

# Robust Equalizer Design for Allpass Transformed DFT Filter Banks with LTI Property

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**Abstract**—Allpass transformed filter banks provide a non-uniform frequency resolution and can be used in mobile speech processing systems, e.g., cellular phones or digital hearing aids. The nominal design of such an allpass transformed analysis-synthesis filter bank (AS FB) with near perfect reconstruction (NPR) is achieved by numerical optimization of finite-impulse response (FIR) equalizers in each subchannel. The underlying nominal optimization problem is an equality constrained least-squares problem. In a robust design, we take into account coefficient uncertainty in a possible implementation of such a filter bank. We will describe this uncertainty by the choice of two simple set-based worst-case uncertainty models, namely a norm bound error model and a coefficient bound error model. When including these error models, both robust designs can be recast as second-order cone programs (SOCP) and solved efficiently by standard numerical optimization methods. Furthermore, we will provide design examples to show that both robust designs maintain a good overall performance with respect to NPR while offering less sensitivity to quantization errors.

## I. INTRODUCTION

Consider a digital filter bank which approximates the non-uniform frequency resolution of the human auditory system. This can be used for different applications in mobile speech processing systems such as subband coding [1] or speech enhancement algorithms [2]–[5]. Using an allpass transformed filter bank is one possibility to achieve this goal [6], [7]. When compared to a tree-structured approaches, allpass transformed filter banks offer a significant lower signal delay, which is crucial for speech processing systems.

Given a prescribed filter bank structure, the design goal is to find a set of equalizer coefficients such that the output signal is a delayed version of the input signal. Thus, the overall performance of the filter bank should be characterized as a perfect reconstruction (PR) system. Several authors proposed an analytical framework for an allpass transformed PR filter bank design, e.g., [1], [8]. One major drawback of this analytical solution is the missing bandpass characteristic at the synthesis filters.

To overcome this problem, the design goal can be relaxed to a near perfect reconstruction (NPR) design based on numerical optimization methods [9]–[11]. An advantage of these methods is that several design criteria can be incorporated easily in

the objective and constraints of a mathematical optimization problem, e.g., stop band attenuation, amount of linear and nonlinear distortion. In addition, if it is possible to express the filter bank design problem as a tractable convex optimization problem, this problem can be solved via efficient numerical methods [12], [13].

In this contribution, we will consider the optimal solution of the numerical allpass transformed filter bank design in [11]. We call this solution the *nominal solution*. The underlying optimization problem consists of an equality constrained least-squares problem. However, when there is at least partial uncertainty in the problem structure, the nominal solution will suffer in a practical hardware implementation. For this reason, we will involve robust optimization methods for the numerical design of allpass transformed filter banks. This leads to so called *robust designs*. It will be shown that these designs are less sensitive to parameter variations while still offering a good overall performance. In particular, we will use ideas from worst-case robust optimization [14]–[16] in order formulate two robust allpass transformed filter bank designs.

The following notation will be used frequently. The complex modulation factor is denoted by  $W_M = e^{-j2\pi/M}$ . Boldface upper-case letters denote matrices, boldface lower-case letters denote vectors.  $\|\cdot\|_p$  denotes the  $p$ -norm of a matrix or vector, respectively.  $\mathbf{I}_M$  is the identity matrix of dimension  $M \times M$ .  $\mathbf{0}_M$  stands for the null vector of dimension  $M \times 1$ . The componentwise inequality between two vectors is given by  $\mathbf{x} \preceq \mathbf{y}$ . The Kronecker product operator is represented as  $\otimes$ .

## II. NOMINAL FILTER BANK DESIGN

The allpass transformed DFT analysis-synthesis filter bank (AS FB) with  $M$  channels and a subsampling factor  $R$  is considered according to Fig. 1 and Fig. 2. In this efficient polyphase network implementation, the analysis and synthesis FIR prototype filters of length  $M$  are denoted as  $h[n]$  and  $g[n]$ , respectively. We will now develop several relations for such an allpass transformed DFT AS FB following [11]. It should be noted, that a generalization of this concept including longer prototype filters was introduced in [10]. However, this generalization leads to essentially the same mathematical optimization problem and is not considered to ease the treatment. The input-output relation is given in the  $z$ -domain as

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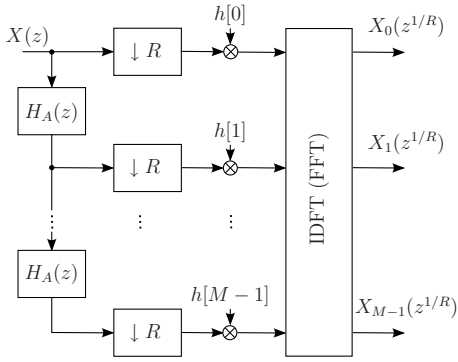


Fig. 1. Allpass transformed DFT analysis filter bank.

