EXIT Functions for Parallel Concatenated Insertion Convolutional Codes

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Abstract—Insertion convolutional codes and the underlying principles attract more and more attention, e.g., in iterative decoding, joint channel coding and cryptography or even in channel estimation. Known bits (dummy bits) are inserted into the information bit sequence before convolutional (turbo) encoding. These bits support the decoding of the information bits resulting in an improved decoding quality. Although this concept is widely employed, there does not exist any theoretical evaluation of these codes. We provide a basis for such an evaluation by means of their EXIT charts. We will analytically derive their EXIT charts as a function of the EXIT charts associated with the underlying convolutional code for a transmission over a BEC as well as a binary input/continuous output AWGN channel. These results can be adopted to all applications where perfect information is exploited by a SISO decoder and provide an excellent basis for the analytical prediction of the expected gains.

I. INTRODUCTION

Insertion convolutional codes (ICCs) have been introduced by Xu and Romme in [1] as a novel technique for adapting the rate or for constructing lower rate codes by inserting known bits (dummy bits) into the information bit sequence before convolutional encoding. It has been observed that the information provided by the dummy bits highly supports the decoding of the information bits.

Besides the application of rate matching, ICCs and the underlying principles attract more and more attention in recent studies. In [2] we consider a packet-switched multimedia transmission based on cross-layer communication where all header bits are fed back after error-free decoding to the SISO channel decoder. These bits are exploited as dummy bits within the turbo decoding resulting in an improved decoding of the payload. The underlying concept has further been applied to joint channel coding and cryptography [3], [4] where iterative decryption is performed based on two information blocks which are individually encrypted but jointly encoded by the channel encoder. After error-free decryption of the first block, this information can be exploited as perfect *a priori* information at the SISO decoder during the iterative decryption process of the second block. In [5], one information block is even substituted by dummy bits which are known in advance and can be exploited at once. In channel estimation, pilot symbols are mostly used for estimation of the current channel state. However, instead of inserting pilots at the modulation stage, in pilot symbol assisted coding schemes, a predetermined fraction of dummy bits is inserted into the information bit sequence before encoding [6]–[8]. These pilots
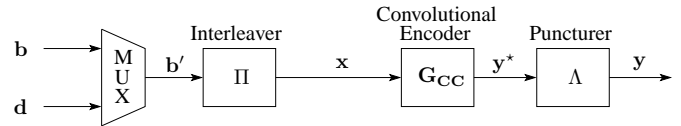
Fig. 1.   Structure of an insertion convolutional encoder.

are sometimes called *encoded pilots*. All these applications have in common that perfect information about a fraction of bits is exploited to improve the decoding of the rest of the bits relying on the concepts presented in [1].

Although there exist many application scenarios for ICCs or their principles, no information theoretical evaluation of their fundamentals has been performed so far. However, a comprehensive theoretical evaluation might be extremely useful to quantify the expected performance gains dependent on the fraction of inserted dummy bits. Such an evaluation can further lead to a deeper understanding of the effect of the dummy bit insertion enabling the analytical optimization of such schemes. Today's transmission systems commonly employ turbo codes consisting of two either parallel or serially concatenated convolutional codes. A widely used tool for the prediction of the convergence behavior of these codes is the EXIT chart analysis which has firstly been introduced by ten Brink in [9]. EXIT charts illustrate the flow of *extrinsic* information through the constituent SISO decoders. For turbo codes, this might be an appropriate measure to quantify the gains achievable by ICCs.

In what follows, EXIT charts for parallel concatenated ICCs are derived based on the EXIT charts of the underlying convolutional code. This enables a theoretical evaluation of the expected gains given by ICCs and provides a basis for mathematical system optimizations.

