

# ON HIERARCHICALLY MODULATED BICM-ID FOR RECEIVERS WITH DIFFERENT COMBINATIONS OF CODE RATE AND MODULATION ORDER

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## ABSTRACT

In this paper, hierarchically modulated *Bit Interleaved Coded Modulation with Iterative Decoding* (BICM-ID) is analyzed. At first, for a set of non-hierarchically modulated BICM-ID configurations it is shown that, while keeping the effective number of data bits per modulation symbol constant, the performance improves when the channel code rate decreases and the modulation order increases.

In a second step, these independently designed non-hierarchically modulated BICM-ID schemes are merged into a single hierarchically modulated one. This new scheme allows using a single common transmitter configuration in combination with different receiver configurations. For instance, even though at the transmitter an  $r=1/6$  code and 64-QAM modulation is applied, a low capable receiver can still use an  $r=1/2$  decoder and QPSK-demodulation. The *Bit Error Rate* (BER) performances of various approaches will be discussed in detail throughout this paper.

## 1. INTRODUCTION

The concept of *Bit Interleaved Coded Modulation with Iterative Decoding* (BICM-ID) has at first been introduced by X. Li and J.A. Ritcey in [1], [2]. Like in BICM (without iterative decoding), a BICM-ID system consists of a serial concatenation of a channel code, a bit interleaver and a digital modulation scheme. At the receiver, the received signal is consecutively demodulated, deinterleaved and decoded. As it has been shown in [1],[2], the performance of a standard BICM system (without iterative decoding) can greatly be improved by introducing a feedback line between the channel decoding and demodulation components. This feedback line allows passing back reliability information, so-called *extrinsic* information, from the decoder to the demodulator according to the well-known *Turbo* principle of digital signal processing [3]. Executing several iterations typically yields several incremental performance improvements, e.g., in terms of a gradually decreasing *Bit Error Rate* (BER). However, BICM-ID schemes need to be carefully designed to allow such performance gains. For

instance, it is well known that the choice of the symbol mapping (i.e. the labelling of signal constellation points) of the digital modulation scheme is a crucial design parameter. While it is the design goal of classical approaches for BICM (without iterative decoding) to minimize the number of differing bits in the symbol labels of neighboring signal constellation points (e.g. typically achieved by a *Gray* labelling), it is the opposite in BICM-ID. That means, the mean *Euclidean* distance between signal constellation points with a similar labelling (i.e. with a *Hamming Distance* of  $d_{\text{ham}}=1$ ) needs to be maximized. Powerful solutions exist already for standard digital modulation techniques.

However, many modern wireless communication systems exploit the benefits of *Hierarchical Modulation* [4][5] (aka *Layered Modulation*). Hierarchical modulation allows multiplexing different data streams in such a way that a common sequence of modulation symbols is used. The so-called *Base Layer* (BL) carries elementary information which enables a coarse reconstruction of the encoded source signal. All *Enhancement Layers* (EL) carry additional information which supports a stepwise refinement of the coarse signal to a high quality signal. The possibility to adapt the reconstructed signal quality makes hierarchical modulation very popular for e.g. digital broadcasting systems where the different receivers might have different constraints on computational resources and/or where the receivers might be subject to different channel conditions. Receivers with higher computational resources and under good channel conditions will be able to reconstruct a high fidelity signal while receivers with lower computational resources and/or under bad channel conditions can only reconstruct the elementary information of the base layer. Famous examples for applications of *Hierarchical Modulation* can be found in the areas of *Digital Video Broadcasting over Satellite* (DVB-S2) [6] or over *Terrestrial Antennas* (DVB-T2) [7].

In literature, digital communication systems utilizing the benefits of BICM-ID as well as systems exploiting the advantages of *Hierarchical Modulation* have typically been discussed separately. Applying BICM-ID to systems with hierarchical modulation has not extensively been studied yet. So far only the new key challenges as well as some first solutions in designing hierarchically modulated BICM-ID

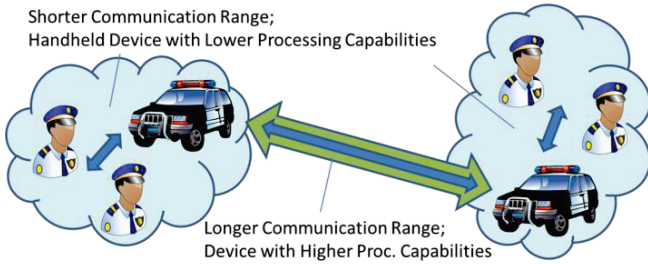
systems have recently been introduced in [8], [9]. Both these contributions aim at optimizing digital broadcasting systems in which the different layers of the hierarchical modulation scheme carry different user data allowing the reconstruction of source signals of different fidelity at the receiver.