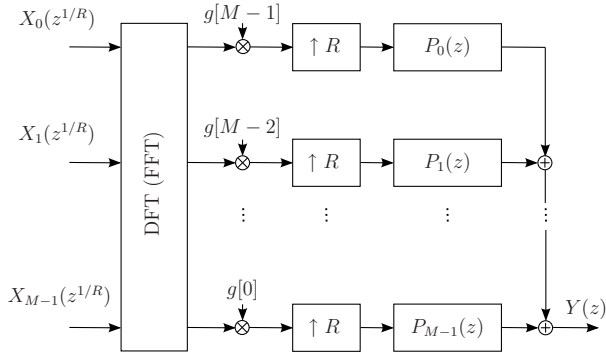


Fig. 2. Allpass transformed DFT synthesis filter bank.

$$Y(z) = \frac{1}{R} \sum_{r=0}^{R-1} X(zW_R^r) \sum_{m=0}^{M-1} \tilde{H}_m(zW_R^r) \bar{G}_m(z). \quad (1)$$

It consists of two filter sections, the analysis and synthesis subband filters, respectively.

The allpass transformed analysis filters read as

$$\tilde{H}_m(zW_R^r) = \sum_{n=0}^{M-1} h[n] W_M^{mn} H_A(zW_R^r)^n \quad (2)$$

where  $H_A(z) = (1 - \alpha z)/(z - \alpha)$  is a stable allpass filter with  $|z| > |\alpha|$ ,  $|\alpha| < 1$  and  $\alpha \in \mathbb{R}$ . Depending on the choice of the allpass coefficient  $\alpha$ , we get a frequency warped filter bank with a non-uniform frequency resolution. Note that the uniform DFT filter bank is included as a special case for  $\alpha=0$ .

The FIR synthesis subband filters are given by

$$\bar{G}_m(z) = \sum_{k=0}^{M-1} g[M-1-k] W_M^{-mk} \sum_{s=0}^{N_p-1} p_k[s] z^{-s} \quad (3)$$

$$= \sum_{k=0}^{M-1} g[M-1-k] W_M^{-mk} P_k(z). \quad (4)$$

We will now design the  $M$  FIR equalizer filter functions  $P_k(z)$  at the end of each channel. In order to get an NPR system, we have to combat two kinds of distortions.

Firstly, we have to minimize the linear distortion. For the considered allpass transformed DFT AS FB, this is equivalent to the conditions

$$\frac{R}{M} \sum_{k=0}^{M-1} h[k] g[M-1-k] \stackrel{!}{=} 1 \text{ and} \quad (5)$$

$$H_A(z)^k P_k(z) \approx z^{-d_0}, k = 0, 1, \dots, M-1. \quad (6)$$

The first condition can easily be met by an appropriate choice of the prototype filter pair, e.g., two rectangular window functions. The second condition can only be fulfilled approximately as it is an equalization of an IIR frequency response by an FIR filter. How to incorporate this condition will be discussed in the next subsection.

Secondly, we have to cancel out the nonlinear distortion, i.e., the alias components. The presence of alias components is caused by subsampling by  $R > 1$ , which is performed to obtain an efficient implementation of the filter bank. Let  $X_l(z) = z^{-l}$ ,  $l = 0, \dots, R-1$  be a sequence of time-shifted unit pulses. The transfer function  $T_l(z) = Y(z)/X_l(z)$  of the filter bank can now be written as

$$\begin{aligned} T_l(z) &= \frac{1}{R} \sum_{r=0}^{R-1} W_R^{-rl} \sum_{m=0}^{M-1} \tilde{H}_m(zW_R^r) \cdot \mathbf{v}_m^T \cdot \mathbf{D}^T(z) \cdot \mathbf{p} \\ &= \mathbf{t}_l^T(z) \mathbf{p} \end{aligned} \quad (7)$$

