Turbo DeCodulation: Iterative Combined Demodulation and Source-Channel Decoding

Thorsten Clevorn, Johannes Brauers, Marc Adrat, and Peter Vary

***Abstract*— We propose the combination of iterative demodulation and iterative source-channel decoding as a multiple Turbo process. The receiver structures of bit-interleaved coded modulation with iterative decoding (BICM-ID), iterative source-channel decoding (ISCD), and iterative source coded modulation (ISCM) are merged to one novel Turbo system, in which in two iterative loops reliability information is exchanged between the three single components, demodulator, channel decoder, and (softbit) source decoder. Simulations show quality improvements compared to the different previously known systems, which use iterative processing only for two components of the receiver.**

***Index Terms*— Turbo DeCodulation, turbo principle, ISCD, BICM-ID, ISCM.**

I. Introduction

With the discovery of Turbo codes [1],[2],[3] channel coding close to the Shannon limit becomes possible with moderate computational complexity. In the past years the Turbo principle of exchanging *extrinsic* information between separate channel decoders has been adapted to other components of the receiver. In our case a receiver consists of demodulator, channel decoder, and source decoder.

In [4],[5] *bit-interleaved coded modulation with iterative decoding* (BICM-ID) is presented, in which the channel decoder exchanges *extrinsic* reliabilities with the demodulator. This technique for *iterative demodulation* is based on non-iterative *bit-interleaved coded modulation* (BICM) [6],[7], the standard transmission scheme for higher order modulations in todays wireless communication systems, e.g., WLAN, due to its simplicity and flexibility and its robustness against fading.

To exploit the residual redundancy in the source coded parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, *iterative source-channel decoding* (ISCD) has been developed in [8],[9],[10]. The *a priori* knowledge on the residual redundancy, e.g., non-uniform probability distribution or auto-correlation, is utilized for *error concealment* by a derivative of a *softbit source decoder* (SBSD) [11], which exchanges *extrinsic* reliabilities with a channel decoder.

In *iterative source coded modulation* (ISCM) [12] no channel coding is needed at all and the *extrinsic* reliabilities are exchanged between the demodulator and the SBSD.

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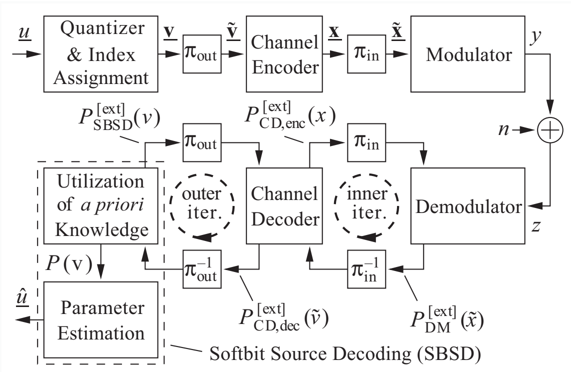

Fig. 1. Baseband model of the Turbo DeCodulation system.

The *Turbo DeCodulation* scheme presented in this letter is a multiple Turbo process, in which the *extrinsic* reliabilities are exchanged between all three receiver components, *demodulator*, *channel decoder*, and *(softbit) source decoder*. The term *Turbo DeCodulation* is chosen according to the short-term *codulation* [13] for the, in our case, source and channel coded modulation. *Turbo DeCodulation* can be either interpreted as serial concatenation of BICM-ID and ISCD via a common channel code or as integration of a single channel code into ISCM.

II. The Turbo DeCodulation System

In Fig. 1 the baseband model of the proposed *Turbo DeCodulation* system is depicted. The *inner* iterative loop corresponds to a BICM-ID system [4],[5], while the *outer* iterative loop is similar to an ISCD system [8],[9],[10].

