Minimum Terms of Residual Redundancy for Successful Iterative Source-Channel Decoding

Marc Adrat, Thorsten Clevorn, Johannes Brauers, and Peter Vary

Abstract— **Iterative source-channel decoding (ISCD) improves the error robustness of a digital communication system by iteratively evaluating natural residual source redundancy and artificial channel coding redundancy in a TURBO-like process. Based on recent results to *extrinsic information transfer* (EXIT) charts we present a novel (experimental) approach to quantify the minimum terms of residual redundancy which are needed for (almost) successful ISCD. Moreover, we clarify why in certain situations the *decoding trajectory* exceeds the EXIT-characteristic of *soft decision source decoding* (SDSD) in an ISCD scheme.**

Index Terms— **Iterative Source-Channel Decoding (ISCD), EXIT chart, Soft Decision Source Decoding (SDSD).**

I. Introduction

Modern speech, audio, and video encoders extract a set of source codec parameters from the original source signal. For instance, most state-of-the-art speech codecs are based on the *code excited linear prediction* (CELP) principle where the set of source codec parameters contains the spectral coefficients, the indices and gain factors of the adaptive as well as the fixed codebooks. Usually, due to complexity limitations in the encoding process and due to design constraints, like the maximum tolerable signal delay, these source codec parameters exhibit considerable amounts of residual redundancy. Such natural residual source redundancy might appear in terms of a non-uniform parameter distribution or in terms of mutual (time) dependencies among consecutive realizations.

Concepts how to benefit from residual redundancy in a digital communication system in order to increase the error robustness can be found (either stand-alone) for channel decoding and for source decoding, or (jointly) for both. The most famous approaches are known as *source-controlled channel decoding* (SCCD) [1] and as *softbit* or *soft decision source-decoding* (SDSD) [2]. The advantages of both concepts can jointly be exploited by *iterative source-channel decoding* (ISCD) [3]–[6]. In ISCD the natural residual source redundancy and artificial channel coding redundancy are evaluated iteratively in a TURBO-like process [7] (see Fig. 1). In order to analyze the convergence behavior of TURBO-processes, *extrinsic information transfer* (EXIT) charts have been introduced [8].

In this letter, we take up recent results to EXIT-charts to state a necessary, but not sufficient condition which can ensure an (almost) error-free speech, audio, or video transmission if using *iterative source-channel decoding*. This condition allows to quantify the minimum terms of residual redundancy which are required to apply ISCD successfully. Last but not least, we analyze why in certain situations the *decoding trajectory* exceeds the EXIT-characteristic of SDSD.

Manuscript received April 10, 2006. The associate editor coordinating the review of this letter and approving it for publication was Dr. Stefano Buzzi. This work was supported by the DFG (Deutsche Forschungsgemeinschaft).

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Digital Object Identifier 10.1109/LCOMM.2006.060544.

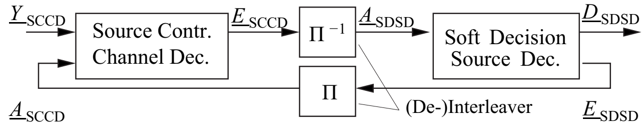

Fig. 1. Exemplary iterative source-channel decoding (ISCD) scheme.

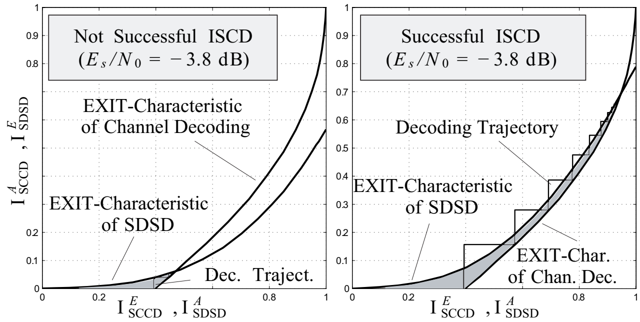

Fig. 2. Exemplary EXIT-charts and decoding trajectories for ISCD.