with

$$\mathbf{v}_m^T = \left( g[M-1] W_M^{-m0}, \dots, g[0] W_M^{-m(M-1)} \right)$$

$$\mathbf{D}(z) = \mathbf{I}_M \otimes \left( 1, z^{-1}, \dots, z^{-(N_p-1)} \right)$$

$$\mathbf{p} = (p_0[0], \dots, p_0[N_p-1], p_1[0], \dots, p_{M-1}[N_p-1]).$$

In order to get an aliasing-free system, it is sufficient to ensure a linear time-invariant (LTI) system. This property can be expressed as [11]

$$T_l(z) = T_0(z) \text{ for } l = 0, \dots, R-1. \quad (8)$$

In what follows now, we will show that (6) and (8) can be modeled approximately in terms of a norm approximation problem with equality constraints. Evaluating these expressions on the unit circle with  $z = e^{j\omega}$  leads to

$$H_A(e^{j\omega})^k \sum_{s=0}^{N_p-1} p_k[s] e^{-j\omega s} = H_A(e^{j\omega})^k \mathbf{w}_{N_p}^T \mathbf{p}_k \approx e^{-j\omega d_0} \quad (9)$$

where  $k = 0, 1, \dots, M-1$ . This can be reformulated as

$$\begin{bmatrix} H_A(e^{j\omega})^0 \mathbf{w}_{N_p}^T & & \mathbf{0}_{(M-1)N_p}^T \\ & \ddots & \\ \mathbf{0}_{(M-1)N_p}^T & & H_A(e^{j\omega})^{M-1} \mathbf{w}_{N_p}^T \end{bmatrix} \cdot \mathbf{p} \approx \begin{bmatrix} e^{-j\omega d_0} \\ \vdots \\ e^{-j\omega d_0} \end{bmatrix} \quad (10)$$

or equivalently as  $\mathbf{V} \cdot \mathbf{p} \approx \mathbf{b}$ . Considering the LTI condition, it follows from (7) and (8) that these conditions can be expressed

as

$$\begin{bmatrix} \mathbf{t}_1(e^{j\omega}) - \mathbf{t}_0(e^{j\omega}) \\ \vdots \\ \mathbf{t}_{R-1}(e^{j\omega}) - \mathbf{t}_0(e^{j\omega}) \end{bmatrix} \cdot \mathbf{p} = \mathbf{T} \cdot \mathbf{p} = \mathbf{0}_{R-1}. \quad (11)$$

With the above expressions, the nominal optimization problem for the considered allpass transformed AS FB can be stated as

$$\begin{aligned} & \underset{\mathbf{p}}{\text{minimize}} && \|\mathbf{V}\mathbf{p} - \mathbf{b}\|_p \\ & \text{subject to} && \mathbf{T}\mathbf{p} = \mathbf{0}_{R-1}, \end{aligned} \quad (12)$$

with the equalizer design vector  $\mathbf{p} \in \mathbb{R}^{MN_p}$ ,  $\mathbf{V} \in \mathbb{C}^{M \times MN_p}$ ,  $\mathbf{b} \in \mathbb{C}^M$  and  $\mathbf{T} \in \mathbb{C}^{(R-1) \times MN_p}$ . After a discretization of  $\omega$  over  $K$  frequency points, the above problem eventually leads to a norm approximation with equality constraints. This is a tractable convex problem which can be solved efficiently [12]. In the following, we will assume the Euclidean vector norm, i.e.,  $p = 2$ .

### III. ROBUST FILTER BANK DESIGN

The implementation of an allpass transformed filter bank in polyphase network structure contains coefficient multiplications in the analysis and synthesis stages. However, due to finite precision, it is possible that rounding errors will cause a deviation between the nominal filter bank and its implemented version. As a consequence, the nominal matrix  $\mathbf{V}$  – which reflects the filter bank structure – is supposed to be subject to uncertainty. In this section we will introduce two robust designs for the nominal optimization problem (12).

