

EXIT Chart Optimized Rate Matching for Wireless Communication Systems

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Abstract—In modern wireless communication systems different services with different bit rates and individual error protection requirements are supported by the same network. Rate matching by repetition is often employed after convolutional turbo encoding in order to achieve flexibly the required decrease of the code rate. This ensures near error-free transmission down to a specific target channel quality. We propose the optimization of the rate matching using EXIT charts. The rate for random repetition required for error-free decoding is determined semi-analytically avoiding extensive simulations. This includes the analytical optimization of the random repetition scheme and the derivation of the overall EXIT chart taking into account the convolutional turbo encoding and rate matching.

I. INTRODUCTION

A vision of mobile communication is that all services as speech, audio, video and data shall be transmitted through the same packet-switched network. The transceiver is located at the physical layer and consists of three major components, namely channel coding, rate matching and modulation. In this contribution, we focus only on the channel coding and rate matching aspects. Conventionally, turbo coding by means of *parallel concatenated convolutional codes* (PCCCs) is employed to protect the payload. Then rate matching is commonly performed by means of puncturing or repetition. This generates the transmission rates which are supported by the transmission system. Furthermore, it accounts for the different error protection requirements and provides near error-free transmission at a specific target channel quality. In order to optimize the latter issue, a theoretical evaluation of the expected decoding gains due to rate matching is extremely useful. Such an analysis has been done only for randomly punctured convolutional codes in [1].

In this contribution, we will provide the theoretical evaluation for rate matching employing random repetition after convolutional turbo encoding. This includes the analytical derivation of the overall EXIT chart [2] after convolutional turbo encoding and repetition as a function of the EXIT chart of the pure convolutional code. Expressions are given for the transmission over a *binary erasure channel* (BEC) as well as a *binary input/continuous output additive white Gaussian noise channel* (BIAWGNC). We will further derive analytically an expression for the optimal design of a random repeater for these cases. A maximization constraint is then formulated by means of the theoretical results which enables a semi-analytical optimization of those systems avoiding extensive

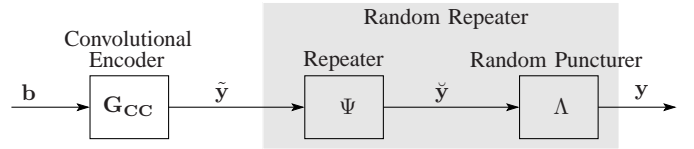


Fig. 1. Structure of a repetition convolutional encoder.

simulations. Furthermore, simulations have shown that the results can be used as a close approximations for standardized deterministic repetition schemes as, e.g., employed in UMTS-LTE [3]. However, this aspect is out of the scope of this paper.

II. REPETITION CONVOLUTIONAL CODES

Repetition convolutional codes (RCCs) are suitable to flexibly decrease the code rate and thus increase the amount of error protection of rate-compatible transmission systems. There are many possible realizations of RCCs which differ in the utilized repeater. Deterministic repeaters which repeat the output bits according to a predefined pattern as well as random repeaters can be employed. In what follows, we restrict the information theoretical evaluation to the evaluation of random repeaters. The structure of the corresponding encoder is shown in Fig. 1. The input bits $\mathbf{b} = (b_1, \dots, b_m, \dots, b_M)$ are encoded by a convolutional encoder of rate r_{CC} resulting in the output bit stream $\tilde{\mathbf{y}} = (\tilde{y}_1, \dots, \tilde{y}_n, \dots, \tilde{y}_N)$ of length $N = M \cdot r_{CC}^{-1}$. Then random repetition of rate r_{RR} is employed yielding the vector $\mathbf{y} = (y_1, \dots, y_k, \dots, y_K)$ of length $K = N \cdot r_{RR}^{-1}$. The random repeater can be realized by a serially concatenation of two components. At first, a deterministic repeater Ψ with code rate

$$r_{\Psi} = \frac{1}{\psi}, \quad \psi \in \mathbb{N} \quad (1)$$


is employed which repeats all output bits of the convolutional encoder $\psi - 1$ times. In a second step, a puncturer Λ randomly eliminates bits within $\tilde{\mathbf{y}}$ according to the puncturing rate r_{Λ} ($r_{\Psi} < r_{\Lambda} \leq 1$) in order to provide any fractional repetition rates. Consequently, the puncturer introduces the random nature of the repetition scheme.