II. EXIT-Chart and Decoding Trajectory

In an EXIT-chart each component decoder of the TURBO-process is represented by an EXIT-characteristic which depicts the transfer of *extrinsic information* from the *a priori* input $\underline{A}$ of the decoder to its *extrinsic* output $\underline{E}$ [8]–[10] (see Fig. 1). For this purpose, the bitwise mutual information $\mathcal{I}^A = \mathcal{I}(V^j; A^j)$ between the originally sent bits V^j, $j = 1, \dots N$ (with N being the number of bits spent to encode a source codec parameter), and the *a priori* information A^j, with $\underline{A} = A^1, \dots A^N$, is evaluated (resp. $\mathcal{I}^E = \mathcal{I}(V^j; E^j)$ in case of the *extrinsic* output $\underline{E}$). If the TURBO-process consists of two components, the two respective EXIT-characteristics are plotted into a common EXIT-chart regarding swapped axis (because the *extrinsic* output of the one decoder serves as *a priori* input for the other one, e.g. $\underline{A}_{SCCD} \leftrightarrow \underline{E}_{SDSD}$, and vice versa). Both EXIT-characteristics determine the bounds for the so-called *decoding trajectory* [8]. The *decoding trajectory* is a step-curve which visualizes the increase of *extrinsic information* of both decoder components within the iterations.

Fig. 2 shows two exemplary EXIT-charts and *decoding trajectories* if these are applied to the *iterative source-channel* *decoding* problem [5], [6], [10]–[12]. In both sub-plots the EXIT-characteristic of the same terminated rate $r = 1/2$ *recursive non-systematic convolutional* (RNSC) code with generator polynomial $\mathbf{G}(13/17, 15/17)_8$ is shown. Note, the EXIT-characteristic of the channel code depends on the transmission channel quality E_s/N_0 due to the channel-related input $\underline{Y}_{\text{SCCD}}$. Therein, E_s denotes the energy per BPSK modulated (*binary phase shift keying*) channel encoded bit and $N_0/2$ determines the power spectral density of the AWGN (*additive white Gaussian noise*) transmission channel.

Moreover, each sub-plot shows an EXIT-characteristic of a *soft decision source decoder*. In both cases, we considered source codec parameters which are encoded using an $Q = 16$ level Lloyd-Max quantizer and an *EXIT-optimized* bit mapping to $N = 4$ bit/level [6]. In the left case we assumed that the source codec parameters exhibit residual redundancy in terms of a Gaussian distribution (with zero mean and variance $\sigma^2_{\underline{V}} = 1$) and auto-correlation $\rho = 0.7$. In the right case, auto-correlation is assumed to be $\rho = 0.85$. Obviously, the intersection of the EXIT-characteristics, which marks a limiting factor for the overall error correcting capability of the TURBO-process, is at different $(\mathcal{I}^E_{\text{SCCD}}, \mathcal{I}^E_{\text{SDSD}})$ values. The higher these values are, the higher the error correcting capability can be. Thus, we will call the right sub-plot as (an almost) successful approach to ISCD because there exists a small tunnel through which the *decoding trajectory* can pass to (relatively) high $(\mathcal{I}^E_{\text{SCCD}}, \mathcal{I}^E_{\text{SDSD}})$ values. However, absolute success, i.e., $(\mathcal{I}^E_{\text{SCCD}}, \mathcal{I}^E_{\text{SDSD}}) = (1, 1)$ can not be obtained by ISCD because the EXIT-characteristic of SDSD is always upper bounded to values $\mathcal{I}^E_{\text{SDSD,max}} < 1$. The theoretical bounds for different settings of quantization concepts, bit mapping strategies, and terms of auto-correlation have been derived in [5], [6], [10], [11].

Two questions arise from the exemplary EXIT-charts and *decoding trajectories* depicted in Fig. 2:

- Firstly, how much residual redundancy in terms of auto-correlation is needed to allow (almost) successful ISCD?
- Secondly, why does in some situations the *decoding trajectory* exceed the EXIT-characteristic of SDSD?

The answers will be given in the following.

III. Minimum Terms of Residual Redundancy

Motivated by A. Ashikhmin's analysis of universal properties of EXIT-charts [9] some additional, specific properties have been derived for the EXIT-characteristic of the *soft decision source decoding* component in an ISCD scheme in [10]. One key observation of [10] was that the area under the EXIT-characteristic of SDSD grows almost linearly with the potential for data rate reduction ΔN. The data rate [10]

$$\Delta N = -\frac{1}{2}\log_2\left(1 - \rho^2\right) \quad (1)$$

can basically be saved if more efforts are spent to eliminate all terms of auto-correlation ρ in the source codec parameter flow, e.g, by linear prediction.

