]. Due to the statistical independence of the original source codec parameters $u_{\kappa,\tau}$, $u_{\check{\kappa},\tau}$ with $\kappa, \check{\kappa} \in [1, M], \kappa \neq \check{\kappa}$, the estimation of a particular $\hat{u}_{\kappa,\tau}$ exhibiting a fixed but arbitrary position κ in the frame $\hat{\underline{u}}_{\tau}$ is based on past and present observations for that specific position κ only. The observations for the other M - 1 positions remain unexploited.

The solid curves show the simulation results if the proposed *smearing filter* is applied at the transmitter and if the resulting explicit artificial dependency and the implicit residual redundancy of the source codec parameters are evaluated iteratively at the receiver. The BCJR-channel decoder serves as component decoder for the RNSC code. If both decoding steps are executed only once, the performance of the proposed scheme is always inferior to the conventional approach due to channel encoding of the non-

Table 1: Theoretical bounds on $I_{\text{SBSD,max}}^{[\text{out}]}$ (in bits) of *softbit source decoding* [10, 11] w.r.t. correlation and index assignments: *natural binary, folded binary, Gray encoded*, and *optimized* [15, 16]; K = 3, 4 and 5 bit/parameter

		Auto-Correlation Property ρ				
		ho = 0.0	ho = 0.7	ho = 0.8	ho = 0.9	ho = 0.95
	natural binary	0.123	0.330	0.429	0.577	0.684
	folded binary	0.036	0.213	0.293	0.430	0.551
3 bit	Gray encoded	0.054	0.226	0.299	0.415	0.515
	optimized [15, 16]	0.111	0.465	0.588	0.732	0.817
	natural binary	0.127	0.298	0.380	0.507	0.612
	folded binary	0.043	0.190	0.260	0.388	0.517
4 bit	Gray encoded	0.068	0.208	0.270	0.374	0.473
	optimized [15, 16]	0.201	0.529	0.649	0.785	0.857
	natural binary	0.118	0.259	0.326	0.430	0.532
	folded binary	0.044	0.165	0.225	0.335	0.471
5 bit	Gray encoded	0.069	0.183	0.234	0.323	0.421
	optimized [15, 16]	0.207	0.574	0.691	0.808	0.864



Figure 4: Simulation Results (Left: Parameter SNR; Right: EXIT-chart and Decoding Trajectory at $E_s/N_0 = -1$ dB)

systematic form (see Section 3). But, if the reliability gain of SBSD is fed back to the BCJR-channel decoder and if a second iteration is carried out, the reference *conventional* SBSD is outperformed. Up to 5 iterations reveal significant quality gains in the most interesting range of channel conditions.

The convergence after approximately 5 iterations can be confirmed by an EXIT-chart analysis [10, 11, 19]. The right hand side of Figure 4 shows the corresponding EXIT-chart and a *decoding trajectory* at a channel quality of $E_s/N_0 = -1$ dB. The EXIT characteristic of SBSD is taken from Figure 3 (lower subplot on the left), but with swapped axes. The *decoding trajectory* denotes the step curve which visualizes the increase in *mutual information* gainable in the single iterations.

Decoding starts with the BCJR-decoder while the *a priori* knowledge amounts $\mathcal{I}_{\text{BCJR}}^{[\text{in}]} = 0$ bit. Due to the reliability gain of softoutput decoding, the decoder is able to provide $\mathcal{I}_{\text{BCJR}}^{[\text{out}]} = 0.09$ bit. The new information serves as *a priori* knowledge for SBSD,

The new information serves as *a priori* knowledge for SBSD, i.e., $\mathcal{I}_{BCIR}^{[out]} \rightarrow \mathcal{I}_{SBSD}^{[in]}$, and thus its *extrinsic mutual information* is determinable $\mathcal{I}_{SBSD}^{[out]} = 0.12$ bit. The instantaneous value of the *decoding trajectory* is labeled "Iteration 1". Iteratively processing of both decoding steps allows to increase the information content step-by-step.

No further information is gainable, when an intersection in the enveloping EXIT characteristics is reached. In ISCD schemes intersections typically appear due to the upper bound $\mathcal{I}_{SBSD,max}^{[out]}$. With respect to the flatness of SBSD's EXIT characteristic such an intersection is reached quite close after "Iteration 5".