The code rate of the RCC can then be computed by

$$r_{RCC} = r_{CC} \cdot r_{RR} = \frac{M}{N} \cdot \frac{r_{\Psi}}{r_{\Lambda}}, \quad (2)$$

taken into account the code rate of the convolutional code $r_{CC} = M/N$ and the repetition rate of the random repeater $r_{RR} = r_{\Psi}/r_{\Lambda}$.

From a decoding point of view, the puncturing can be described by a BEC with error probability $P_{e,\Lambda} = 1 - r_{\Lambda}$ and capacity $C_{\Lambda} = 1 - P_{e,\Lambda} = r_{\Lambda}$.

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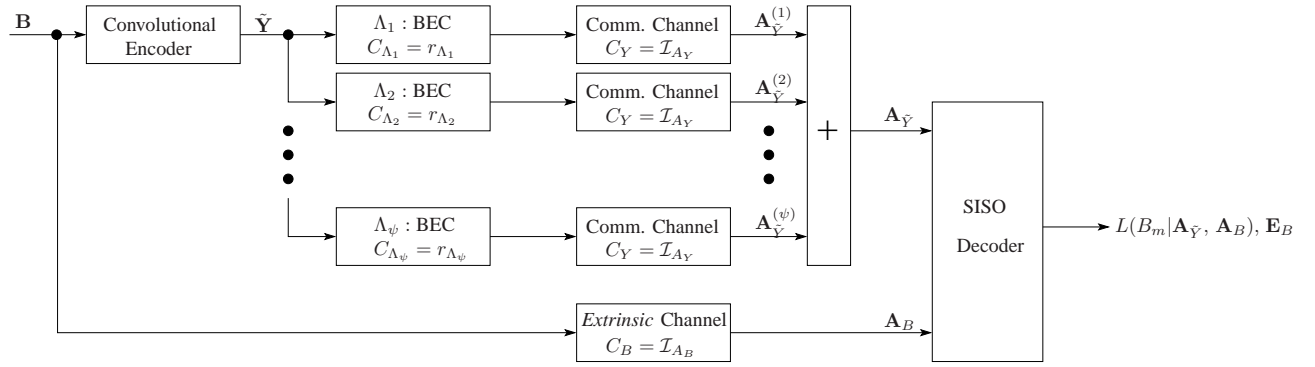


Fig. 2. Information theoretical model for the repetition convolutional decoder employed in a parallel concatenated turbo scheme. $\mathbf{A}_{\tilde{Y}}^{(q)}$ and \mathbf{A}_B signify log-likelihood values.

III. EXIT FUNCTIONS FOR RCCS

The invention of *EXtrinsic Information Transfer* (EXIT) charts by ten Brink [2] has been a breakthrough in the optimal design of iterative decoders. EXIT charts describe the flow of *extrinsic* information through concatenated *Soft-In/Soft-Out* (SISO) decoders and are a powerful tool to predict the convergence behavior of the iterative decoder, particularly at low channel qualities. A general *information theoretical model* for analyzing the behavior of the decoder has been presented in [4] which is adopted in Fig. 2 to the scenario considered here. In what follows, capital letters are applied for random variables and small letters signify their realizations.

The information bits $\mathbf{B} = (B_1, \dots, B_m, \dots, B_M)$ are encoded by the convolutional encoder resulting in the bit stream $\tilde{\mathbf{Y}} = (\tilde{Y}_1, \dots, \tilde{Y}_n, \dots, \tilde{Y}_N)$. These bits are then transmitted over $\psi = r_{\tilde{\Psi}}^{-1}$ independent memoryless communication channels modeling the influence of the repeater Ψ . This assumption is valid for systems employing sufficiently large interleavers. The impact of the puncturer Λ is modeled by ψ potentially different BECs with capacity

$$C_{\Lambda_q} = 1 - P_{\Lambda_q} = r_{\Lambda_q} \quad \text{and} \quad 1 \leq q \leq \psi, \quad (3)$$

where r_{Λ_q} denotes the puncturing rate associated with branch q . The overall puncturing rate r_{Λ} is thus given by

$$r_{\Lambda} = \frac{1}{\psi} \cdot \sum_{q=1}^{\psi} r_{\Lambda_q}. \quad (4)$$