A. Transmitter

At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ of K_S source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \dots K_S$ denoting the position in the frame. The single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}_\kappa^{(\xi)}$, $\xi = 1, \dots 2^{M_\kappa}$. To each quantizer reproduction level $\bar{u}_\kappa^{(\xi)}$ selected at time instant τ a unique bit pattern $\mathbf{v}_{\kappa,\tau}$ of M_κ bits is assigned according to the index assignment Γ, $\mathbf{v}_{\kappa,\tau} = \Gamma(\bar{u}_\kappa^{(\xi)})$. For simplicity we assume $M_\kappa = M$ for all κ. The single bits of a bit pattern $\mathbf{v}_{\kappa,\tau}$ are indicated by $v_{\kappa,\tau}^{(m)}$, $m = 1, \dots M$. The frame of bit patterns is denoted by $\underline{\mathbf{v}}_\tau$.

The first and outer bit interleaver π_{out} scrambles the incoming frame $\underline{\mathbf{v}}_\tau$ of data bits to $\underline{\tilde{\mathbf{v}}}_\tau$ in a deterministic manner. Notice, bit interleaving as well as channel encoding need not to be limited to the present set $\underline{\mathbf{v}}_\tau$ of bit patterns resp. $\underline{\tilde{\mathbf{v}}}_\tau$. Both routines can also be realized for a time sequence of $\Lambda+1$

consecutive sets, e.g., $\underline{\mathbf{v}}_{\tau-\Lambda}, \ldots \underline{\mathbf{v}}_{\tau}$, if a delay of (in maximum) Λ time instants is tolerable in a practical application. However, to simplify matters, in the following we consider only a single time instant τ.

For channel encoding of a frame $\underline{\tilde{\mathbf{v}}}_{\tau}$ of bits $v_{n,\tau}$, $n=1\ldots N$, we use a standard terminated *recursive systematic convolutional* (RSC) code of constraint length $J+1$ and a code rate (close to) $r=1/2$. In general, any channel code can be used as long as the respective decoder is able to provide the required *extrinsic probabilities*. For termination of the RSC code J tail bits are appended to the frame $\underline{\tilde{\mathbf{v}}}_{\tau}$. The resulting codeword is denoted by $\underline{\mathbf{x}}_{\tau}$ with systematic bits $x_{n,\tau}^{(1)}$ and parity bits $x_{n,\tau}^{(2)}$ and $n=1,\ldots N+J$.

The second and inner bit-interleaver π_{in} permutes this codeword $\underline{\mathbf{x}}_{\tau}$ to $\underline{\tilde{\mathbf{x}}}_{\tau}$. Besides ensuring uncorrelated *extrinsic* information just as the outer interleaver, this inner interleaver is additionally responsible for breaking up burst errors on the transmission link.

The interleaved codeword $\underline{\tilde{\mathbf{x}}}_{\tau}$ is divided into bit patterns $\tilde{\mathbf{x}}_{k,\tau}$, $k=1,\ldots K_C$, with I single bits $\tilde{x}_{k,\tau}^{(i)}$, $i=1,\ldots I$. In case the last bit pattern $\tilde{\mathbf{x}}_{K_C,\tau}$ is not completely filled the remaining positions are padded by zeros. The modulator maps each pattern $\tilde{\mathbf{x}}_{k,\tau}$ according to a mapping rule μ to a complex modulated symbol $y_{k,\tau}$ out of the *signal constellation set* $\mathcal{Y}$, $y_{k,\tau} = \mu(\tilde{\mathbf{x}}_{k,\tau})$. The respective inverse relation is denoted by μ^{-1}, with $\tilde{x}_{k,\tau}^{(i)}=\mu^{-1}(y_{k,\tau})^{(i)}$. The modulated symbols are normalized to an average energy of $\mathrm{E}\{\|y_{k,\tau}\|^2\}=1$.

On the channel complex additive white Gaussian noise (AWGN) $n_{k,\tau} = n'_{k,\tau} + jn''_{k,\tau}$ with a known power spectral density of $\sigma_n^2=N_0$ ($\sigma_{n'}^2=\sigma_{n''}^2=N_0/2$) is applied, i.e., $z_{k,\tau}=y_{k,\tau}+n_{k,\tau}$.

B. Receiver

The received symbols $z_{k,\tau}$ are evaluated in a multiple Turbo process, which exchanges *extrinsic* reliabilities between demodulator and channel decoder in the *inner* iterations, and between channel decoder and *softbit source decoder* in the *outer* iterations. Such reliability information can either be evaluated in terms of probabilities $P(\cdot)$ or in so-called *log-likelihood* ratios, or short L-values [2].