In this paper, we will address another scenario in which the enhancement layers do not carry additional user data, but additional error protection information. This scenario as well as the new system design will be introduced in Section 2. In Section 3 we will at first analyze the behavior of BICM-ID schemes with different combinations of code rate and modulation order. In this section, all these schemes are individually optimized, but all have in common that the effective amount of user data per modulation symbol is identical. In Section 4 all these schemes will be merged by using the same hierarchical digital modulation scheme. The performance loss if compared to the individually designed schemes will be analyzed. Finally, in Section 5, we will demonstrate that these losses can be considerably reduced by relaxing some design constraints. In Section 6 we will summarize the key conclusions of our contribution.

## 2. HIERARCHICALLY MODULATED BICM-ID

### 2.1 Operational Scenario & System Design Objectives

In this paper, we propose to apply a hierarchically modulated BICM-ID scheme to a communication scheme whose operational scenario is illustrated in Fig. 1.



**Figure 1:** Operational Scenario for Hierarchical Modulated BICM-ID

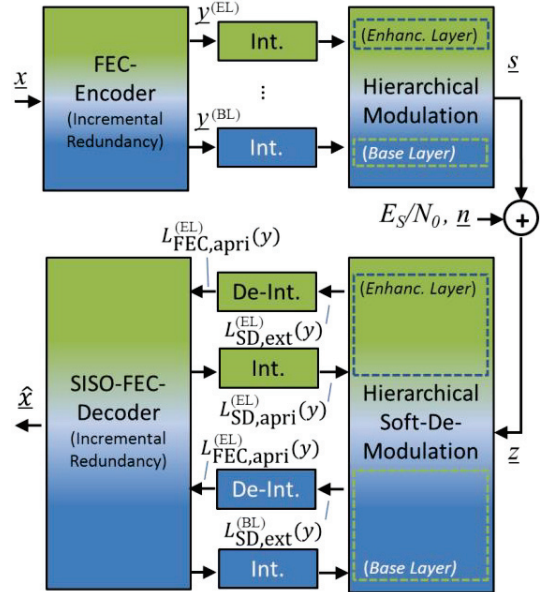
Similar to digital broadcasting systems, we are also looking at operational scenarios in which the radios of the users offer different capabilities and in which the communication links are subject to different amounts of channel noise. However, in our scenario the amount of user data (e.g., voice call) carried on all communication links shall be the same, but the amount of error protection needs to adapt to, e.g., the range.

Fig. 1 shows two groups of users. It is assumed that the group leader wants to communicate with all his group members as well as the other group leader at the same time. The group leaders have a radio with extended SWAP capabilities (*Size, Weight, and Power*) if compared to their group members (e.g. *vehicular mounted radio* instead of a *handheld* or *manpack*). Typically the group members are in close distance to their group leader while the distance between the two groups might be large.

With this operational scenario in mind, we have to design a communication system which allows carrying the same amount of user data (see *blue arrows* in Fig. 1) with different levels of error protection. The longer the range is, the more error protection is needed. However, it can be assumed, that the users at a larger distance have higher computational resources at hand making use of the additional error protection information (see *green arrow* enclosing the *blue arrow* in Fig. 1).

### 2.2 System Design

In order to design a communication system which fulfills the above mentioned objectives we propose a hierarchically modulated BICM-ID scheme which also exploits the concept of *Incremental Redundancy* [10],[11]. The *Base Layer* carries the user data including some basic error protection. The *Enhancement Layer(s)* carry additional error protection information which can be exploited by the radios with the higher processing capabilities. The corresponding block diagram is shown in Fig. 2.



