# EXIT-Optimized Quantizer Design for Iterative Source-Channel Decoding

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## Abstract

In digital mobile communications the input speech, audio, or video signal has to be quantized before transmission. In this paper, we propose a novel quantizer design strategy which is optimized for the case that *iterative source-channel decoding* (ISCD) is applied at the receiver site. The combination of the new quantizer with the ISCD scheme reveals a substantially increased error robustness in bad channel conditions at the cost of a tolerable loss of baseline performance in almost error-free situations. This trade-off can smoothly be adjusted.

## 1 Introduction

In every digital mobile speech, audio, or video communication system two kinds of noise in the reconstructed source signal after channel transmission are (more or less) inevitable. Firstly, there is always quantization noise, and secondly, there are often distortions due to transmission errors.

In early approaches to quantizer design error-free channel conditions are assumed and the quantizer codebooks are chosen such that the *minimum mean squared error* (MMSE) between the input source signal and its quantized representation is minimized [1]. More sophisticated techniques like N. Favardin's *channel optimized vector quantizers* (COVQ) [2] include the channel condition in the quantizer design. But, even COVQs are not the optimal solution for future transmission systems where at the receiver site reliability information will be exchanged between different decoding components.

For instance, in *iterative source-channel decoding*[1] (ISCD) [3–7] a derivative of *softbit source-decoding* [8] and a soft-input/soft-output channel decoder are concatenated according to the Turbo-principle [9, 10]. *Softbit source-decoding* exploits natural residual source redundancy and the channel decoder utilizes the artificial redundancy due to channel coding.

The convergence behavior of ISCD can be analyzed using EXIT-charts [5, 11–13]. Besides that, EXIT-charts often reveal new design guidelines to optimize Turbo-processes. For instance, the channel coding component and the index assignment of a transmission system with ISCD have been optimized in [7]. Both advancements reveal substantial improvements in the error robustness of the overall transmission link.

In this paper, we pick up the results of [7] and extend them to the EXIT-optimized quantizer (EOQ) design. The new EOQ will improve the error correcting capabilities of ISCD further.

## 2 Transmission System with ISCD

Figure 1 shows a block diagram of a transmission system with an *iterative source-channel decoder* at the receiving end.

### A) Source Encoding

Let us assume that we have a parametric source encoder that operates on a frame-by-frame basis. The $M$ source codec parameters $u_{\mu,\tau}$, $\mu = 1, \ldots, M$, determined at time $\tau$ are individually quantized to $\bar{u}_{\mu,\tau} \in \mathbb{U}_\mu$. The quantizer codebooks $\mathbb{U}_\mu$ of size $2^{K_\mu}$ are time invariant. $K_\mu$ quantifies the length of the bit pattern $\mathbf{x}_{\mu,\tau} = \Phi_\mu(\bar{u}_{\mu,\tau})$ after index assignment and bit mapping. A single data bit of $\mathbf{x}_{\mu,\tau}$ is denoted by $x^{(\kappa)}_{\mu,\tau}$ with $\kappa = 1, \ldots, K_\mu$. A term like $\mathbf{x}^{(\backslash k)}_{\mu,\tau}$ stands for a bit pattern

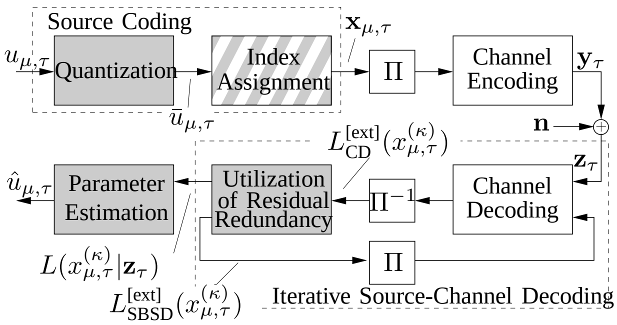


Fig. 1. Transmission System with *Iterative Source-Channel Decoding* (ISCD) at the receiving end

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[1]Note, in the literature the term *iterative source-channel decoding* is used twice. It is also used in connection with the decoding of *variable length codes*.