II. INSERTION CONVOLUTIONAL CODES

The structure of an insertion convolutional encoder is depicted in Fig. 1. The multiplexed vector $\mathbf{b}' = [\mathbf{b}\ \mathbf{d}]$ containing the dummy bits $\mathbf{d} = (d_1 \ldots d_\ell \ldots d_L)$ $(1 \le \ell \le L)$ and the information bits $\mathbf{b} = (b_1 \ldots b_m \ldots b_M)$ $(1 \le m \le M)$ is either deterministically or even randomly interleaved by an interleaver Π. The resulting sequence $\mathbf{x} = (x_1 \ldots x_k \ldots x_K)$ of length $K = M + L$ is then encoded by a convolutional encoder with generator polynomial(s) $\mathbf{G_{CC}}$ and code rate r_{CC} obtaining the output vector $\mathbf{y}^\star = (y_1^\star \ldots y_n^\star \ldots y_{N^\star}^\star)$ of length $N^\star = K \cdot r_{CC}^{-1}$. If $\mathbf{G_{CC}}$ is systematic and if the ICC is either a component code of a *parallel concatenated convolutional code* (PCCC) or the inner code of a *serially concatenated convolutional code* (SCCC), puncturing Λ is introduced in
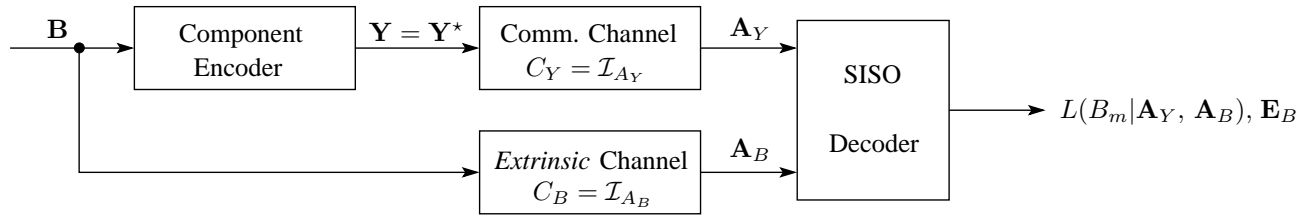
Fig. 2. Information theoretical decoding model for one of the components of a parallel concatenated turbo code.

order to eliminate all systematic dummy bits. These bits are known in advance and, thus, do not have to be transmitted over the channel saving energy and bandwidth. Accordingly, the length of the output vector $\mathbf{y} = (y_1 \ldots y_n \ldots y_N)$ is reduced by the number of dummy bits L resulting in $N = K \cdot r_{CC}^{-1} - L$. For non-systematic convolutional codes, Λ is neglected, i.e., $\mathbf{y} = \mathbf{y}^\star$ and $N = N^\star$. Please note that there are some applications where the transmission of the systematic dummy bits are reasonable (e.g. pilot symbols for channel estimation).

Hence, the code rate of the insertion convolutional encoder amounts to

$$r_{ICC} = \begin{cases} \frac{M}{M+L} \cdot r_{CC} & : \quad \mathbf{G_{CC}} \text{ non-systematic} \\ \frac{M}{(M+L) \cdot r_{CC}^{-1} - L} & : \quad \mathbf{G_{CC}} \text{ systematic} \end{cases} \quad . \quad (1)$$

III. EXIT CHART FUNDAMENTALS

In order to analyze the flow of mutual information through a SISO decoder, a general decoding model has been presented in [10] and is shown in Fig. 2. A successful application to randomly punctured convolutional codes is given in [11]. In what follows, capital letters are applied for random variables and small letters signify their realizations. For the sake of simplicity, the puncturing introduced in Fig. 1 is not considered explicitly in the following information theoretical evaluation since no additional information is provided by the transmission of the systematic dummy bits. Throughout this contribution we consider *parallel concatenated convolutional codes* (PCCCs). Nevertheless, all results can easily be adapted to serially concatenated convolutional codes (SCCCs) as we will describe in Sec. VI.