Formulating an exact uncertainty model for the multiplicative effect of several coefficient errors might be a futile task. For example, it has been reported in [17] that even the simple case of a multiplicative ellipsoidal uncertainty of two quantities involves some advanced concepts of ellipsoidal calculus. For this reason, we will resort to a simple set-based worst-case uncertainty model  $\mathcal{V}$ , such that  $\mathbf{V} \in \mathcal{V}$ . The robust worst-case design can be reformulated as the following bi-criterion problem

$$\begin{aligned} & \underset{\mathbf{p}, \varepsilon}{\text{minimize}} && \sup_{\mathbf{V} \in \mathcal{V}} \|\mathbf{V}\mathbf{p} - \mathbf{b}\|_2 \\ & \text{subject to} && \|\mathbf{T}\mathbf{p}\|_2 \leq \varepsilon. \end{aligned} \quad (13)$$

It is reasonable to model uncertain equality constraints within a prescribed range [18]. Therefore, we have introduced a new epigraph variable  $\varepsilon \in \mathbb{R}$  and replaced the equality constraints by a second-order cone constraint. In order to get a computationally tractable optimization problem, we have to get rid of the  $\sup(\cdot)$ . We will now introduce two common choices for the uncertainty set  $\mathcal{V}$  [12], [15], such that a tractable problem can be found more easily.

#### A. Norm bound error

The first model is the so called norm bound error model. We describe the uncertainty in  $\mathbf{V}$  by the set

$$\mathcal{V}_{\text{nb}} = \{\mathbf{V} + \Delta\mathbf{V} \mid \|\Delta\mathbf{V}\|_2 \leq a_{\text{nb}}\}, \quad (14)$$

where  $\|\cdot\|_2$  denotes the spectral norm of the matrix  $\Delta\mathbf{V}$ . The parameter  $a_{\text{nb}}$  controls the worst-case uncertainty about the nominal matrix  $\mathbf{V}$ . It can be verified, that the robust worst-case problem with a norm bound error is equivalent to

$$\begin{aligned} & \underset{\mathbf{p}, \varepsilon, t_1, t_2}{\text{minimize}} && t_1 + a_{\text{nb}}t_2 \\ & \text{subject to} && \|\mathbf{V}\mathbf{p} - \mathbf{b}\|_2 \leq t_1 \quad \|\mathbf{p}\|_2 \leq t_2 \\ & && \|\mathbf{T}\mathbf{p}\|_2 \leq \varepsilon, \end{aligned} \quad (15)$$

with the design vector  $\mathbf{p} \in \mathbb{R}^{MN_p}$  and epigraph variables  $t_1, t_2, \varepsilon \in \mathbb{R}$ . This optimization problem is a second-order cone program (SOCP) which can be solved efficiently.

#### B. Coefficient bound error

The norm bound error model lacks in exploiting the special matrix structure in the nominal matrix  $\mathbf{V}$ . In fact,  $\mathbf{V}$  is a sparse matrix by definition (10). The coefficient bound error model accounts for this special structure. It enables us to adjust every entry in the nominal matrix, which also allows to preserve the sparsity pattern in  $\mathbf{V}$ . The coefficient bound error model is explicitly given by

$$\mathcal{V}_{\text{cb}} = \{\hat{\mathbf{V}} \in \mathbb{R}^{m \times n} \mid l_{ij} \leq \hat{v}_{ij} \leq u_{ij}, i = 1, \dots, m, j = 1, \dots, n\}, \quad (16)$$

where each coefficient  $\hat{v}_{ij}$  is lower bounded by  $l_{ij}$  and upper bounded by  $u_{ij}$ . We will now use the following equivalent real formulation for complex matrices and vectors

$$\hat{\mathbf{V}} = \begin{bmatrix} \text{Re}(\mathbf{V}) & -\text{Im}(\mathbf{V}) \\ \text{Im}(\mathbf{V}) & \text{Re}(\mathbf{V}) \end{bmatrix}, \quad \hat{\mathbf{b}} = \begin{bmatrix} \text{Re}(\mathbf{b}) \\ \text{Im}(\mathbf{b}) \end{bmatrix}. \quad (17)$$