**Figure 2:** Block Diagram for Hierarchical Modulated BICM-ID

Let a binary random source (e.g. a voice coder) provide a sequence  $\underline{x}$  of source data of length  $N$  (e.g., frame size representing a speech segment of 20 ms). This sequence  $\underline{x}$  is channel encoded by a rate  $r$  FEC-encoder (*Forward Error Correction*) which supports the concept of *Incremental Redundancy* [10],[11]. The resulting sequence  $\underline{y}$  of channel encoded bits can be split into the sub-sequence  $\underline{y}^{(BL)}$  with *Base Layer* information as well as sub-sequences  $\underline{y}^{(EL)}$  with *Enhancement Layer* information. To keep the illustration comprehensible we just show one *Enhancement Layer* in Figure 2, even though several *Enhancement Layers* might exist. The sub-sequences  $\underline{y}^{(BL)}$  and  $\underline{y}^{(EL)}$  are individually bit-interleaved and then processed by a hierarchical digital modulation scheme.

The sequence  $\underline{s}$  of modulation symbols is transmitted over an *Additive White Gaussian Noise* (AWGN) channel with known channel quality  $E_s/N_0$ . The term  $E_s$  determines the mean energy per modulated symbol  $s$  and  $N_0/2$  the single-sided noise spectral density of the AWGN.

At the receiving end, the noisy received sequence  $\underline{z}$  is demodulated. The hierarchical soft-demodulator provides soft-decision information, so-called *extrinsic information*, for every reconstructed channel encoded bit  $y$ . In iterative decoding, such soft-decision information is typically expressed in terms of so-called *Log-Likelihood Ratios* (LLRs, or *L-Values*) [3]. Thus, the hierarchical soft-demodulator provides  $L_{SD,ext}^{(BL)}(y)$  and  $L_{SD,ext}^{(EL)}(y)$  separately for both the *base layer* as well as the *Enhancement Layers*. After de-interleaving of these *L-values*, they serve as *a priori* information  $L_{FEC,apri}^{(BL)}(y)$  and  $L_{FEC,apri}^{(EL)}(y)$  for the SISO-FEC Decoder (*Soft-Input/Soft-Output*). The SISO-FEC Decoder exploits all this *a priori* information to determine an estimate  $\hat{x}$  for the original data sequence  $x$  as well as to determine the *extrinsic information*  $L_{FEC,ext}^{(BL)}(y)$  and  $L_{FEC,ext}^{(EL)}(y)$  which is fed back to the demodulator.

The comparison of the originally sent sequence  $x$  and its estimated reconstruction  $\hat{x}$  allows to determine the *Bit Error Rate* (BER) as a function of the channel quality  $E_s/N_0$ .

Please note, if compared to the hierarchically modulated BICM-ID scheme proposed in [8], [9], the key difference in the block diagram is on the left side of Fig. 2. In [8],[9] separate FEC-encoders for separate input data sequences  $x$  are used. Consequently, separate SISO-FEC-decoders are also applied at the receiver to reconstruct the sequences  $x$ .

### 3. PERFORMANCE OF A NON-HIERARCHICAL BICM-ID REFERENCE SYSTEM

Before demonstrating the performance of hierarchically modulated BICM-ID schemes in Section 4, it is of interest to also know the performance of classic non-hierarchical BICM-ID systems (i.e. *Base Layer* only, no *Enhancement Layers*). The performance of such systems will serve as reference for all subsequent considerations.

Let us consider an overall BICM-ID system design which offers three modes. All modes are designed in a way that the effective number of data bits per modulation symbol is identical. Table 1 summarizes the key system configuration settings for all three modes. All three modes ensure that effectively one data bit per modulation symbol is sent.

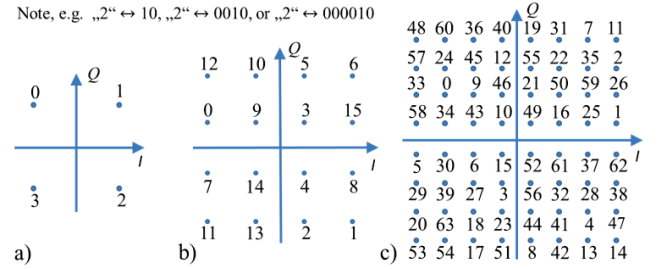
The length  $N$  of the input data sequences  $x$  is set to  $N=1000$ . Non-recursive convolutional codes of different code rate  $r$ , but with identical constraint length  $K=3$  are used [12],[13]. Termination is applied, i.e.  $K-1=2$  bits are appended to each input sequence  $x$ . Please note, due to the design of the generator polynomials  $G$ , the channel encoded output sequence  $\underline{y}$  of Mode A is included in the one of Mode B. Analogously, the sequence of Mode B is contained in the one

of Mode C. This satisfies already the basic prerequisites for incremental redundancy.