$\mathbf{x}_{\mu,\tau}$ excluding bit $x^{(k)}_{\mu,\tau}$, i.e., $\mathbf{x}^{(\backslash k)}_{\mu,\tau}$ is of size $K_\mu - 1$. The residual redundancy of the bit patterns $\mathbf{x}_{\mu,\tau}$ can be measured in terms of $P(\mathbf{x}_{\mu,\tau})$ resp. $P(\mathbf{x}_{\mu,\tau}|\mathbf{x}_{\mu,\tau-1})$. For convenience, we assume in the following that the codebooks and index assignments are the same for all $M$ source codec parameters, i.e. $\mathbb{U}_\mu = \mathbb{U}$, $K_\mu = K$ and $\Phi_\mu(\cdot) = \Phi(\cdot)$ for all $\mu = 1, \ldots, M$.

### B) Bit Interleaving

A bit interleaver $\Pi$ permutes the $M \cdot K$ data bits of a frame $\mathbf{x}_\tau = \mathbf{x}_{1,\tau}, \ldots, \mathbf{x}_{M,\tau}$ in a deterministic manner. In conventional transmission systems without ISCD bit interleaving is realized with respect to the subjective relevance of the individual bits in order to prepare the data frame $\mathbf{x}_\tau$ for *unequal error protection*. In contrast, in systems with ISCD bit interleaving has to be realized such that (at the receiver site) independent terms of *extrinsic* information can be extracted from source resp. channel decoding (see below).

### C) Channel Encoding and Modulation

Regarding channel decoding, we restrict our considerations to *recursive non-systematic convolutional* (RNSC) codes. In respect of ISCD the usage of these codes turned out to be beneficial [7]. If a terminated memory $J$ RNSC code with rate $r$ is used then the bit interleaved data frame $\mathbf{x}_\tau$ will be expanded to a code sequence $\mathbf{y}_\tau$ of size $1/r \cdot (M \cdot K + J)$. The single elements of $\mathbf{y}_\tau$ are transmitted by BPSK (*binary phase shift keying*) modulation over an AWGN channel (*additive white Gaussian noise*) with known $E_s/N_0$. The term $E_s$ denotes the energy of each BPSK-modulated element of $\mathbf{y}_\tau$ and $N_0/2$ specifies the power spectral density of the AWGN. After channel transmission the possibly noisy sequence $\mathbf{z}_\tau$ is received.

### D) Iterative Source-Channel Decoding

At the receiver, the aim of *iterative source-channel decoding* is to determine *a-posteriori* likelihood values (short: $L$-values) $L(x^{(\kappa)}_{\mu,\tau}|\mathbf{z}_\tau)$ for each data bit $x^{(\kappa)}_{\mu,\tau}$ given the noisy received sequence $\mathbf{z}_\tau$. For this purpose, at first the artificial redundancy which has been introduced by channel encoding is exploited to determine so-called *extrinsic* information $L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})$ of channel decoding (CD). The precise determination rule $\mathcal{F}_{\text{CD}}(\cdot)$ for

$$L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau}) = \mathcal{F}_{\text{CD}}\left(\mathbf{z}_\tau, L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau})\right) \tag{1}$$

given the received values $\mathbf{z}_\tau$ (and in higher numbers of iteration the *extrinsic* information $L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau})$ of *softbit source decoding* (SBSD)) is well-known and thus, we refer the reader to the literature, e.g., [9, 10].

The *extrinsic* information $L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})$ of channel decoding serves as input for the utilization of natural residual source redundancy. The rule $\mathcal{F}_{\text{SBSD}}(\cdot)$ how to combine these $L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})$ with the source statistics $P(\mathbf{x}_{\mu,\tau})$ resp. $P(\mathbf{x}_{\mu,\tau}|\mathbf{x}_{\mu,\tau-1})$ in order to quantify the *extrinsic* information

$$L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau}) = \mathcal{F}_{\text{SBSD}}\left(L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})\right) \tag{2}$$

of SBSD is also well-known and can be found in, e.g., [3–7].