The communication channel in each branch q provides log-likelihood ratios (LLRs) $\mathbf{A}_{\tilde{Y}}^{(q)}$ for the SISO decoder. The addition of all partial LLR vectors $\mathbf{A}_{\tilde{Y}}^{(q)}$ yields the overall channel-related LLR vector $\mathbf{A}_{\tilde{Y}}$:

$$\mathbf{A}_{\tilde{Y}} = \sum_{q=1}^{\psi} \mathbf{A}_{\tilde{Y}}^{(q)}. \quad (5)$$

According to [4], the LLRs \mathbf{A}_B at the *a priori* input of the SISO decoder can be modeled by an *extrinsic* channel with capacity $C_B = \mathcal{I}_{A_B}$, where $\mathcal{I}_{A_B} := \mathcal{I}(A_{B_m}; B_m)$ denotes the mutual information between B_m and A_{B_m} . This information models the *extrinsic* information provided by the constituent SISO decoder which is taken as *a priori* information by the considered SISO decoder.

The SISO decoder exploits \mathbf{A}_B and $\mathbf{A}_{\tilde{Y}}$ and determines the *extrinsic* LLRs E_{B_m} as well as the *a posteriori* LLRs $L(B_m | \mathbf{A}_{\tilde{Y}}, \mathbf{A}_B)$ for each input bit B_m . According to [5], the *extrinsic* LLRs E_{B_m} for *parallel concatenated convolutional codes* (PCCCs) are given by means of the *a posteriori* LLRs and the *a priori* LLRs:

$$E_{B_m} = L(B_m | \mathbf{A}_{\tilde{Y}}, \mathbf{A}_B) - A_{B_m} - Z_{B_m}. \quad (6)$$

In case of systematic PCCCs, the LLR Z_{B_m} corresponds to the channel observation regarding the information bit B_m provided by the underlying communication channel.

By means of the mutual *a priori* information \mathcal{I}_{A_B} , the EXIT characteristic $\mathcal{T}_B(\mathcal{I}_{A_B} | \mathcal{I}_{A_{\tilde{Y}}})$ is defined according to

$$\mathcal{T}_B(\mathcal{I}_{A_B} | \mathcal{I}_{A_{\tilde{Y}}}) = \mathcal{I}(E_{B_m}; B_m) =: \mathcal{I}_{E_B}, \quad (7)$$

where $\mathcal{I}_{A_{\tilde{Y}}}$ is considered as arbitrary but fixed.

In general, the numerical computation of \mathcal{I}_{E_B} is impractical and Monte Carlo simulations have to be carried out. However, for a system employing the random repeater described in Sec. II the EXIT function $\mathcal{T}_B^{(\text{RCC})}(\mathcal{I}_{A_B} | \mathcal{I}_{A_{\tilde{Y}}})$ can be derived by means of the EXIT characteristic $\mathcal{T}_B^{(\text{CC})}(\mathcal{I}_{A_B} | \mathcal{I}_{A_{\tilde{Y}}})$ of the pure convolutional code by using the *information theoretical model* of Fig. 2.

In this contribution we perform the analysis exemplarily for the BEC and the BIAWGNC. The evaluation of the *binary symmetric channel* (BSC) is out of our scope but can easily be obtained with the results given in what follows.

A. BEC Case

According to [4], the *extrinsic* information $\mathcal{I}_{A_{\tilde{Y}}}$ generated by a repetition code of rate $r_{\Psi} = 1/\psi$ for a transmission over a BEC with capacity $C_Y = \mathcal{I}_{A_Y}$ is given by

$$\mathcal{I}_{A_{\tilde{Y}}} = 1 - (1 - \mathcal{I}_{A_Y})^{\psi} \quad (8)$$

$$= 1 - \prod_{q=1}^{\psi} (1 - \mathcal{I}_{A_Y}). \quad (9)$$

The individual and possibly different puncturing of the branches q (see Fig. 2) can now be incorporated by substituting \mathcal{I}_{A_Y} with $\mathcal{I}_{A_Y}^{(q)} = C_{\Lambda_q} \cdot C_Y = r_{\Lambda_q} \cdot \mathcal{I}_{A_Y}$ for all $1 \leq q \leq \psi$:

$$\mathcal{I}_{A_{\tilde{Y}}} = 1 - \prod_{q=1}^{\psi} (1 - \mathcal{I}_{A_Y}^{(q)}). \quad (10)$$