The SISO decoder receives *extrinsic* log-likelihood ratios (LLRs) $\mathbf{A}_B = (A_{B_1}, \ldots, A_{B_m}, \ldots, A_{B_M})$ about the information bits $\mathbf{B} = (B_1, \ldots, B_m, \ldots, B_M)$ from the other component of the PCCC which can be exploited by the SISO decoder as *a priori* information. This can be modeled by an *extrinsic* channel with capacity $\mathcal{I}_{A_B}$. Furthermore, channel-related LLRs $\mathbf{A}_Y = (A_{Y_1}, \ldots, A_{Y_n}, \ldots, A_{Y_N})$ about the output bits $\mathbf{Y} = (Y_1, \ldots, Y_n, \ldots, Y_N)$ are provided to the SISO decoder by the communication channel with capacity $\mathcal{I}_{A_Y}$. Note that the output bits may contain also the information bits if systematic component codes are employed. Both types of information are then exploited in order to determine the *extrinsic* LLRs E_{B_m} as well as the *a posteriori* LLRs $L(B_m|\mathbf{A}_Y, \mathbf{A}_B)$ of the input bits B_m. According to [9], the *extrinsic* LLRs E_{B_m} for PCCCs are given by means of the *a posteriori* LLRs and the *a priori* LLRs:

$$E_{B_m} = L(B_m|\mathbf{A}_Y, \mathbf{A}_B) - A_{B_m} - Z_{B_m}. \quad (2)$$

For systematic PCCCs, the LLR Z_{B_m} corresponds to the channel observation regarding the information bit B_m provided

by the underlying communication channel. For non-systematic PCCCs, there is no channel observation, i.e., $Z_{B_m} = 0$.

By means of the mutual *a priori* information

$$\mathcal{I}_{A_B} := \mathcal{I}(A_{B_m}; B_m) \text{ and} \quad (3)$$
$$\mathcal{I}_{A_Y} := \mathcal{I}(A_{Y_n}; Y_n), \quad (4)$$

the EXIT characteristic $\mathcal{T}_B(\mathcal{I}_{A_B}|\mathcal{I}_{A_Y})$ can be defined according to

$$\mathcal{T}_B(\mathcal{I}_{A_B}|\mathcal{I}_{A_Y}) = \mathcal{I}(E_{B_m}; B_m) =: \mathcal{I}_{E_B}, \quad (5)$$

where $\mathcal{I}_{A_Y}$ in (5) is considered as arbitrary but fixed. Since all bits B_m and Y_n can be assumed as i.i.d., we skip the subscripts in what follows unless there is a risk of confusion.

In order to obtain the desired EXIT characteristics $\mathcal{T}_B(\mathcal{I}_{A_B}|\mathcal{I}_{A_Y})$, at first, the mutual *a priori* information $\mathcal{I}_{A_B}$ is computed analytically by choosing an appropriate *extrinsic* channel. Then the *extrinsic* information $\mathcal{I}_{E_B}$ is determined. In general, the computation of $\mathcal{I}_{E_B}$ is impractical needing to rely on Monte Carlo simulation. In this contribution we focus on the *binary erasure channel* (BEC) and the *binary input/continuous output additive white Gaussian noise channel* (BIAWGNC) for the communication channel as well as the *extrinsic* channel. The evaluation of the *binary symmetric channel* (BSC) is out of our scope but can be easily derived from the results given in what follows.

Mutual A Priori Information for the BEC and the BIAWGNC

The following considerations are valid for the communication channel as well as the *extrinsic* channel. Therefore, we will skip the subscript associated with the respective channel unless there is a risk of confusion.

In the BEC case, the capacity $C = \mathcal{I}_A$ is simply given by means of the erasure probability P_e as

$$\mathcal{I}_A = 1 - P_e. \quad (6)$$

Considering a transmission over a BIAWGNC, the distribution of the LLRs at the input of the SISO decoder can be modeled by an independent Gaussian random variable A with conditional probability density function

$$p_A(A = \xi|B = b) = \frac{1}{\sqrt{2\pi}\sigma_A} \cdot \exp\left\{-\frac{(\xi - \frac{\sigma_A}{2}b)^2}{2\sigma_A^2}\right\}. \quad (7)$$