Figure 3 shows the EXIT-characteristics of SDSD for different values $N \in \{3, 4, 5\}$ of bits/level and various values $\rho \in \{0.0, 0.7, 0.8, 0.85, 0.9, 0.95\}$ of auto-correlation. In all cases we used the EXIT-optimized bit mappings as proposed in [6]. These mappings are designed such that they provide a higher maximum value for $\mathcal{I}^E_{\text{SDSD,max}}$ (at the right border of the EXIT-chart) than every other possible bit mapping. Note, the EXIT-characteristics for $N = 4$ bit/level and $\rho = 0.7$ resp. $\rho = 0.85$ correspond to the curves plotted in Fig. 2.

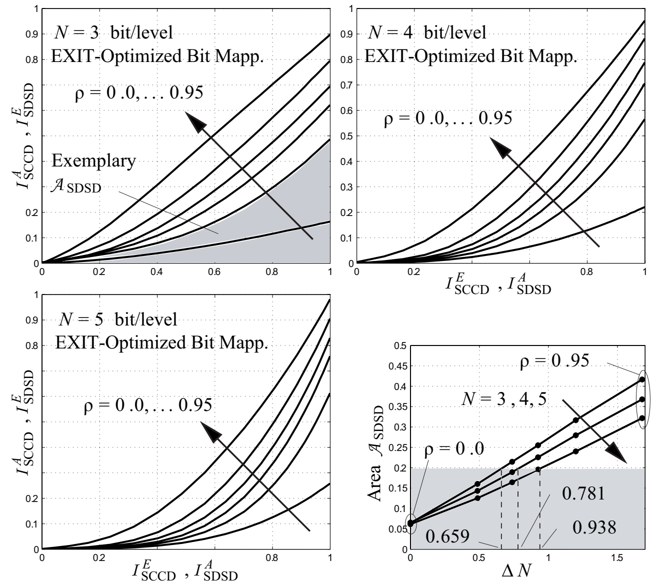

Fig. 3. EXIT-characteristics of SDSD for $\rho \in \{0.0, 0.7, 0.8, 0.85, 0.9, 0.95\}$ and $N = 3, 4, 5$ bit/level; areas under these EXIT-characteristics.

The sub-plot down right shows the linear dependencies on ΔN of the areas $\mathcal{A}_{\text{SDSD}}$ under the EXIT-characteristics of SDSD for the different settings of N and ρ.

> A necessary, but not sufficient condition to ensure (almost) error-free *iterative source-channel decoding* is that the area under the EXIT-characteristic of SDSD is greater than the area under the swapped EXIT-characteristic of SCCD, i.e., $\mathcal{A}_{\text{SDSD}} > 1 - \mathcal{A}_{\text{SCCD}}$ (see Fig. 2).

In other words, if the area $1 - \mathcal{A}_{\text{SCCD}}$ is greater than $\mathcal{A}_{\text{SDSD}}$ then ISCD can never be successful. Thus, if a specific channel coding scheme and a specific channel quality are given, e.g., the rate $r = 1/2$ RNSC code with $\mathbf{G}(13/17, 15/17)_8$ at an $E_s/N_0 = -3.8$ dB (see Fig. 2) then we can determine the area $1 - \mathcal{A}_{\text{SCCD}} = 0.198$. As a consequence, for successful ISCD EXIT-characteristics of SDSD are required which exhibit an area $\mathcal{A}_{\text{SDSD}} > 0.198$. The unsuitable region is marked gray in the sub-plot down right in Fig. 3. The minimum residual redundancy in terms of auto-correlation can be determined to $\rho_{\text{min}} = 0.77$ (resp. $\Delta N = 0.659$) for $N = 3$ bit/level, to $\rho_{\text{min}} = 0.81$ (resp. $\Delta N = 0.781$) for $N = 4$ bit/level, and to $\rho_{\text{min}} = 0.85$ (resp. $\Delta N = 0.938$) for $N = 5$ bit/level.