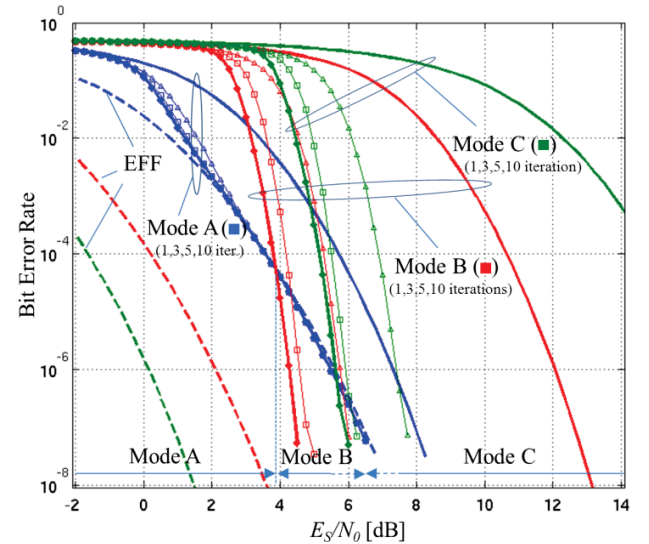
Before digital modulation, random bit-interleavers of size  $(N+K-1) / r$  are used to scramble the channel encoded bit sequence  $\underline{y}$ . The symbol mappings for the non-hierarchical BICM-ID scheme can be found in Figure 3.

**Table 1:** System configuration settings for the three Modes under consideration

Modes	Channel Code [12], [13]	Modulation Scheme
Mode A	$r=1/2$ with $G(5,7)$ , $d_{free}=5$	QPSK (see Figs. 3a, 5a, 8a)
Mode B	$r=1/4$ with $G(5,7,5,7)$ , $d_{free}=10$	16-QAM (see Figs. 3b, 5b)
Mode C	$r=1/6$ with $G(5,7,5,7,7,7)$ , $d_{free}=16$	64-QAM (see Figs. 3c, 5c, 8c)



**Figure 3:** Labelling of Signal Constellation Points in a Non-Hierarchical BICM-ID scheme



**Figure 4:** Bit Error Rate (BER) Performance of the Non-Hierarchical BICM-ID Modes

At the receiving end, BICM-ID is applied. Figure 4 shows the simulation results for all three Modes for different

numbers of iteration  $i=1,3,5,10$ . The ones for  $i=1$  and  $i=10$  are shown in bold. Here  $i=1$  means that no information is fed back from SISO-FEC-Decoding to Soft-Demodulation, i.e. all processing steps at the receiver are executed only once. In addition, the curves labelled EFF (*error-free feedback*) indicate the best possible theoretically attainable performance if error-free information is passed back on the feed-back line. For each mode, the corresponding curve can be considered as a bound.

From the simulation results it can be seen that for each mode BICM-ID can provide considerable gains in BER performance for a certain number of iterations. All BER curves are able to approach the EFF bound quite closely as soon as the so-called *waterfall region* is reached.

More importantly, it can also be seen that there are dedicated regions in which each Mode has its specific benefit. Mode A provides the lowest BER for  $E_s/N_0 < 3.87$  dB. At this  $E_s/N_0$ , the BER curve of Mode B with 10 iterations crosses the EFF curve of Mode A. It can be assumed from the BER curves that another intersection of Mode C with 10 iterations and the EFF curve of Mode B will exist in very low BER regions at  $E_s/N_0 \approx 5.90 \dots 6.30$  dB. Thus, Mode A has the lowest BER for  $E_s/N_0 < 3.87$  dB. Mode B outperforms the other Modes in between  $3.87 < E_s/N_0 < 5.90 \dots 6.30$  dB, and Mode C is best for  $E_s/N_0 > 5.90 \dots 6.30$  dB. Consequently, it makes sense to adapt the mode according to the channel quality. Of course, going from Mode A to Mode B and finally Mode C, also means that the computational complexity increases. Therefore, a receiver with better SWAP capabilities might be necessary.