Notice, sometimes the *decoding trajectory* exceeds the EXIT characteristic of the RNSC code. The reason is that the probability distribution of the *extrinsic* output *L*-values $L_{\text{SBSD}}^{[\text{ext}]}(x_{\kappa,\tau}(\lambda))$ of SBSD is usually non-Gaussian, in particular, if SBSD gains no direct access to the channel-related reliability information. Thus, the model which is used to determine the EXIT characteristic of the RNSC code (see Section 4.1) does not hold strictly anymore. However, even if the EXIT characteristic of the RNSC code is not best suited to describe the envelope for the attainable region of the *decoding trajectory*, the EXIT-chart is still appropriate to analyze the convergence behavior.

If in a practical application a baseline *parameter SNR* of, e.g., 13.5 dB is accepted to provide a tolerable reconstruction quality (see "Design Constraint" in Figure 4), the higher robustness permits to decrease the lower limit for an acceptable performance from $E_s/N_0 = 3.06$ dB (conventional SBSD) down to $E_s/N_0 = 0.27$ dB (after 5 iterations). The reference remains superior if E_s/N_0 drops below -1.01 dB.

Second Experiment

The results shall be confirmed by a second experiment. For this purpose, the source codec parameters $u_{\kappa,\tau}$ are assumed to exhibit less correlation $\rho = 0.9$. The resolution of the Lloyd-Max quantizer is increased to K = 4 bit/parameter and the *optimized* bit mapping [15, 16] is applied for index assignment.

Due to the higher quantizer resolution a larger interleaver Φ of $K \cdot M = 2000$ is needed. Again, we use a random interleaver design to ensure independence of both *extrinsic* reliability gains.

Channel encoding is realized by a memory J = 5, terminated RNSC code with generator polynomial $\mathbf{G} = (\frac{1}{1+D+D^2+D^3+D^5})$. The effective coderate amounts to $r = 2000/2005 \approx 0.9975$.

Figure 5 shows the simulation results. Again, the *conventional* SBSD approach [4] without *smearing filter* can be outperformed in the most interesting range of channel conditions (see left part of Figure 5). After only 2 iterations the new TURBO *error concealment* scheme becomes superior. Up to 9 iterations make further substantial quality improvements possible.

If a baseline *parameter SNR* of 18.5 dB provides a tolerable reconstruction quality the range of acceptable performance is extended from $E_s/N_0 = 5.1$ dB (conventional SBSD) down to $E_s/N_0 = 1.35$ dB (after 9 iterations). Conventional SBSD remains best if $E_s/N_0 < 0.55$ dB.

The convergence after approximately 9 iterations can be confirmed by the EXIT-chart analysis which is shown in the right part of Figure 5. With respect to the applied parameter configuration of correlation ρ , quantizer resolution 2^{K} , and bit mapping, the EXIT characteristic of *softbit source decoding* has to be taken from the center subplot in the upper row of Figure 3. After 9 iterations the *decoding trajectory* reaches the intersection in the EXIT characteristics quite close.



Figure 5: Simulation Results (Left: Parameter SNR; Right: EXIT-chart and Decoding Trajectory at $E_s/N_0 = 1$ dB)

6. CONCLUSIONS

In this paper, error concealment by *softbit source decoding* has been improved in the most interesting range of channel conditions. At the transmitter a modulo-2 *smearing filter* is used which adds artificial dependencies to sets of statistically independent source codec parameters. The encoding scheme exhibits a code rate (very close to) r = 1 and is especially designed to assist SBSD. At the receiver an iterative, TURBO-like utilization of this artificial dependency and of the residual redundancy of the source symbols permits step-wise robustness gains in several iterations. The convergence behavior has been confirmed by an EXIT-chart analysis. For this purpose, new EXIT characteristics of SBSD have been measured which are required if *softbit source decoding* is concatenated with a *non-systematic* channel code.

7. ACKNOWLEDGEMENTS

The authors want to thank Ulrich von Agris for many fruitful comments and inspiring discussions and Norbert Görtz for providing the *optimized* bit mappings [15, 16].