Figure 4 shows the simulation results for the error correcting capabilities. As quality criterion serves the *parameter signal-to-noise ratio* (SNR) between the originally determined source codec parameters and their reconstructed versions after ISCD. For this purpose, the set of originally determined source codec parameters is modeled by $M = 500$ independent 1st-order Gauss-Markov processes with filter coefficient ρ. Thus, each source codec parameter exhibits auto-correlation ρ, but there is no cross-correlation between different parameters. Every set of M codec parameters is individually channel encoded. The

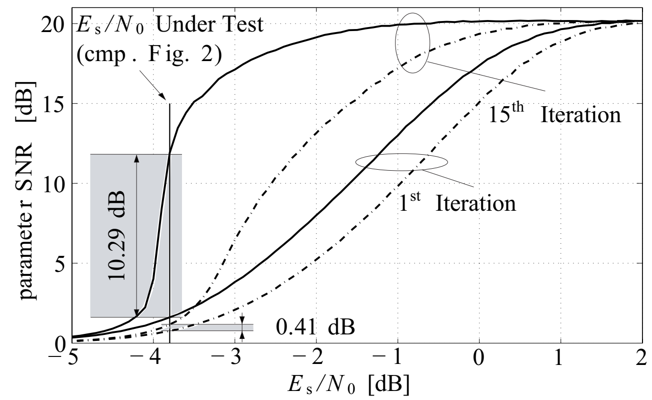

Fig. 4. Simulation results for $\rho = 0.7$ (dash-dotted) and $\rho = 0.85$ (solid).

remaining simulation settings for source and channel coding correspond to the settings of Fig. 2. For details to the ISCD algorithm we refer the reader to the literature, e.g. [3]–[6].

The dashed-dotted curves are the simulation results for $\rho = 0.7$ and the solid curves for $\rho = 0.85$. In both cases, significant improvements can be obtained by ISCD (lower curve: 1st iteration, upper curve: 15^{th} iteration). However, at the channel condition under consideration, i.e., at $E_s/N_0 = -3.8$ dB, only the solid curves reveal noteworthy SNR improvements by the iterations. The reason is that $\rho = 0.85$ lies above the threshold $\rho_{\text{min}} = 0.81$. In contrast, an auto-correlation of $\rho = 0.7$ is by far not enough to make ISCD successful. Notice, the corresponding EXIT-charts are shown in Fig. 2.

IV. Decoding Trajectory Exceeds EXIT-Characteristics

Finally, it should be clarified why in certain situations the *decoding trajectory* exceeds the EXIT-characteristic of the *soft decision source decoder* (see right sub-plot of Fig. 2). The reason can be found in the determination rule for the *extrinsic information* of SDSD. The determination rule for a single bit $E^j_{\text{SDSD},\tau}$ at time τ (corresponds to set index) reads [3], [5], [6]

$$E^j_\tau = \log \frac{\sum\limits_{\underline{V}^{[j]}_\tau} \theta(\underline{V}^{[j]}_\tau) \sum\limits_{\underline{V}_{\tau-1}} P(\underline{V}^{[j]}_\tau | \underline{V}_{\tau-1}, V^j_\tau = +1)\alpha(\underline{V}_{\tau-1})}{\sum\limits_{\underline{V}^{[j]}_\tau} \theta(\underline{V}^{[j]}_\tau) \sum\limits_{\underline{V}_{\tau-1}} P(\underline{V}^{[j]}_\tau | \underline{V}_{\tau-1}, V^j_\tau = -1)\alpha(\underline{V}_{\tau-1})} \tag{2}$$

For convenience, we skipped the lower index "SDSD" of $E^j_{\text{SDSD},\tau}$. The upper index $[j]$ in squared brackets means that the bit under test V^j_τ is excluded from $\underline{V}_\tau$, i.e. $\underline{V}^{[j]}_\tau = \underline{V}_\tau \setminus V^j_\tau$. The outer summation in Eq. (2) has to be realized over all 2^{N-1} possible permutations of $\underline{V}^{[j]}_\tau$ of the present set τ and the inner summation runs over all 2^N permutations of $\underline{V}_{\tau-1}$ at time $\tau - 1$. The term $\theta(\underline{V}^{[j]}_\tau)$ determines the input information

$$\theta(\underline{V}^{[j]}_\tau) = \exp \sum_{i=1,\ldots N,\, i\neq j} \frac{V^i_\tau}{2} A^i_{SDSD,\tau} \quad . \tag{3}$$

The term $P(\underline{V}^{[j]}_\tau | \underline{V}_{\tau-1}, V^j_\tau)$ of Eq. (2) represents the residual redundancy. It specifies the probability for a sent pattern $\underline{V}_\tau$ at time τ given the preceding pattern $\underline{V}_{\tau-1}$ as well as a fixed value for the bit under test $V^j_\tau = \pm 1$. The reliability for the preceding $\underline{V}_{\tau-1}$ is taken into account by the rightmost term $\alpha(\underline{V}_{\tau-1})$. This term can be determined recursively over the time instants in order to include the entire history [3], [5], [6].