Thus, as the first important result we can conclude that, if the channel quality becomes good enough, adding more error protection (note, a lower code rate  $r$  typically results in a higher free distance  $d_{free}$ , see Table 1) can over-compensate the loss induced by a more vulnerable modulation scheme (i.e. a modulation scheme of higher order). Of course, this is true if all modes are separately designed. It still needs to be proven if this statement is also fulfilled for a hierarchically modulated BICM-ID scheme.

#### 4. PERFORMANCE OF A HIERARCHICALLY MODULATED BICM-ID SCHEME

In a hierarchically modulated BICM-ID scheme, it needs to be ensured that the bits assigned to a signal constellation point of the lower order modulation scheme are maintained in the higher order modulation scheme.

For a *straightforward* approach, Figure 5 shows the symbol mappings for the three modes which satisfy this side constraint. All the other system configuration settings mentioned in Section 3 and Table 1 remain the same.

##### 4.1 Using same Mode at Transmitter and Receiver

Figure 6 shows the *Bit Error Rate* (BER) performance of the hierarchically modulated BICM-ID modes with the *Base Layer* and two *Enhancement Layers*. All results have been

simulated for identical modes at the transmitter (TX) as the receiver (RX), i.e., TX: A  $\Rightarrow$  RX: A, B  $\Rightarrow$  B, and C  $\Rightarrow$  C. In Section 4.2 we will present the results for the case where we exploit the hierarchical modulation in a way that we use Mode C at the transmitter and different Modes at the receiver (TX: C  $\Rightarrow$  RX: A, C  $\Rightarrow$  B, and C  $\Rightarrow$  C).

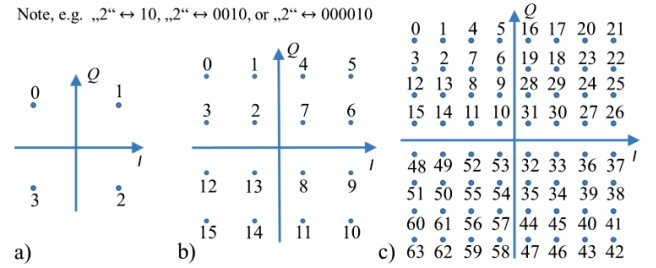


Figure 5: Labelling of Signal Constellation Points in a Hierarchically Modulated BICM-ID scheme

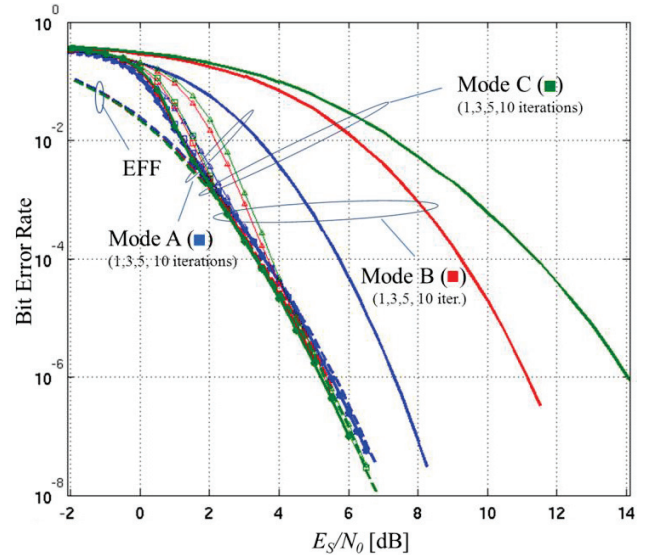


Figure 6: Bit Error Rate (BER) Performance of the Hierarchically Modulated BICM-ID Modes

Because the settings for Mode A have not changed, the BER performance of this mode remains the same if compared to the non-hierarchical BICM-ID scheme (compared to Fig. 4). However, it can easily be seen that the BER performances of the other two Modes B and C, i.e. for the higher order modulation schemes 16-QAM and 64-QAM, suffer considerably from the additional constraint in the design of the symbol mapping. Even though some noteworthy gains can be realized by the iterations, none of these modes is able to outperform Mode A (QPSK) in low BER regions.