The dimension of  $\hat{\mathbf{V}}$  is given by  $m = 2M$  and  $n = 2MN_p$ . To find a tractable formulation for the robust worst-case problem with coefficient bounds, we will follow [12] (exercise 6.8). Note that

$$\sup_{l_{ij} \leq \hat{v}_{ij} \leq u_{ij}} |\hat{\mathbf{v}}_i^T \hat{\mathbf{p}} - \hat{b}_i| = \left| \sum_{j=1}^n \hat{v}_{ij}^+ \hat{p}_j - \hat{b}_i \right| + \left| \sum_{j=1}^n \hat{v}_{ij}^- |\hat{p}_j| \right|, \quad (18)$$

with

$$\hat{v}_{ij}^+ = (u_{ij} + l_{ij})/2, \quad \hat{v}_{ij}^- = (u_{ij} - l_{ij})/2. \quad (19)$$

With the above expression, we can eliminate the supremum in the objective, that is

$$\sup_{\mathbf{V} \in \mathcal{V}_{\text{cb}}} \|\hat{\mathbf{V}}\hat{\mathbf{p}} - \hat{\mathbf{b}}\|_2 = \sum_{i=1}^m \left( \left| \sum_{j=1}^n \hat{v}_{ij}^+ \hat{p}_j - \hat{b}_i \right| + \sum_{j=1}^n \hat{v}_{ij}^- |\hat{p}_j| \right)^2 \quad (20)$$

The robust worst-case problem with coefficient bounds can now be recast as

$$\begin{aligned} & \underset{\hat{\mathbf{p}}, \mathbf{x}, \mathbf{y}, t, \varepsilon}{\text{minimize}} && t \\ & \text{subject to} && \|\hat{\mathbf{V}}^- \mathbf{x} + \mathbf{y}\|_2 \leq t, \quad \|\hat{\mathbf{T}}\hat{\mathbf{p}}\|_2 \leq \varepsilon, \\ & && -\mathbf{y} \preceq \hat{\mathbf{V}}^+ \hat{\mathbf{p}} - \hat{\mathbf{b}} \preceq \mathbf{y}, \quad -\mathbf{x} \preceq \hat{\mathbf{p}} \preceq \mathbf{x}. \end{aligned} \quad (21)$$

The variables are  $\hat{\mathbf{p}}, \mathbf{x} \in \mathbb{R}^n$ ,  $\mathbf{y} \in \mathbb{R}^m$  and  $t, \varepsilon \in \mathbb{R}$ . Each entry in  $\hat{\mathbf{V}}^+$  and  $\hat{\mathbf{V}}^-$  is given by (19). Again, this problem is an SOCP, which in turn can be solved efficiently.

#### IV. DESIGN EXAMPLES

In the whole section we will employ the following filter bank specification:  $M = 16$  channels,  $N_p = 36$  coefficients for each  $P_k(z)$ , subsampling by  $R = 4$ , allpass coefficient  $\alpha = 0.4$  and  $K = 256$  discrete frequency points. Furthermore, every optimization problem is solved by CVX, a package for specifying and solving convex programs [13].

Consider first the worst-case problem with a norm bound error (15). Solving this problem for different values of  $a_{nb}$  leads to different design vectors  $\mathbf{p}_{nb}$ , which depend on  $a_{nb}$ . The worst-case objective function value for this problem is given by

$$f_{nb}(\mathbf{p}) = \|\mathbf{V}\mathbf{p} - \mathbf{b}\|_2 + a_{nb}\|\mathbf{p}\|_2 \quad (22)$$

Now insert the design vectors  $\mathbf{p}_{nom}$  and  $\mathbf{p}_{nb}$  into the above expression. Evaluating this yields the plot in Fig. 3a. The robust design performs always better in the worst-case sense than the nominal design.

The same can be done for the robust worst-case problem (21) with coefficient bounds (19). The worst-case objective is given by

$$f_{cb}(\mathbf{p}) = \sum_{i=1}^m \left( \left| \sum_{j=1}^n \hat{v}_{ij}^+ \hat{p}_j - \hat{b}_i \right| + \sum_{j=1}^n \hat{v}_{ij}^- |\hat{p}_j| \right)^2. \quad (23)$$

We will use the following values  $u_{ij} = (1 + a_{cb})v_{ij}$  and  $l_{ij} = (1 - a_{cb})v_{ij}$ . This leads to  $\hat{v}_{ij}^+ = v_{ij}$  and  $\hat{v}_{ij}^- = a_{cb}v_{ij}$ , respectively. Note that this simple choice preserves the sparsity pattern in the nominal matrix. The result is depicted in Fig. 3b. Again, the robust design performs better than the nominal design.