Optimal Random Repetition

Random repetition is realized by a deterministic repeater of rate $r_\Psi = 1/\psi$, $\psi \in \mathbb{N}$, followed by a random puncturer. Optimal random repetition is achieved if the mutual information $\mathcal{I}_{A_{\bar{Y}}}$ at the output of the parallel concatenated communication channels is maximized.

Proposition 3.1: The mutual information $\mathcal{I}_{A_{\bar{Y}}}$ at the output of ψ parallel concatenated BECs each with capacity $\mathcal{I}_{A_Y^{(q)}} = C_{\Lambda_q} \cdot C_Y = r_{\Lambda_q} \cdot \mathcal{I}_{A_Y}$ ($1 \leq q \leq \psi$) is maximized by the puncturing rates

$$r_{\Lambda_q} = \begin{cases} 1 & , \quad 1 \leq q \leq \psi - 1 \\ \psi \cdot r_\Lambda - (\psi - 1) & , \quad q = \psi \end{cases} \quad (11)$$

subject to $0 < r_{\Lambda_q} \leq 1$ and under the constraint of an equal average puncturing rate (equal target code rate) r_Λ ($r_\psi < r_\Lambda \leq 1$).

Proof: Instead of maximizing $\mathcal{I}_{A_{\bar{Y}}}$ according to (10), we minimize

$$\Phi(\mathbf{r}) := 1 - \mathcal{I}_{A_{\bar{Y}}} = \prod_{q=1}^{\psi} (1 - r_{\Lambda_q} \cdot \mathcal{I}_{A_Y}), \quad \mathbf{r} = (r_{\Lambda_1} \dots r_{\Lambda_\psi})^T \quad (12)$$

subject to

$$r_\Lambda = \frac{1}{\psi} \cdot \sum_{q=1}^{\psi} r_{\Lambda_q} \quad , \quad 0 < r_{\Lambda_q}, r_\Lambda \leq 1, \quad (13)$$

where r_Λ is arbitrary within the domain but fixed. Substitution of r_{Λ_ψ} in (12) by means of (13) results in

$$\Phi(\mathbf{r} \setminus \psi) = \left(1 - \left(\psi r_\Lambda - \sum_{q=1}^{\psi-1} r_{\Lambda_q} \right) \mathcal{I}_{A_Y} \right) \prod_{q=1}^{\psi-1} (1 - r_{\Lambda_q} \mathcal{I}_{A_Y}) \quad (14)$$

with $\mathbf{r} \setminus \psi = (r_{\Lambda_1} \dots r_{\Lambda_{\psi-1}})$. In order to determine all local extrema for functions of more than one variable, the first partial derivatives have to be zero. Due to the symmetry of $\Phi(\mathbf{r} \setminus \psi)$ with respect to each r_{Λ_q} , the first derivatives of all variables are given according to

$$\frac{\partial \Phi(\mathbf{r} \setminus \psi)}{\partial r_{\Lambda_k}} = \left[\prod_{\substack{q=1 \\ q \neq k}}^{\psi-1} (1 - r_{\Lambda_q} \mathcal{I}_{A_Y}) \right] \cdot (\mathcal{I}_{A_Y} \cdot (1 - r_{\Lambda_k} \mathcal{I}_{A_Y})) - \mathcal{I}_{A_Y} \cdot \left(1 - \mathcal{I}_{A_Y} \cdot \left(\psi \cdot r_\Lambda - \sum_{q=1}^{\psi-1} r_{\Lambda_q} \right) \right) \stackrel{!}{=} 0 \quad (15)$$

$$\Leftrightarrow 2r_{\Lambda_k} + \sum_{\substack{q=1 \\ q \neq k}}^{\psi-1} r_{\Lambda_q} = \psi \cdot r_\Lambda \quad , \quad 1 \leq k \leq \psi - 1. \quad (16)$$