Hence, the mutual information $\mathcal{I}_A$ between the transmitted bipolar information bits B and the LLRs A can be expressed

according to [12]:

$$\mathcal{I}_A := \mathcal{I}(A = \xi; B = b)$$

$$= \frac{1}{2} \cdot \sum_{b=-1,1} \int_{-\infty}^{\infty} p_A(A = \xi | B = b) \cdot$$

$$\cdot \log_2 \frac{2 \cdot p_A(A = \xi | B = b)}{p_A(A = \xi | B = -1) + p_A(A = \xi | B = 1)} d\xi. \quad (8)$$

Substituting (7) into (8) results in ten Brink's well known J-function

$$J(\sigma_A) := \mathcal{I}_A = 1 - \int_{-\infty}^{\infty} \frac{\exp\left\{ -\frac{(\xi - \frac{\sigma_A^2}{2})^2}{2\sigma_A^2} \right\}}{\sqrt{2\pi}\sigma_A} \cdot \log_2\left(1 + e^{-\xi}\right) d\xi,$$
$$(9)$$

which cannot be expressed in closed-form and, thus, has to be numerically approximated. This function provides an interrelation between the mutual information $\mathcal{I}_A$ and the variance σ_A of the related LLRs. Hence, Monte-Carlo simulations can be carried out for approximating the flow of *extrinsic* information by simply varying σ_A.

IV. EXIT FUNCTIONS FOR ICCS

In order to derive analytically an expression for the EXIT function of the insertion convolutional decoder by means of the EXIT function of the pure convolutional decoder, we assume a random interleaver in Fig. 1. It can easily be shown that this EXIT function slightly underestimate that one associated with an ICC employing a deterministic interleaver. However, this is out of the scope of this paper.

The transformed information theoretical decoding model for an insertion convolutional decoder with random interleaving is depicted in Fig. 3. In the ICC decoding model, the vector $\mathbf{X}$ rather than the information bit vector $\mathbf{B}$ is considered since dummy bit insertion using a random interleaver Π has been employed before convolutional encoding. The *extrinsic* channel models the distribution of the *extrinsic* LLRs $\mathbf{A}_B$ regarding the information bits $\mathbf{B}$ provided by the other component of the considered PCCC. They are exploited as *a priori* information in the SISO decoding process. However, no information is given by the *extrinsic* channel about the dummy bits $\mathbf{D}$. The perfect *a priori* information provided by the dummy bits can be modeled by a parallel concatenated BEC with erasure probability

$$P_{\text{e}} = 1 - P_D = 1 - \frac{L}{K} = \frac{M}{K} \quad (10)$$

and capacity C_D according to (6). The BEC secures that only the dummy bit fraction $P_D = L/K$ is exploited by the SISO decoder as perfect *a priori* information $\mathbf{A}_D$, while no additional information is generated for the information bits $\mathbf{B}$. Consequently, $\mathbf{A}_B$ equals zero at each dummy bit position and $\mathbf{A}_D$ equals zero at each information bit position. Hence, the vectors $\mathbf{A}_B$ and $\mathbf{A}_D$ can be added resulting in the vector $\mathbf{A}_X$ which is exploited as overall *a priori* information by the SISO decoder.

However, in order to obtain the mutual *a priori* information $\mathcal{I}_{A_X} := \mathcal{I}(A_B A_D; X)$ at the input of the SISO decoder, a simple addition of the mutual information $\mathcal{I}_{A_B} := \mathcal{I}(A_B; X)$ and $\mathcal{I}_{A_D} := \mathcal{I}(A_D; X)$ is not possible. According to [13] the mutual a priori information $\mathcal{I}_{A_X}$ at the output of two parallel concatenated channels is given in general by

$$\mathcal{I}_{A_X} := \mathcal{I}(A_B A_D; X)$$
$$= \mathcal{I}(A_B; X) + \mathcal{I}(A_D; X) - \mathcal{I}(A_B; A_D)$$
$$= \mathcal{I}_{A_B} + \mathcal{I}_{A_D} - \mathcal{I}(A_B; A_D), \quad (11)$$

where $\mathcal{I}(A_B; A_D)$ is called the *information defect* with respect to $\mathcal{I}_{A_X} \leq \mathcal{I}_{A_B} + \mathcal{I}_{A_D}$.