The *extrinsic* information $L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau})$ is fed back to the channel decoder where it can serve as additional input knowledge to recalculate the $L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})$ values. Thus, the feedback allows refinements of $L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau})$ as well as $L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau})$ until the maximum number of permitted iterations is reached.

After the final iteration the *a-posteriori* $L$-value $L(x^{(\kappa)}_{\mu,\tau}|\mathbf{z}_\tau)$ can directly be calculated from both terms of *extrinsic* information. As we restrict our considerations to *non*-systematic channel codes the determination rule reads [3–7]

$$L(x^{(\kappa)}_{\mu,\tau}|\mathbf{z}_\tau) = L^{[\text{ext}]}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau}) + L^{[\text{ext}]}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau}) \quad . \tag{3}$$

### E) Parameter Estimation

Finally, the bitwise *a-posteriori* $L$-values are provided to parameter estimation where the individual source codec parameters $\hat{u}_{\mu,\tau}$ are reconstructed. For this purpose the bitwise $L(x^{(\kappa)}_{\mu,\tau}|\mathbf{z}_\tau)$ are transformed into *a-posteriori* reliability information $P(\bar{u}_{\mu,\tau}|\mathbf{z}_\tau)$ of complete bit patterns $\mathbf{x}_{\mu,\tau}$ resp. quantizer reproduction levels $\bar{u}_{\mu,\tau}$ (for details see e.g., [3–7]). Parameter reconstruction is carried out by *minimum mean squared error* (MMSE) estimation

$$\hat{u}_{\mu,\tau} = \sum_{\bar{u}_{\mu,\tau} \in \mathbb{U}} \bar{u}_{\mu,\tau} \cdot P(\bar{u}_{\mu,\tau}|\mathbf{z}_\tau) \quad . \tag{4}$$

## 3 EXIT-Optimized Quantization

Usually, quantization of the input source parameters $u_{\mu,\tau}$ is realized such that the mean squared error $\text{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$ is minimized [1]. As an alternative, we propose a new quantizer design guideline which increases the error correcting capabilities of *iterative source-channel decoding*. The improved error robustness in bad channels conditions is achieved at the cost of a slightly higher $\text{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$ and thus, a weaker baseline quality in almost error-free channel situations.

Notice, the new optimized quantizer codebook $\bar{u}^\star_{\mu,\tau} \in \mathbb{U}^\star$ (the $\star$ indicates the optimization) has a direct impact on all the blocks marked gray in Fig. 1. Besides quantization itself also parameter estimation according to Eq. (4) considers a different codebook $\mathbb{U}^\star$. In addition, the utilization of residual source redundancy is based on different statistics $P(\mathbf{x}^\star_{\mu,\tau})$ resp. $P(\mathbf{x}^\star_{\mu,\tau}|\mathbf{x}^\star_{\mu,\tau-1})$ with $\mathbf{x}^\star_{\mu,\tau} = \Phi(\bar{u}^\star_{\mu,\tau})$. Moreover, with respect to the latter relation and in view of the end-to-end distortion $\text{E}\{|u_{\mu,\tau} - \hat{u}_{\mu,\tau}|^2\}$ in many situations it is beneficial to re-optimize the index assignment $\Phi(\cdot)$ as well.

### A) EXIT-Charts

The optimization criterion for the new quantizer design guideline will be derived from the *extrinsic information transfer* (EXIT) charts [11]. Two exemplary EXIT-charts for ISCD systems are shown in Fig. 2.