Consider now a fixed parameter setting  $a_{nb} = 13.125$  and  $a_{cb} = 0.7875$  which leads to fixed robust design vectors  $\mathbf{p}_{nb}$  and  $\mathbf{p}_{cb}$ , respectively. Both vectors are inserted into the adjustable nominal objective

$$f_{adj}(\mathbf{p}) = \|(\mathbf{V} + \mathbf{V}_{adj})\mathbf{p} - \mathbf{b}\|_2. \quad (24)$$

We will now adjust the amount of uncertainty in this objective by changing the influence of  $\mathbf{V}_{adj}$ . In the case of the norm bound error model,  $\mathbf{V}_{adj}$  is an i.i.d. central complex gaussian random matrix with a spectral norm given by the values between  $a_{nb,min} = 0$  and  $a_{nb,max} = 15$ . In the case of the coefficient bound error model, the entries of  $\mathbf{V}_{adj}$  are adjusted by uniformly distributed perturbations within  $[-a_{cb}, +a_{cb}]$  for  $a_{cb,min} = 0$  and  $a_{cb,max} = 0.9$ . On the one hand, when there is almost no uncertainty present, i.e., for low values of  $a_{nb}$  and  $a_{cb}$ , the nominal design will outperform both robust designs. On the other hand, with increasing uncertainty the robust designs will behave better and better when compared to the nominal design. These relations can be seen in Fig. 4.

For practical reasons we will consider a 16-bit quantized version of the matrix  $\mathbf{V}$ . Compare now the nominal design and the norm bound error robust design for different values of  $a_{nb}$ . The results can be seen in Fig. 5. Notice that the robust design clearly outperforms the nominal design.

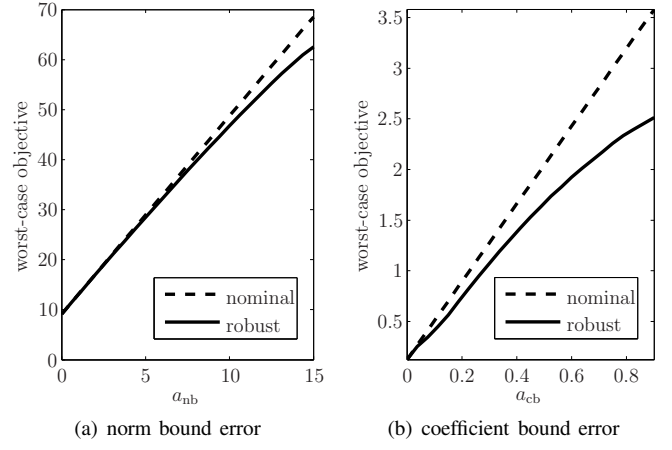


Fig. 3. Worst-case performance.

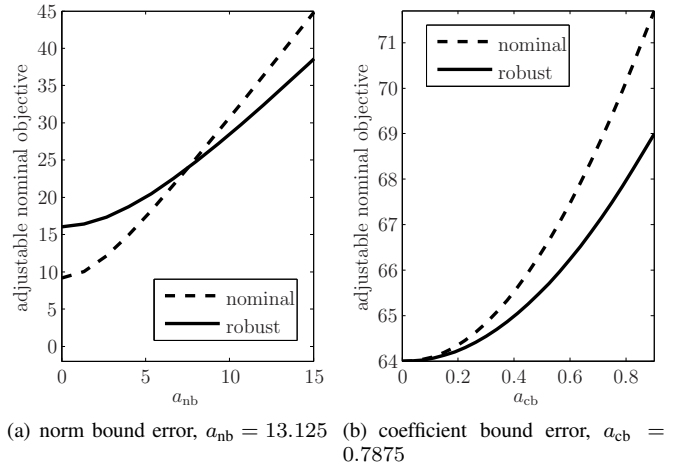


Fig. 4. Comparison between the nominal and two fixed robust designs, varied over different levels of uncertainty.