In this contribution, the extrinsic channel as well as the communication channel is either a BEC or a BIAWGNC. For these types of channels (11) can be computed using the following proposition.

Proposition 4.1: Consider the parallel concatenation of two channels, where the first channel corresponds to a BEC with mutual information $\mathcal{I}_{A_D} = P_D$ and the second one either to a BEC or to a BIAWGNC with mutual information $\mathcal{I}_{A_B}$. Then, the mutual Information $\mathcal{I}_{A_X}$ at the output of the parallel concatenated channels can be expressed as

$$\mathcal{I}_{A_X} = \mathcal{I}_{A_B} + \mathcal{I}_{A_D} - \mathcal{I}_{A_B} \cdot \mathcal{I}_{A_D} \quad (12)$$
$$= P_D + (1 - P_D) \cdot \mathcal{I}_{A_B}. \quad (13)$$

Proof: See [13] for the BEC case.
Proof for the BIAWGNC case:
The conditional probability density function $p_{A_X}(A_X = \xi | X = x) =: p_{A_X}(\xi | x)$ of the information A_X at the output of the parallel concatenation is given by

$$p_{A_X}(\xi | x) = (1 - P_D) \cdot p_{A_B}(\xi | x) + P_D \cdot \delta_{x \cdot \infty}, \quad (14)$$

with $\delta_{x \cdot \infty}$ being the Dirac-delta function shifted to $\xi \to x \cdot \infty$ and $p_{A_B}(\xi | x)$ given according to (7). The first term describes the influence of the *extrinsic* channel which is modeled as BIAWGNC, while the second term accounts for the effect of the dummy bit insertion. Due to the symmetry of $p_{A_X}(A_X = \xi | X = x)$, $\mathcal{I}_{A_X}$ is given by (8) according to

$$\mathcal{I}_{A_X} = \int_{-\infty}^{\infty} p_{A_X}(\xi | x = 1)$$

$$\cdot \underbrace{\log_2 \frac{2 \cdot p_{A_X}(\xi | x = 1)}{p_{A_X}(\xi | x = -1) + p_{A_X}(\xi | x = 1)}}_{=: g(\xi | x)} d\xi. \quad (15)$$

The contribution of $g(\xi | x)$ to the integration result can be determined by computing the auxiliary function

$$g(\xi | x) - \log_2 \frac{2 \cdot p_{A_B}(\xi | x = 1)}{p_{A_B}(\xi | x = -1) + p_{A_B}(\xi | x = 1)} \quad (16)$$

$$= \log_2 \frac{p_{A_X}(\xi | x = 1) \cdot (p_{A_B}(\xi | x = -1) + p_{A_B}(\xi | x = 1))}{(p_{A_X}(\xi | x = -1) + p_{A_X}(\xi | x = 1)) \cdot p_{A_B}(\xi | x = 1)}$$

$$= \log_2 \frac{f_1(\xi | x) + f_2(\xi | x)}{f_1(\xi | x) + f_3(\xi | x)}. \quad (17)$$