In conjunction with (13), equation (16) defines a linear system of equations consisting of ψ equations and ψ variables. It is easy to prove that it is solved for

$$r_{\Lambda_q} = r_\Lambda \quad , \quad 1 \leq q \leq \psi. \quad (17)$$

However, this is only a necessary condition for the existence of an extremum. The sufficient condition for a maximum and a minimum can be verified by means of the Hesse matrix which contains all permutations of the second partial derivatives.

If the Hesse matrix is negative definite there exist a local maximum at the considered position, otherwise this position corresponds to a local minimum. It is easy to show that the second partial derivatives evaluated at the considered point result in

$$\left. \frac{\partial^2 \Phi(\mathbf{r} \setminus \psi)}{\partial r_{\Lambda_k} \partial r_{\Lambda_j}} \right|_{r_{\Lambda_q} = r_\Lambda} < 0, \quad \forall j, k \quad (18)$$

under the constraints of $0 < r_\Lambda < 1$ and $0 \leq \mathcal{I}_{A_Y} < 1$. The root of the first derivatives is, thus, associated with a local maximum. However, we want to figure out the global minimum in order to obtain the optimal random repetition. Nevertheless, this implicates that the desired minimum is located at the boundary of the considered domains $0 < r_{\Lambda_q} \leq 1$. The points at the boundary which have to be considered are given by

$$\mathbf{r}' = (\mathbf{r}'_1, \dots, \mathbf{r}'_i, \dots, \mathbf{r}'_{\psi-1}) = \{(1, r_{\Lambda_2}, \dots, r_{\Lambda_\psi})^T, \dots, (1, \dots, 1, r_{\Lambda_\psi})^T\}, \quad (19)$$

where r_{Λ_ψ} is determined according to (13). In principle, all permutations of 'one' entries are possible. Nevertheless, it is sufficient to analyze the set of vectors given in (19) due to the identical nature of all branches in Fig. 2. We prove in the following that $\Phi(\mathbf{r}'_i)$ is monotonically decreasing in i :

$$\begin{aligned} \Phi(\mathbf{r}'_i) &\geq \Phi(\mathbf{r}'_{i+1}) \quad , \quad 1 \leq i \leq \psi - 2 \quad (20) \\ &\Leftrightarrow (1 - \mathcal{I}_{A_Y})^i \left(1 - \mathcal{I}_{A_Y} \left(\psi r_\Lambda - i - \sum_{q=i+1}^{\psi-1} r_{\Lambda_q} \right) \right) \\ &\quad \cdot \prod_{q=i+1}^{\psi-1} (1 - r_{\Lambda_q} \mathcal{I}_{A_Y}) \\ &\geq (1 - \mathcal{I}_{A_Y})^{i+1} \left(1 - \mathcal{I}_{A_Y} \left(\psi r_\Lambda - (i+1) - \sum_{q=i+2}^{\psi-1} r_{\Lambda_q} \right) \right) \\ &\quad \cdot \prod_{q=i+2}^{\psi-1} (1 - r_{\Lambda_q} \mathcal{I}_{A_Y}). \quad (21) \end{aligned}$$

This can be simplified to

$$r_{\Lambda_{i+1}} \leq 1 \quad (22)$$

which is fulfilled per definition. Therefore, it can be concluded that $\Phi(\mathbf{r}'_i)$ is monotonically decreasing with increasing index i . Consequently, the optimal puncturing is obtained for the last entry in \mathbf{r}' proving Proposition 3.1. ■

Corollary 3.2: The worst case scenario corresponds to the set of puncturing rates which minimizes the mutual information $\mathcal{I}_{A_{\bar{Y}}}$ at the output of the parallel concatenated BECs. This is fulfilled by the puncturing rates

$$r_{\Lambda_q} = r_\Lambda. \quad (23)$$

Consequently, the mutual information $\mathcal{I}_{A_{\bar{Y}}}$ can be bounded for an arbitrary puncturing scheme by

$$1 - (1 - r_\Lambda \mathcal{I}_{A_Y})^\psi \leq \mathcal{I}_{A_{\bar{Y}}} \leq 1 - (1 - \mathcal{I}_{A_Y})^{\psi-1} (1 - r_{\Lambda_\psi} \mathcal{I}_{A_Y}),$$

with $r_{\Lambda_\psi} = \psi \cdot r_\Lambda - (\psi - 1)$.