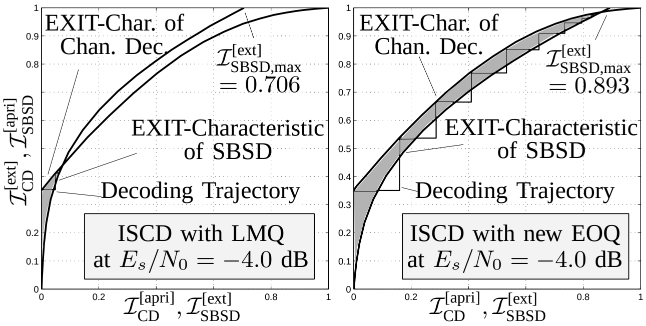


Fig. 2. Exemplary EXIT-Charts and Decoding Trajectories for ISCD (left: with conventional Lloyd-Max quantization (LMQ), right: with new EXIT-optimized quantization (EOQ), for simulation details see Section 4)

An EXIT-chart contains two EXIT-characteristics and a *decoding trajectory*. Each EXIT-characteristic depicts the mutual information $\mathcal{I}(\cdot;\cdot)$ between the originally sent data bit $x^{(\kappa)}_{\mu,\tau}$ and the corresponding $L$-values at the *a priori* input resp. *extrinsic* output of the Turbo-component. For ISCD we have

- Channel decoding:
$$\mathcal{I}^{\text{[apri]}}_{\text{CD}} = \mathcal{I}(x^{(\kappa)}_{\mu,\tau}; L^{\text{[ext]}}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau}))$$
$$\to \mathcal{I}^{\text{[ext]}}_{\text{CD}} = \mathcal{I}(x^{(\kappa)}_{\mu,\tau}; L^{\text{[ext]}}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau}))$$
- Softbit source decoding:
$$\mathcal{I}^{\text{[apri]}}_{\text{SBSD}} = \mathcal{I}(x^{(\kappa)}_{\mu,\tau}; L^{\text{[ext]}}_{\text{CD}}(x^{(\kappa)}_{\mu,\tau}))$$
$$\to \mathcal{I}^{\text{[ext]}}_{\text{SBSD}} = \mathcal{I}(x^{(\kappa)}_{\mu,\tau}; L^{\text{[ext]}}_{\text{SBSD}}(x^{(\kappa)}_{\mu,\tau})).$$

Notice, in case of channel decoding the mapping from $\mathcal{I}^{\text{[apri]}}_{\text{CD}}$ to $\mathcal{I}^{\text{[ext]}}_{\text{CD}}$ is based on the relation $\mathcal{F}_{\text{CD}}(\cdot)$ of Eq. (1). Thus, the resulting EXIT-characteristic depends on the channel quality $E_s/N_0$ due to the dependence on $\mathbf{z}_\tau$. In contrast, the EXIT-characteristic of SBSD is independent of $E_s/N_0$ (see Eq. (2)).

The step-curve in between the two EXIT-characteristics is called *decoding trajectory*. It visualizes the increase in reliability information throughout the iterations. If possible, this reliability information shall increase until the *decoding trajectory* reaches the upper right corner of the EXIT-chart. But, the error correcting capabilities of all *iterative source-channel decoding* processes are generally limited due to an inevitable intersection of the EXIT-characteristics. This intersection results from the fact that the EXIT-characteristic of SBSD is upper bounded by [14]

$$\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}} = \frac{1}{2^K} \sum_{k=1,\ldots,2^K} H(x^{(k)}_{\mu,\tau}) - H(x^{(k)}_{\mu,\tau}|\mathbf{x}^{(\backslash k)}_{\mu,\tau}, \mathbf{x}_{\mu,\tau-1}) \tag{5}$$

The (conditional) entropy measures $H(\cdot)$ and $H(\cdot|\cdot)$ can immediately be determined as the required statistics $P(x^{(k)}_{\mu,\tau})$ resp. $P(x^{(k)}_{\mu,\tau}|\mathbf{x}^{(\backslash k)}_{\mu,\tau}, \mathbf{x}_{\mu,\tau-1})$ can be measured from the parameter statistics $P(\mathbf{x}_{\mu,\tau})$ resp. $P(\mathbf{x}_{\mu,\tau}|\mathbf{x}_{\mu,\tau-1})$. Remember, $\mathbf{x}^{(\backslash k)}_{\mu,\tau}$ means bit pattern $\mathbf{x}_{\mu,\tau}$ excluding bit $x^{(k)}_{\mu,\tau}$.