The precision of the term $\alpha(\underline{V}_{\tau-1})$ depends on the number of iterations which were carried out in the preceding time instants. Strictly speaking, this precision also depends on the capabilities of the other constituent decoder in the TURBO-scheme. To make the computation of the EXIT-characteristics of SDSD independent from the channel coding component it is common to assume, that only a single iteration will be done. However, if larger numbers of iterations are performed the precision of $\alpha(\underline{V}_{\tau-1})$ is higher and thus, more *extrinsic mutual information* $\mathcal{I}^E_{\text{SDSD}}$ is available. As the *decoding trajectory* in Fig. 2 was depicted after the 15^{th} iteration, it carries more *extrinsic mutual information* and thus, it exceeds the EXIT-characteristic of SDSD. We have verified by simulation that the *decoding trajectory* meets the EXIT-characteristic of SDSD if only a single iteration is carried out.

V. Conclusions

Based on the area properties of EXIT-charts we presented a novel (experimental) approach to quantify the minimum terms of residual redundancy being required to make ISCD successful. Moreover, we explained that due to the recursive determination rule for the *extrinsic information* of SDSD its EXIT-characteristic does not mark a tight upper bound for the *decoding trajectory*. The latter aspect facilitates that even (slightly) less residual redundancy can make ISCD successful.

References

[1] J. Hagenauer, "Source-controlled channel decoding," *IEEE Trans. Commun.*, vol. 43, no. 9, pp. 2449–2457, Sept. 1995.
[2] T. Fingscheidt and P. Vary, "Softbit speech decoding: a new approach to error concealment," *IEEE Trans. Speech Audio Proc.*, vol. 9, pp. 240–251, Mar. 2001.
[3] M. Adrat, P. Vary, and J. Spittka, "Iterative source-channel decoder using extrinsic information from softbit-source decoding," in *Proc. IEEE ICASSP 2001*, vol. 4, pp. 2653–2656.
[4] N. Görtz, "On the iterative approximation of optimal joint source-channel decoding," *IEEE J. Sel. Areas Commun.*, vol. 19, pp. 1662–1670, Sept. 2001.
[5] M. Adrat, "Iterative source-channel decoding for digital mobile communications," PhD thesis, Aachener Beiträge zu Digitalen Nachrichtensystemen (ed. P. Vary), ISBN 3-86073-835-6, RWTH Aachen, 2003.
[6] M. Adrat and P. Vary, "Iterative source-channel decoding: improved system design using EXIT charts," *EURASIP J. Applied Signal Processing*, special issue on turbo processing, vol. 6, pp. 928-941, May 2005.
[7] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: turbo-codes," *IEEE Trans. Commun.*, vol. 44, no. 10, pp. 1261-1271, Oct. 1996.
[8] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 40, pp. 1727–1737, Oct. 2001.
[9] A. Ashikhmin, G. Kramer, and S. ten Brink, "Extrinsic information transfer functions: model and erasure channel properties," *IEEE Trans. Inf. Theory*, vol. 50, pp. 2657–2673, Nov. 2004.
[10] M. Adrat, J. Brauers, T. Clevorn, and P. Vary, "The EXIT-characteristic of softbit-source decoders," *IEEE Commun. Lett.*, vol. 9, pp. 540–542, June 2005.
[11] M. Adrat, U. von Agris, and P. Vary, "Convergence behavior of iterative source-channel decoding" in *Proc. IEEE ICASSP 2003*, vol. 4, pp. 269–272.
[12] N. Görtz and A. Schaefer, "The use of EXIT charts in iterative source-channel decoding," in *Proc. EUSIPCO 2004*, pp. 1549–1552.