According to Proposition 3.1, the transformed mutual information $\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})}$ exploited by the SISO decoder after repetition and puncturing is given by

$$\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})} := \mathcal{I}_{A_{\bar{Y}}} = 1 - (1 - \mathcal{I}_{A_Y})^{\psi^{-1}} \cdot (1 - r_{\Lambda_{\psi}} \cdot \mathcal{I}_{A_Y}). \quad (24)$$

B. BIAWGNC Case

In the BIAWGNC case, $\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})}$ can be expressed by the J -function introduced by ten Brink according to [2], [6]:

$$\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})} = \mathcal{I}_{A_{\bar{Y}}} = J(\sigma_A), \quad (25)$$

where σ_A denotes the variance of the LLRs $A_{\bar{Y}}$ at the input of the SISO decoder and has to be determined in what follows. For repetition codes it is well-known that equal decoding performance can be achieved by transmitting the information bits without redundancy but with the same overall energy. This implies that the energy per modulation symbol E'_s (without repetition) is increased by $r_{\Psi}^{-1} = \psi$ (repetition rate) resulting in the modified energy per modulation symbol

$$E_s = r_{\Psi}^{-1} \cdot E'_s. \quad (26)$$

By means of $E_s/N_0 = 1/(2\sigma^2)$ and $\sigma = 2/\sigma_A$ [2] this can be converted to

$$\sigma_A = \sqrt{r_{\Psi}^{-1}} \cdot \sigma'_A. \quad (27)$$

Neglecting the puncturing at first, (27) directly delivers the desired relation

$$J(\sqrt{r_{\Psi}} \cdot \sigma_A) = J(\sigma'_A) \quad (28)$$

between the variances of the LLRs of each branch and the LLRs at the output of the parallel concatenated channels. $J(\sigma'_A)$ corresponds to the mutual information about the code bits of each branch q according to Fig. 2. The puncturing with overall rate r_{Λ} causes a linear reduction of the channel-related mutual information $J(\sigma'_A)$. Consequently, it can be incorporated in (28) as a multiplicative factor:

$$J(\sqrt{r_{\Psi}} \cdot \sigma_A) = r_{\Lambda} \cdot J(\sigma'_A) \quad (29)$$

$$\Leftrightarrow \sigma_A = \frac{1}{\sqrt{r_{\Psi}}} \cdot J^{-1}(r_{\Lambda} \cdot J(\sigma'_A)). \quad (30)$$

C. Derivation of the EXIT Functions

The expressions for the channel-related mutual information derived for the BEC and the BIAWGNC case are utilized for the prediction of the behavior of the SISO decoder. The repetition convolutional decoder utilizes the SISO decoder related to the employed convolutional code. Consequently, it generates equal *a posteriori* and *extrinsic* LLRs if assuming equal input information.

Hence, the EXIT function of the repetition convolutional decoder can be constructed from the EXIT function of the employed convolutional decoder according to

$$\mathcal{T}_B^{(\text{RCC})}(\mathcal{I}_{A_B} | \mathcal{I}_{A_Y}) = \mathcal{T}_B^{(\text{CC})}(\mathcal{I}_{A_B}^{(\text{CC})} | \mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})}), \quad (31)$$

with $\mathcal{I}_{A_B}^{(\text{CC})} = \mathcal{I}_{A_B}$, $0 \leq \mathcal{I}_{A_B}, \mathcal{I}_{A_Y} \leq 1$, and $\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})}$ given by (24) and (25), respectively. Consequently, only the EXIT characteristic for the pure convolutional code has to be simulated, while the influence of the random repeater can then be determined analytically. This provides a semi-analytical

procedure for generating the EXIT characteristic of repetition convolutional codes required for the evaluation of repetition convolutional turbo codes.