From a pure mathematical viewpoint, the Dirac-delta function is not strictly a function, but can be formally defined as an irregular distribution. For the sake of simplicity, we use the notation $\delta_{x \cdot \infty} := \lim_{\substack{\epsilon \to 0 \\ \xi_0 \to \infty}} \delta_\epsilon(\xi - x \cdot \xi_0)$, where $\delta_\epsilon(\bullet)$ is sometimes called a nascent delta function.
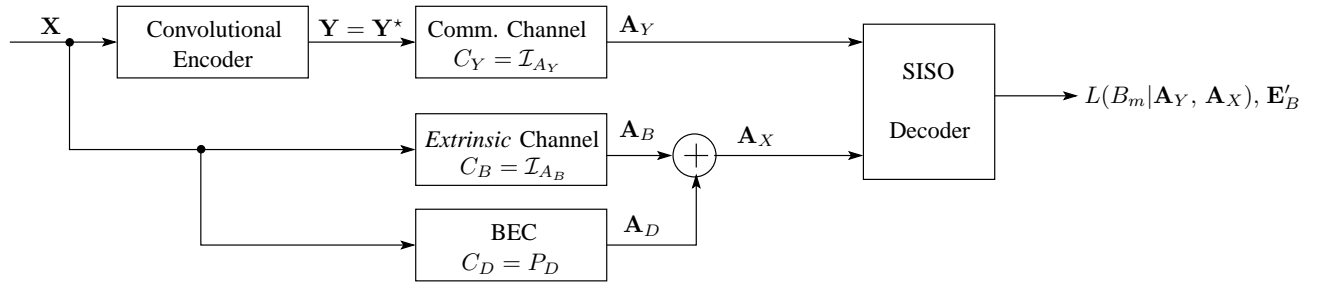
Fig. 3. Transformed information theoretical decoding model for the insertion convolutional decoder.

The auxiliary functions $f_1(\xi|x)$, $f_2(\xi|x)$ and $f_3(\xi|x)$ can be computed by means of (14) to

$$f_1(\xi|x) = (1 - P_D) \cdot p_{A_B}(\xi|x = 1)$$
$$\cdot (p_{A_B}(\xi|x = -1) + p_{A_B}(\xi|x = 1)) \quad (18)$$
$$f_2(\xi|x) = P_D \cdot \delta_\infty \cdot (p_{A_B}(\xi|x = -1)$$
$$+ p_{A_B}(\xi|x = 1)) = 0 \quad (19)$$
$$f_3(\xi|x) = P_D \cdot \delta_\infty \cdot p_{A_B}(\xi|x = 1)$$
$$+ P_D \cdot \delta_{-\infty} \cdot p_{A_B}(\xi|x = 1) = 0, \quad (20)$$

where $f_2(\xi|x)$ and $f_3(\xi|x)$ can easily be derived by using the suppression property of the Dirac-delta function. This results in

$$g(\xi|x) - \log_2 \frac{2 \cdot p_{A_B}(\xi|x = 1)}{p_{A_B}(\xi|x = -1) + p_{A_B}(\xi|x = 1)} = 0 \quad (21)$$

Hence, Equation (15) simplifies to

$$\mathcal{I}_{A_X} = \int_{-\infty}^{\infty} p_{A_X}(\xi|x = 1) \cdot$$

$$\cdot \log_2 \frac{2 \cdot p_{A_B}(\xi|x = 1)}{p_{A_B}(\xi|x = -1) + p_{A_B}(\xi|x = 1)} d\xi$$

$$= \int_{-\infty}^{\infty} ((1 - P_D) \cdot p_{A_B}(\xi|x = 1) + P_D \cdot \delta_\infty) \cdot \log_2(\bullet) d\xi$$

$$= P_D + (1 - P_D) \cdot \mathcal{I}_{A_B} \quad (22)$$

$$= \mathcal{I}_{A_D} + \mathcal{I}_{A_B} - \mathcal{I}_{A_D} \cdot \mathcal{I}_{A_B}, \quad (23)$$

exploiting the Dirac-delta property again. ∎

Proposition 4.1 provides the desired expression for the equivalent mutual information $\mathcal{I}_{A_B}^{(CC)}$ exploited by the SISO decoder in the system with no dummy bit insertion as *a priori* information:

$$\mathcal{I}_{A_B}^{(CC)} := \mathcal{I}_{A_X} = \mathcal{I}_{A_B} + \mathcal{I}_{A_D} - \mathcal{I}_{A_B} \cdot \mathcal{I}_{A_D} \quad (24)$$

$$= \mathcal{I}_{A_B} + P_D \cdot (1 - \mathcal{I}_{A_B}) \geq \mathcal{I}_{A_B}. \quad (25)$$

Consequently, dummy bits provide additional mutual *a priori* information to the SISO decoder resulting in a more reliable decoding.