One reason for the poor performance can be found by analyzing the so-called *Harmonic Mean Distance*  $d_h^2$  values [2]. Simply speaking, the higher the *Harmonic Mean Distance* is, the better the BER performance of the EFF curves can be. Table 2 summarizes the *Harmonic Mean Distance*  $d_h^2$  values for all the modes of the different non-

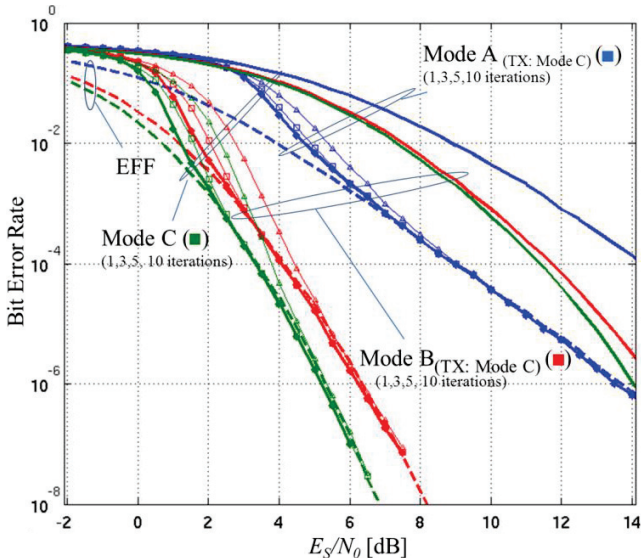
hierarchically and hierarchically modulated BICM-ID scheme discussed in this paper. From the  $d_h^2$  values it can be seen that there is a considerable reduction for Modes B and C if we change the symbol mapping from a non-hierarchical BICM-ID scheme to a hierarchically modulated one (*straightforward* approach, i.e. changing the labels only).

**Table 2:** Harmonic Mean Distances  $d_h^2$  for the Modes under consideration

Modes	Non-Hierarchical BICM-ID (see Fig. 3)	Hierarchical BICM-ID – Straight-forward (see Fig. 5)	Hierarchical BICM-ID – Relaxed Constraints (see Fig. 8)
Mode A	2.6667	2.6667	2.6667
Mode B	2.7190	0.8533	---
Mode C	2.8742	0.2902	0.7917

#### 4.2 Using always Mode C at Transmitter and different Modes at Receiver

Before relaxing some of the design constraints in Section 5, let us have a closer look at the BER performance for the case in which 64-QAM modulation is applied at the transmitter and different modulation types are used at the receiver. Remember, according to the operational scenario shown in Fig.1, the key motivation for introducing hierarchical modulation was based on the fact that a high capable transmitter sends the same data with different amounts of error protection to various receivers with different capabilities.



**Figure 7:** Bit Error Rate (BER) Performance of the Hierarchically Modulated BICM-ID Modes

Fig. 7 shows the simulation results. If compared to Fig. 6, the BER performance of using a rate  $r=1/6$  channel code and 64-QAM modulation on both sides of the communication scheme, i.e. Mode C at the transmitter (TX) and the receiver (RX), does not change.

In addition, it can be seen from Fig. 7 that if we change the signal processing at the RX to Mode A ( $r=1/2$  and QPSK) or Mode B ( $r=1/4$  and 16-QAM) the performance losses are significant. The losses are so high, that it would be beneficial in any channel condition to apply Mode C on both sides of the communication scheme.

Thus, as the second important result we can conclude that the performance loss is considerable if we apply the concepts of *Hierarchical Modulation* and *Incremental Redundancy* in a *straightforward* approach. In this context, *straightforward* approach means that we only change the symbol mapping, i.e. the bit labels of the signal constellation points, to ensure that the prerequisites for *Hierarchical Modulation* are given. A more sophisticated approach will be discussed in the following section.

#### 5. PERFORMANCE OF A HIERARCHICALLY MODULATED BICM-ID SCHEME WITH RELAXED CONSTRAINTS

In this section, we will present a second approach for a hierarchically modulated BICM-ID scheme in which some of the design constraints of the *straightforward* approach (see Section 4) are relaxed. That means,

- instead of considering the three Modes A, B, and C we will consider the two Modes A and C only.
- instead of a classic 64-QAM scheme with one bit label per signal constellation point each, we will use an 8-PSK scheme with eight bit labels per point.