IV. EVALUATION

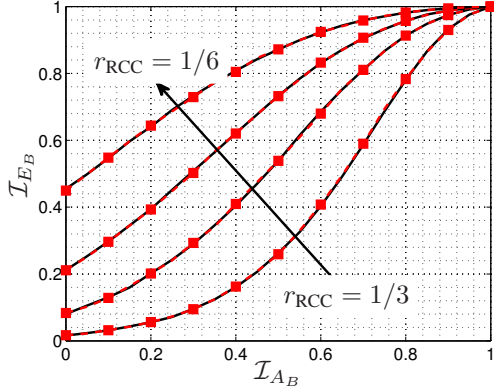
As an example, the systematic rate-1/2 component code of the UMTS-LTE turbo encoder with generator polynomials $\mathbf{G}_{\text{CC}} = (1, 15/13)_8$ is considered. The EXIT function of the resulting RCC is analytically computed by means of (31) for a communication channel modeled by either a BEC with erasure probability $P_e = 0.8$ or a BIAWGNC with channel quality $E_s/N_0 = -6$ dB. The corresponding EXIT functions are depicted in Fig. 3 for different target code rates $r_{\text{RCC}} \in \{1/3, 1/4, 1/5, 1/6\}$. The analytically derived EXIT functions (bold lines) are compared with their measured versions (dashed lines with filled squares). The influence of the bit repetition can be interpreted as a transmission without repetition over a communication channel with increased mutual information $\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})} \geq \mathcal{I}_{A_Y}$. Considering Fig. 3, it has been observed for the BEC case that the analytically derived EXIT functions exactly match their measured versions, while for the BIAWGNC case the analytical derivation provides a very close approximation due to the numerical computation of the J -function.

V. EXIT CHART OPTIMIZED REPETITION CONVOLUTIONAL TURBO CODES

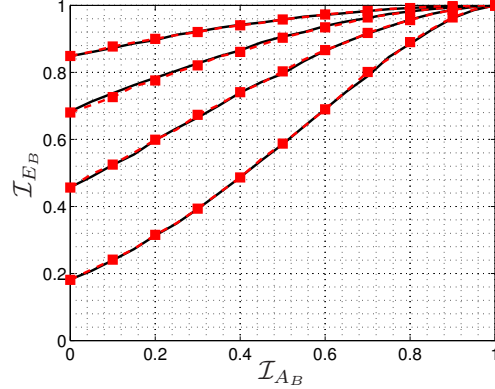
As a part of rate matching, RCCs construct lower rate codes based on a higher rate convolutional code in order to adjust the error robustness for the transmission of data with different error protection requirements through the same network. Often there is a requirement to allow for a near error-free transmission down to one distinct channel quality E_s/N_0 . In order to limit the energy and bandwidth consumption, it is reasonable to transmit as few additional bits as possible. Consequently, we are interested in discovering the required repetition rate meeting the formulated channel quality constraint. So far, this task could only be solved by carrying out a high number of Monte-Carlo simulations. A less time-consuming semi-analytical solution can now be formulated mathematically by means of the results derived in the previous section. Hence, only the EXIT function of the convolutional code has to be determined by simulations, while the optimal repetition rate can then be analytically derived relying on this EXIT function.

Exemplarily consider two parallel concatenated repetition convolutional codes with equal generator polynomials. The sampled EXIT characteristics of both SISO decoders, which contain K sample points each, are denoted by $\mathcal{C}_{\text{RCC}}(k)$ and $\mathcal{C}_{\text{RCC}}^{-1}(k)$ with $0 \leq k < K$. The latter signifies the sampled version of the inverse EXIT characteristic. The EXIT characteristic $\mathcal{T}_B^{(\text{RCC})}(\mathcal{I}_{A_B} | \mathcal{I}_{A_Y}) = \mathcal{T}_B^{(\text{CC})}(\mathcal{I}_{A_B} | \mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})})$ can be analytically computed by means of the EXIT characteristic of the underlying convolutional decoder and with $\mathcal{I}_{A_{\bar{Y}}}^{(\text{CC})}$ given according to (24) and (25), respectively. Decoding is possible if an open decoding tunnel exists. The maximum repetition rate $r_{\text{RR}} = r_{\Psi}/r_{\Lambda}$ can thus be calculated by solving the following maximization problem:

$$\max(r_{\text{RR}}) \quad (32)$$



(a) component code of PCCC scheme + BEC



(b) component code of PCCC scheme + BIAWGNC

Fig. 3. Comparison of analytically derived (bold lines) and measured EXIT functions (dashed lines with filled squares) of the repetition convolutional decoder based on the systematic rate-1/2 UMTS-LTE component code with generator polynomials $\mathbf{G}_{\text{CC}} = (1, 15/13)_8$ for different code rates $r_{\text{RCC}} \in \{1/3, 1/4, 1/5, 1/6\}$. Transmission has been performed over a BEC with erasure probability $P_e = 0.8$ and a BIAWGNC at $E_s/N_0 = -6$ dB.

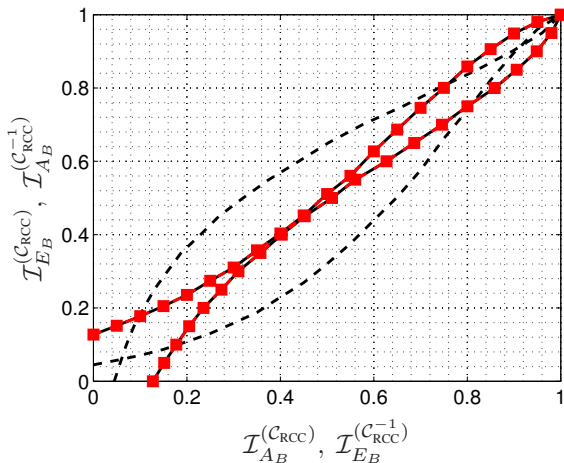


Fig. 4. EXIT chart optimization at $E_s/N_0 = -6$ dB for the systematic rate-1/3 UMTS-LTE turbo code. Three EXIT charts are plotted: EXIT chart for the UMTS-LTE decoder without rate matching (dashed lines), analytically derived optimized EXIT chart (bold lines) and measured optimized EXIT chart (dashed lines with filled squares) for the convolutional code with rate matching employing random repetition.

subject to

$$0 < r_{\text{RR}} \leq 1 \quad (33)$$

$$\mathcal{C}_{\text{RCC}}(k) - \mathcal{C}_{\text{RCC}}^{-1}(k) > \Omega, \quad 0 \leq k < K, \quad (34)$$

where Ω controls the openness of the decoding tunnel introduced for practical reasons.

As an example, the parallel concatenated systematic rate-1/3 convolutional turbo code employed in the UMTS-LTE standard is considered. Transmission is performed over a BIAWGNC at $E_s/N_0 = -6$ dB. The minimum openness of the tunnel is set to $\Omega = 0$. Solving (32), a maximum repetition rate of $r_{\text{RR}} = r_{\Psi}/r_{\Lambda} = 0.738$ has to be employed in order to allow for near error-free decoding. The resulting EXIT chart of the corresponding RCC is depicted by bold lines in Fig. 4 while the measured version is marked by dashed lines with filled squares. The EXIT chart of the underlying convolutional turbo code is plotted in the same diagram using dashed lines.

The decoding tunnel of the parallel concatenated RCC is extremely narrow which would require a huge block length and many iterations for reaching the upper right corner (1, 1). In order to relax these requirements, the openness Ω of the tunnel can be enforced to be greater than zero at the expense of a lower code rate.

VI. CONCLUSION

We have proposed an EXIT chart optimization of convolutional turbo coding and rate matching employing random repetition for any given channel quality without carrying out many time-consuming EXIT chart simulations. This includes the computation of the EXIT characteristic after convolutional encoding and rate matching by means of the EXIT characteristic of the employed convolutional code. At first, an analytical derivation of the optimal random repetition scheme is presented. Furthermore, a semi-analytical procedure for calculating the EXIT characteristics for BEC and BIAWGNC transmission with convolutional turbo coding and rate matching is derived and verified by an example. This result can then be utilized to optimize the system with respect to the required repetition rate for near error-free decoding at any target channel qualities as exemplarily shown for the UMTS-LTE turbo code.

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