Thus, in order to improve the error robustness of ISCD systems we define the following

**Design Criterion for the new EXIT-Optimized Quantizer (EOQ):**
The EXIT-optimized quantizer codebook $\bar{u}^\star_{\mu,\tau} \in \mathbb{U}^\star$ of a given size $2^K$ maximizes the *extrinsic* mutual information $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}}$. At the same time an upper bound $N_{Q,\text{max}}$ for the quantization noise $\text{E}\{|u_{\mu,\tau} - \bar{u}^\star_{\mu,\tau}|^2\}$ is preserved.

Notice, from an upper bound $N_{Q,\text{max}}$ for $\text{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$ roughly[2] follows a lower bound for the baseline reconstruction quality in terms of the parameter signal-to-noise ratio (SNR),

$$\text{parameter SNR} = 10 \cdot \log \frac{\text{E}\{|u_{\mu,\tau}|^2\}}{\text{E}\{|u_{\mu,\tau} - \hat{u}_{\mu,\tau}|^2\}} \quad , \tag{6}$$

in good channel conditions where the transmission channel can be considered as error-free, i.e. $\text{E}\{|\bar{u}_{\mu,\tau} - \hat{u}_{\mu,\tau}|^2\} \approx 0$.

### B) Design of an EXIT-Optimized Quantizer

Unfortunately, a closed form determination of the EXIT-optimized quantizer is an impractically complex task as many factors (i.e. all block marked gray in Fig. 1) have an impact on the overall optimization process, as besides the optimization of the quantizer codebook itself also the search for a proper index assignment needs to be incorporated in the optimization process. Anyhow, in order to keep the optimization manageable we propose a two-staged process. First, we optimize the index assignment for the initial quantizer codebook $\mathbb{U}$ according to [7], and afterwards, we search for the optimal $\mathbb{U}^\star$ using an iterative optimization algorithm. Having found the optimal quantizer codebook $\mathbb{U}^\star$ the index assignment can be re-optimized during an optional post-processing.

The iterative optimization algorithm for $\mathbb{U}^\star$ is based on the following observations and assumptions.

1) **Observations:** We have observed that the *extrinsic* mutual information $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}}$ attains higher values if the probability $P(\mathbf{x}_{\mu,\tau})$ of unlikely quantizer reproduction levels $\bar{u}_{\mu,\tau} \in \mathbb{U}$ is made even less probable. In turn, the probability $P(\mathbf{x}_{\mu,\tau})$ of likely quantizer reproduction levels $\bar{u}_{\mu,\tau} \in \mathbb{U}$ has to be made even more probable.
2) **Assumptions:** For the sake of simplicity, we assume that the original source codec parameters $u_{\mu,\tau}$ exhibit a symmetric probability density function which monotonically increases from the margins to the center.

[2] Notice, $\text{E}\{|u_{\mu,\tau} - \hat{u}_{\mu,\tau}|^2\} \approx \text{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\} + \text{E}\{|\bar{u}_{\mu,\tau} - \hat{u}_{\mu,\tau}|^2\}$. Equality holds for those quantizers which minimize $\text{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$ [15].

Hence, we propose the following iterative search algorithm for the EXIT-optimized quantizer codebook $\bar{u}^{\star}_{\mu,\tau} \in \mathbb{U}^{\star}$:

1) **Side Condition:** Define the maximum acceptable value $N_{Q,\text{max}}$ for the quantization noise $\mathrm{E}\{|u_{\mu,\tau} - \bar{u}^{\star}_{\mu,\tau}|^2\}$.
2) **Initialization:** Initialize the quantizer $\bar{u}_{\mu,\tau} \in \mathbb{U}$ as well as the index assignment $\Phi(\cdot)$ and determine the source statistics $P(\mathbf{x}_{\mu,\tau})$ resp. $P(\mathbf{x}_{\mu,\tau}|\mathbf{x}_{\mu,\tau-1})$. For instance, start with Lloyd-Max quantization [1] and the corresponding EXIT-optimized index assignment [7].
3) **Initial Values:** Compute the initial value $\mathcal{I}^{[\text{ext}]}_{\text{SBSD,max}}$ according to (5) and confirm that the initial quantization noise fulfills $\mathrm{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\} \leq N_{Q,\text{max}}$.
4) **Find EXIT-Optimized Quantizer Codebook:** In order to find the EXIT-optimized quantizer codebook $\mathbb{U}^{\star}$ the initial codebook $\mathbb{U}$ must be modified in a reasonable way. For this purpose, we propose the following approach as an exemplary solution. Notice, with respect to the observation, $\mathbb{U}$ is modified indirectly by varying the probability distribution $P(\mathbf{x}_{\mu,\tau})$:
   a) set quantizer interval counter to $q = 1$
   b) if $q \geq 2^{K-1}$ then stop optimization process
   c) initialize step-size to e.g. $\Delta P = 1/2^K$
   d) determine temporary $\tilde{P}(\mathbf{x}_{\mu,\tau}) = P(\mathbf{x}_{\mu,\tau}) - \Delta P$ for the $q$-th as well as $(2^K - q + 1)$-th interval (w.r.t. the symmetry, see assumption)
   e) if these $\tilde{P}(\mathbf{x}_{\mu,\tau}) < 0$ undo step 4-d and repeat step 4-d with a smaller step-size, e.g. $\Delta P = \Delta P/2$. Do the repetitions only as long as the step-size is above a given lower limit, e.g., $\Delta P > 10^{-9}$. If the step-size falls below this limit freeze the last valid realization of the $q$-th resp. $(2^K - q + 1)$-th interval, $\tilde{P}(\mathbf{x}_{\mu,\tau}) = \tilde{P}(\mathbf{x}_{\mu,\tau}) + \Delta P$, increase the interval counter $q = q + 1$ and return to step 4-b. Otherwise, if the temporary $\tilde{P}(\mathbf{x}_{\mu,\tau}) > 0$, proceed with step 4-f.
   f) add the spare $2\Delta P$ (see step 4-d) equally distributed to the $P(\mathbf{x}_{\mu,\tau})$ of the inner $2^K - 2q$ intervals, i.e., $\tilde{P}(\mathbf{x}_{\mu,\tau}) = P(\mathbf{x}_{\mu,\tau}) + \frac{2\Delta P}{2^K - 2q}$
   g) recompute the quantizer codebook $\bar{u}_{\mu,\tau} \in \mathbb{U}$ from $\tilde{P}(\mathbf{x}_{\mu,\tau})$. Determine $\tilde{P}(\mathbf{x}_{\mu,\tau}|\mathbf{x}_{\mu,\tau-1})$, $\mathcal{I}^{[\text{ext}]}_{\text{SBSD,max}}$, and $\mathrm{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$
   h) if $\mathrm{E}\{|u_{\mu,\tau} - \bar{u}_{\mu,\tau}|^2\}$ is lower than the maximum upper bound $N_{Q,\text{max}}$ check if $\mathcal{I}^{[\text{ext}]}_{\text{SBSD,max}}$ is greater than any other value measured before. If so, save the $\bar{u}_{\mu,\tau}$ as optimized quantizer codebook $\mathbb{U}^{\star}$. In any case, return to step 4-d.
5) **Optional Post-Processing:** After the new quantizer codebook $\mathbb{U}^{\star}_{\mu}$ is found it might be beneficial to re-optimize the index assignment as well. Moreover, the entire optimization process might be restarted with step 2 using the optimized codebook $\mathbb{U}^{\star}_{\mu}$ and the re-optimized index assignment as new initial setting.

# 4 Simulation Results

The benefits of the new EXIT-optimized quantizer shall be demonstrated by simulation. For this purpose, $M = 500$ source codec parameters $u_{\mu,\tau}$, $\mu = 1, \ldots, M$ are modeled by $M$ independent $1^{\text{st}}$-order Gauss-Markov processes with zero mean, unit variance, and filter coefficient $\rho = 0.8$. The filter coefficient determines the auto-correlation of $u_{\mu,\tau}$. Quantization is realized using a 16-level Lloyd-Max quantizer (LMQ) [1] resp. a 16-level EXIT-optimized quantizer (EOQ). The EOQ is determined as described in Section 3. The upper bound $N_{Q,\text{max}} = 0.014$ on the quantization noise is chosen such that a minimal parameter SNR of $\text{SNR}_{\text{min}} = 18.5$ dB is guaranteed in (almost) error-free channel conditions (see Eq. (6)). The codebooks $\mathbb{U}$ of the LMQ and $\mathbb{U}^{\star}$ of the EOQ are listed in Table I.