REFERENCES

- T. Hindelang, S. Heinen, P. Vary, and J. Hagenauer, "Two Approaches to Combined Source-Channel Coding: A Scientific Competition in Estimating Correlated Parameters," *International Journal of Electronics and Communications (AEÜ)*, vol. 54, Dec. 2000.
- [2] T. Fingscheidt, T. Hindelang, R. Cox, and N. Seshadri, "Joint Source-Channel (De-) Coding for Mobile Communication," *IEEE Trans. on Comm.*, vol. 50, pp. 200–212, Feb. 2002.
- [3] J. Hagenauer, 'Source-Controlled Channel Decoding," *IEEE Trans.* on Communications, pp. 2449–2457, Sept. 1995.
- [4] T. Fingscheidt and P. Vary, 'Softbit Speech Decoding: A New Approach to Error Concealment," in IEEE Trans. on Speech and Audio Processing, pp. 240–251, Mar. 2001.
- [5] S. Heinen, "An Optimal MMSE-Estimator For Source Codec Parameters Using Intra-Frame and Inter-Frame Correlation," in *Proc.* of *ISIT*, (Washington, USA), 2001.
- [6] C. Berrou and A. Glavieux, 'Near Optimum Error Correcting Coding and Decoding: Turbo-Codes," *IEEE Trans. on Communications*, vol. 44, pp. 1261–1271, 1996.

- [7] J. Hagenauer, E. Offer, and L. Papke, 'Iterative Decoding of Binary Block and Convolutional Codes,' *IEEE Trans. on Information The*ory, pp. 429–445, Mar. 1996.
- [8] M. Adrat, P. Vary, and J. Spittka, 'Iterative Source-Channel Decoder Using Extrinsic Information from Softbit-Source Decoding," in *Proc.* of *ICASSP*, (Salt Lake City, Utah, USA), May 2001.
- [9] M. Adrat, J.-M. Picard, and P. Vary, "Analysis of Extrinsic Information from Softbit-Source Decoding Applicable to Iterative Source-Channel Decoding," in *Proc. of the 4th Intern. ITG Conf. on Source* and Channel Coding, (Berlin, Germany), pp. 239–244, Jan. 2002.
- [10] M. Adrat, U. von Agris, and P. Vary, "Convergence Behavior of Iterative Source-Channel Decoding," in *Proc. of IEEE ICASSP*, (Hongkong, China), Apr. 2003.
- [11] M. Adrat, Iterative Source-Channel Decoding for Digital Mobile Communications. PhD thesis, Aachener Beiträge zu Digitalen Nachrichtensystemen (ed. P. Vary), ISBN 3-86073-835-6, RWTH Aachen, 2003.
- [12] N. Görtz, "Analysis and Performance of Iterative Source-Channel Decoding," in *Proc. of 2nd Int. Symp. on TURBO Codes & Related Topics*, (Brest, France), pp. 251–254, 2000.
- [13] N. Görtz, 'On the Iterative Approximation of Optimal Joint Source-Channel Decoding," *IEEE Journal on Selected Areas in Communications*, vol. 19, pp. 1662–1670, Sept. 2001.
- [14] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, 'Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate," *IEEE Trans. on Inform. Theory*, pp. 284–287, Mar. 1974.
- [15] J. Hagenauer and N. Görtz, "The Turbo Principle in Joint Source-Channel Coding," in *Proc. of IEEE ITW 2003*, (Paris, France), pp. 275–278, Mar. 2003.
- [16] N. Görtz, 'Optimization of Bit Mappings for Iterative Source-Channel Coding," in *Proc. of International Symposium on Turbo Codes and Related Topics*, (Brest, France), pp. 255–258, Sept. 2003.
- [17] S. ten Brink, 'Convergence of Iterative Decoding," *Electronics Letters*, vol. 35, pp. 806–808, May 1999.
- [18] S. ten Brink, 'Designing Iterative Decoding Schemes with the Extrinsic Information Transfer Chart," *International Journal of Electronics* and Communications (AEÜ), vol. 54, pp. 389–398, Dec. 2000.
- [19] S. ten Brink, 'Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes," *IEEE Trans. on Communications*, vol. 49, pp. 1727–1737, Oct. 2001.