In both systems, the same SISO decoder is used. Consequently, both systems generate equal *a posteriori* LLRs and *extrinsic* LLRs if assuming equal input information.

Hence, the EXIT function of the insertion convolutional decoder can be constructed from the EXIT functions of the applied convolutional decoder according to

$$\mathcal{T}_B^{(ICC)}(\mathcal{I}_{A_B}|\mathcal{I}_{A_Y}) = \mathcal{T}_B^{(CC)}(\mathcal{I}_{A_B}^{(CC)}|\mathcal{I}_{A_Y}^{(CC)}), \quad (26)$$

with $\mathcal{I}_{A_Y}^{(CC)} = \mathcal{I}_{A_Y}$ and $0 \leq \mathcal{I}_{A_B}, \mathcal{I}_{A_Y} \leq 1$. Obviously, the dummy bit insertion entails a shift in the operation point towards higher mutual *a priori* information, namely from $(\mathcal{I}_{A_B}, \mathcal{I}_{A_Y})$ to $(\mathcal{I}_{A_B}^{(CC)}, \mathcal{I}_{A_Y}^{(CC)})$.

V. EVALUATION

As an example, the systematic rate-1/2 component code of the UMTS-LTE turbo encoder with generator polynomials $\mathbf{G_{CC}} = (1, 15/13)_8$ is employed as convolutional code by the insertion convolutional encoder. The EXIT function of the resulting ICC is then analytically computed by means of (26) for different channels:

(a) Communication channel and *extrinsic* channel are modeled by a BEC.
(b) Communication channel and *extrinsic* channel are modeled by a BIAWGNC.

The corresponding EXIT functions are depicted in Fig. 4. They are analytically derived for the insertion convolutional decoder (bold lines) according to (26) and compared with their measured EXIT functions (dashed lines with filled squares) for different dummy bit fractions $L/K \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$. The communication channel is either a BEC with erasure probability $P_e = 0.7$ or a BIAWGNC with channel quality $E_s/N_0 = -6\,\mathrm{dB}$.

As we have stated in the previous section, $\mathcal{T}_B^{(ICC)}(\mathcal{I}_{A_B}|\mathcal{I}_{A_Y})$ can be derived by means of the EXIT function related to the underlying convolutional decoder by shifting the operation point from $(\mathcal{I}_{A_B}, \mathcal{I}_{A_Y})$ to $(\mathcal{I}_{A_B}^{(CC)}, \mathcal{I}_{A_Y}^{(CC)})$. This leads to a stretching of the diminished interval $[L/K, 1]$ to $[0, 1]$ due to the mapping of $\mathcal{I}_{A_B}$ to $\mathcal{I}_{A_B}^{(CC)}$.