TABLE I
QUANTIZER CODEBOOKS $\mathbb{U}$ AND $\mathbb{U}^{\star}$ OF SIZE $2^4 = 16$

| | |
|---|---|
| LMQ | $\pm2.732590$, $\pm2.069017$, $\pm1.618046$, $\pm1.256231$, $\pm0.942341$, $\pm0.656759$, $\pm0.388048$, $\pm0.128395$ |
| EOQ | $\pm4.526537$, $\pm2.884547$, $\pm2.232524$, $\pm1.766491$, $\pm1.267845$, $\pm0.843296$, $\pm0.498190$, $\pm0.161962$ |

As index assignment serves in both cases the EXIT-optimized bit mapping [7] which has been determined for the LMQ (i.e. we skipped the optional post-processing). Bit interleaving is realized using a pseudo random interleaver of size $K \cdot M = 2000$. For channel encoding we used a terminated rate $r = 1/2$, memory 3 RNSC code with generator polynomial $G(13/17, 15/17)_8$ (see also [7]).

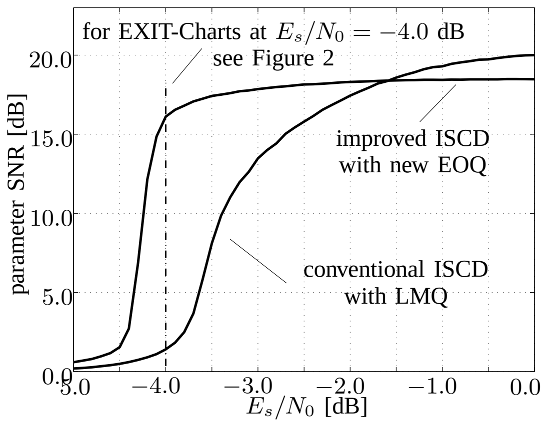


Fig. 3. Simulation Results

Figure 3 shows the parameter SNR (see Eq. (6)) as a function of the channel quality $E_s/N_0$. The simulation results have been obtained for 20 iterations. It can be seen that the conventional approach using a Lloyd-Max quantizer is superior in channel conditions $E_s/N_0 > -1.58$ dB. The parameter reconstruction quality is higher because in both approaches ISCD can correct most of the transmission errors and the parameter SNR is upper limited by the quantization noise. This quantization noise, however, is minimal in case of Lloyd-Max quantization. Nevertheless, the approach with the EOQ converges to the predefined parameter signal-to-noise ratio of $SNR = 18.5$ dB. Moreover, the new approach is able to outperform the reference approach in channel conditions $E_s/N_0 < -1.58$ dB. The baseline reconstruction quality can be retained without significant quality degradations down to $E_s/N_0 \approx -4.0$ dB. The new system with EOQ exhibits a substantially improved error robustness.

Notice, the corresponding EXIT-charts for both approaches at a channel condition of $E_s/N_0 = -4.0$ dB are shown in Fig. 2. The new system with EOQ obviously benefits from the fact that the *extrinsic* mutual information has been improved from $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}} = 0.706$ (LMQ) to $\mathcal{I}^{\text{[ext]}}_{\text{SBSD,max}} = 0.893$ (EOQ).

## 5 Conclusions

In this paper, we presented the design guideline for a new quantizer which improves the error correcting capabilities of *iterative source-channel decoding*. The design guideline is derived from the modern EXIT-chart tool. After an exemplary solution for finding an EXIT-optimized quantizer had been presented, the improved error robustness in heavily disturbed channel conditions was demonstrated by simulation. The loss of baseline reconstruction quality in (almost) error-free channel situations can smoothly be adjusted by the system designer.

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