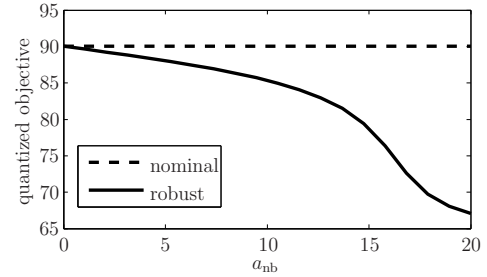


Fig. 5. Comparison of the quantized objective between the nominal and the norm bound error robust design using a 16-bit quantization of the matrix  $\mathbf{V}$ .

The above analysis showed some qualitative and quantitative behaviour of the objective function for both robust designs. However, it gives almost no insight into the overall performance of the filter bank. This issue can be addressed by the analysis of the magnitude of the bifrequency system function  $T(e^{j\omega_1}, e^{j\omega_2})$  [7]. It consists of the 2D Fourier transform of the filter bank system response function. The main design objectives are the minimization of linear distortion and the elimination of alias components. The former condition corresponds to

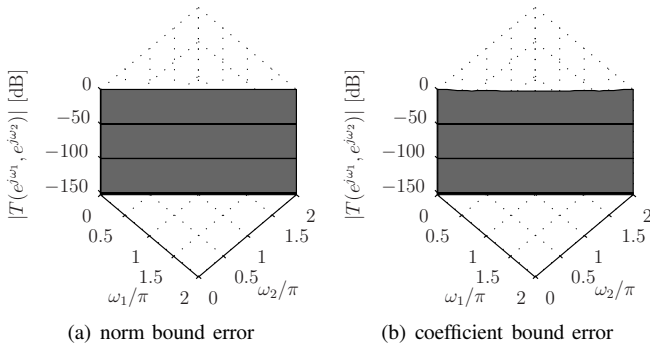


Fig. 6. Bifrequency system function for both robust designs. The parameter setting is fixed and given by  $a_{nb} = 13.125$  and  $a_{cb} = 0.7875$ .

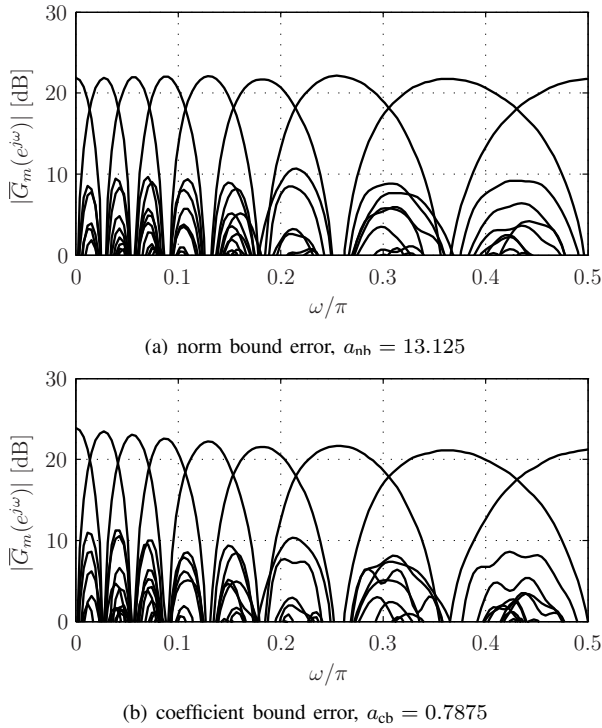


Fig. 7. Synthesis filters for both robust designs and a fixed parameter setting.

the main diagonal in the bifrequency system function, which is constant to 0 dB when there is no linear distortion. The latter condition can be analyzed by the occurrence of parallel side-diagonals in the bifrequency system function. Both robust designs yield almost no linear and nonlinear distortion, as can be seen from Fig. 6. Furthermore, both robust designs offer a bandpass characteristic with respect to the synthesis filters (Fig. 7), which is needed for speech enhancement algorithms. Therefore, the overall performance criteria are fulfilled by both robust designs.

## V. CONCLUSION

We have considered a nominal design of an allpass transformed filter bank with minimized linear and nonlinear distortion. This design can be formulated as a least-norm approximation problem with equality constraints. In fact, this

problem has a fairly simple structure from an optimization point of view. Based on this simple problem structure, we have introduced two robust design approaches. It has been shown, that both approaches can be expressed as convex optimization problems in a reasonable and tractable way. These problems were solved using CVX. Furthermore, their solutions offered a good compromise between robustness and overall filter bank performance with respect to linear and nonlinear distortions.

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