Considering Fig. 4, it has been observed for the BEC case that the analytically derived EXIT functions exactly match their measured versions, while for the BIAWGNC case the analytical derivation provides a very close approximation. The condition for the consistency of both curves is the equality of the amount of mutual *a priori* information and the consistency of the underlying probability distribution. However, the latter condition is not fulfilled due to the increasing divergence of both distributions for an increasing number of dummy bits (c.f. Equation (14)). For a high number of dummy bits, the parallel concatenation of the *extrinsic* BIAWGNC and the BEC modeling the dummy bit insertion converges to a BEC providing a lower bound for the measured EXIT function according to [13]. Nevertheless, both EXIT functions match
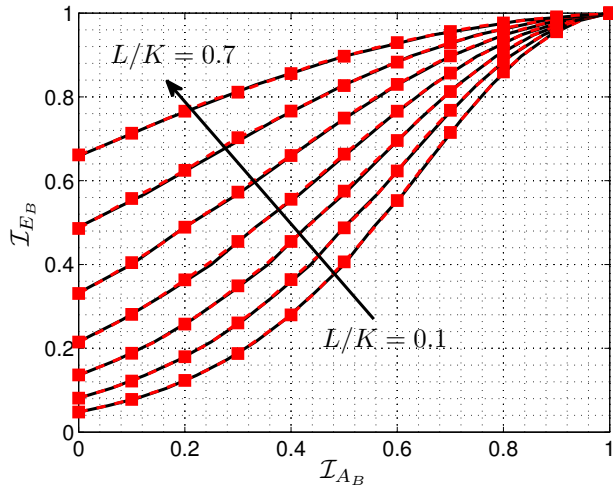
(a) BEC, component code of PCCC scheme
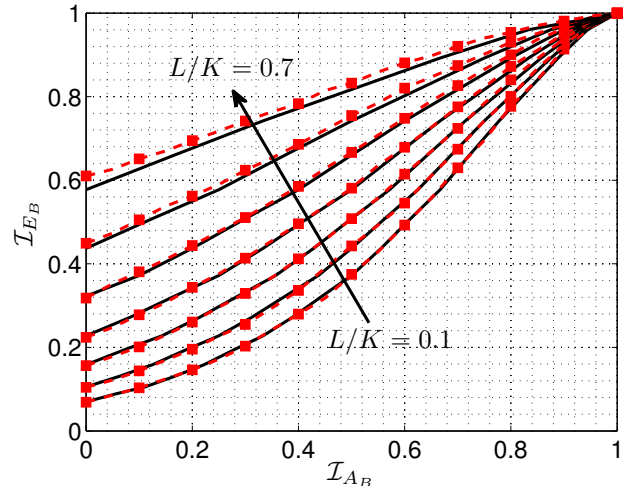
(b) BIAWGNC, component code of PCCC scheme

Fig. 4. Comparison of analytically derived (bold lines) and measured EXIT functions (dashed lines with filled squares) of the insertion convolutional encoder based on the systematic rate-1/2 UMTS-LTE component code with generator polynomials $\mathbf{G_{CC}} = (1, 15/13)_8$ for different dummy bit fractions $L/K \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$. Transmission has been performed over a BEC with erasure probability $P_\mathrm{e} = 0.7$ and a BIAWGNC with channel quality $E_s/N_0 = -6\,\mathrm{dB}$.

almost perfectly for a dummy bit fraction of up to 40 % of the block size and, thus, for many relevant application scenarios.

In order to optimize ICCs with respect to the inserted fraction of dummy bits, many time-consuming EXIT chart simulations have to be carried out so far. However, using the expressions derived in the previous section, this issue can be solved analytically in less time and with higher accuracy.

VI. EXTENSION TO SERIALLY CONCATENATED CONVOLUTIONAL CODES

The results in previous sections can easily be adapted to *serially concatenated convolutional codes* (SCCCs). In this case it has to be distinguished between inner codes and outer codes of SCCCs. Inner codes obey the same decoding model as PCCCs and, consequently, the same mathematical expressions. Only outer codes has to be analyzed separately. For these codes, no channel information is provided and only extrinsic information about the output bits is delivered by the inner decoder yielding $\mathbf{A}_B = \mathbf{0}$. Furthermore, these codes have to generate *extrinsic* information about the output bits rather than the input bits, because the output bits generated by the outer code serves as input for the inner code. However, a full derivation of the corresponding EXIT function for outer codes in SCCCs is out of the scope of this paper.

VII. CONCLUSION

An information theoretical derivation and evaluation based on EXIT functions for insertion convolutional codes employed in parallel concatenated turbo codes has been presented. This provides a basis for the prediction of the expected gains achieved by the insertion of known bits (dummy bits) into the information bit sequence before convolutional encoding. The corresponding EXIT functions were analytically derived for random dummy bit insertion as a function of the EXIT functions of the pure convolutional code and were compared to their measured versions. A transmission over the BEC as well as the binary input/continuous output AWGN channel was

considered. This provides a great basis for evaluating analytically the expected gains achieved by insertion convolutional codes with respect to the inserted dummy bit fraction avoiding extensive simulations.

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