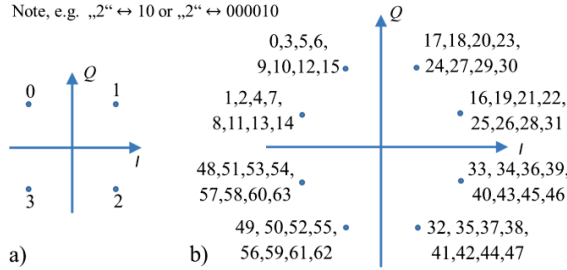
Please note, the first relaxation is fully in-line with the operational scenario under consideration where we have only two kinds of communication links (see Fig. 1). As a result of this relaxation we are free in the design of four out of six bits of the symbol mapping of Mode C.

In addition, the second relaxation allows increasing the distances between the signal constellation points. Both relaxations together permit us to improve the *Harmonic Mean Distance*  $d_h^2$  value of Mode C from 0.2902 to 0.7917.

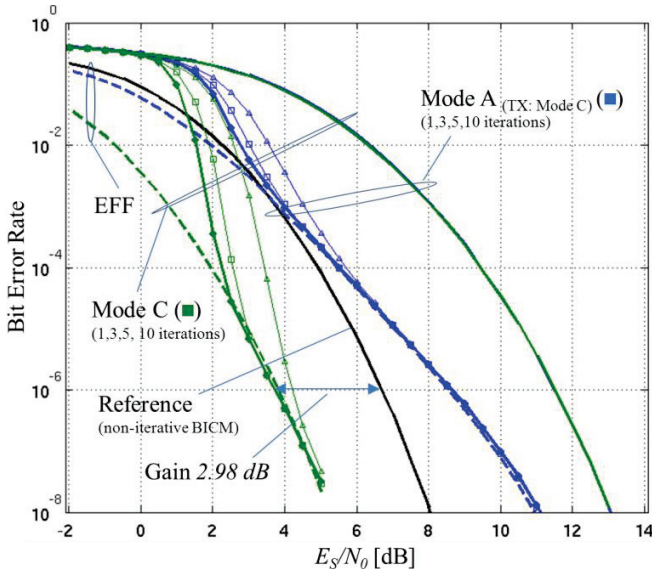
Figure 8 illustrates the symbol labelling of the signal constellation points in a hierarchically modulated BICM-ID scheme with relaxed design constraints (i.e. with the *Base Layer* and only one *Enhancement Layer*). The corresponding simulation results for the BER performance are shown in Figure 9.

If compared to the simulation results of the *straightforward* approach to hierarchically modulated BICM-ID shown in Fig. 7, the results for the approach with relaxed constraints are much better. In addition, if we compare the BER performance with a classic non-iterative BICM design (using

a rate  $r=1/2$  channel code in combination with a *Gray encoded* QPSK-modulation, see curve labelled *Reference*) we can observe a gain in  $E_s/N_0$  of 2.98 dB at a BER of  $10^{-6}$ . This gain is available for a wide range of channel conditions.



**Figure 8:** Labelling of Signal Constellation Points in a Hierarchically Modulated BICM-ID scheme with relaxed Design Constraints.



**Figure 9:** Bit Error Rate (BER) Performance of the Hierarchically Modulated BICM-ID Modes with relaxed Design Constraints

Please note, Modes A and C offer the same performance for the 1<sup>st</sup> iteration because in the initial processing step no information can be extracted from the two subgroups in each quadrant of the I/Q-plane (see Fig. 8).

However, the configuration with Mode C as TX and Mode A as RX still provides a loss if compared to the *Reference*.

Finally, we can conclude as the third important result that considerable parts of the previously mentioned performance loss can be compensated by relaxing some of the initial design constraints. This motivates us to proceed with our research work aiming for a solution in which both receiver designs with Mode C as TX and Modes A as well as C as RX, outperform the non-iterative BICM *Reference*.

## 6. CONCLUSIONS

In this paper, we have presented a novel approach for hierarchically modulated BICM-ID. This approach allows multiple receivers supporting a wide range of processing capabilities (i.e. from low for handhelds up to high for vehicular mounted radios) to decode the same transmit signal on the air with different fidelity. The new approach exploits the fact that the performance of BICM-ID schemes improves when the channel code's rate decreases while at the same time the modulation order is increased in order to keep the effective number of data bits per modulation symbol constant. It has been shown by simulation that changing the symbol labels alone to fulfill the elementary design constraint of hierarchical modulation is not sufficient to provide gains over a classic non-iterative BICM scheme. However, relaxing some of the initial design constraints has already offered a considerable improvement.

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