1) Demodulator (DM): The demodulator computes *extrinsic* probabilities $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ for each bit $\tilde{x}_{k,\tau}^{(i)}$ being $b=0,1$ according to (for details see [4],[5])

$$P_{\text{DM}}^{[\text{ext}]}(\tilde{x}_{k,\tau}^{(i)}{=}b) \sim \sum_{\hat{y}\in\mathcal{Y}_b^i} P(z_{k,\tau}|\hat{y}) \prod_{j=1,j\neq i}^{I} P_{\text{CD,enc}}^{[\text{ext}]}\Big(\tilde{x}_{k,\tau}^{(j)}{=}\mu^{-1}(\hat{y})^{(j)}\Big). \quad (1)$$

Each $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ consists of the sum over all possible channel symbols $\hat{y}$ for which the i^{th} bit of the corresponding bit pattern $\tilde{\mathbf{x}} = \mu^{-1}(\hat{y})$ is b. These channel symbols form the subset $\mathcal{Y}_b^i$ with $\mathcal{Y}_b^i=\{\mu([\tilde{x}^{(1)},\ldots\tilde{x}^{(i)}])|\tilde{x}^{(i)}=b\}$. The conditional probability density $P(z_{k,\tau}|\hat{y})$ is given by $P(z_{k,\tau}|\hat{y}) = (1/\pi\sigma_n^2)\exp(-\|z_{k,\tau}-\hat{y}\|^2/\sigma_n^2)$. $P_{\text{CD,enc}}^{[\text{ext}]}$ is the *extrinsic* information provided by the channel decoder. In the first iteration the $P_{\text{CD,enc}}^{[\text{ext}]}(\tilde{x})$ are set as equiprobable.

2) Channel Decoder (CD): In literature, concepts how to determine bit-wise *extrinsic* probabilities $P_{\text{CD,enc}}^{[\text{ext}]}$ and $P_{\text{CD,dec}}^{[\text{ext}]}$, or their respective L-values, from the artificial dependencies explicitly introduced by channel encoding are well known. The most famous example is denoted as *symbol-by-symbol maximum a posteriori* (MAP) decoder whose detailed rules are discussed, e.g., in [14],[15]. Note, in case of *Turbo DeCodulation* the channel decoder needs to compute *extrinsic* reliabilities for decoded bits $v_{n,\tau}$ as well as for the encoded bits $x_{n,\tau}$. For the $r=1/2$ RSC code considered in this letter with one data bit $v_{n,\tau}(E)$ and two encoded bit $x_{n,\tau}^{(l)}(E)$ per trellis edge E of the trellis section n, we get

$$P_{\text{CD,dec}}^{[\text{ext}]}(v_{n,\tau}{=}b) \sim \quad (2)$$
$$\sum_{E|v_{n,\tau}=b} \alpha_{n-1,\tau}(E)\cdot P_{\text{DM}}^{[\text{ext}]}\Big(x_{n,\tau}^{(1)}(E)\Big)\cdot P_{\text{DM}}^{[\text{ext}]}\Big(x_{n,\tau}^{(2)}(E)\Big)\cdot \beta_{n,\tau}(E),$$

$$P_{\text{CD,enc}}^{[\text{ext}]}(x_{n,\tau}^{(l)}{=}b) \sim \quad (3)$$
$$\sum_{E|x_{n,\tau}^{(l)}=b} \alpha_{n-1,\tau}(E)\cdot P_{\text{SBSD}}^{[\text{ext}]}\Big(v_{n,\tau}(E)\Big)\cdot P_{\text{DM}}^{[\text{ext}]}\big(x_{n,\tau}^{[\text{ext}]}(E)\big)\cdot \beta_{n,\tau}(E),$$

with $x_{n,\tau}^{[\text{ext}]}(E)$ being the extrinsic encoded bit, i.e., the other encoded bit, of edge E with respect to $x_{n,\tau}^{(l)}(E)$. The probabilities for the starting point $\alpha_{n-1,\tau}(E)$ and the end point $\beta_{n,\tau}(E)$ of each edge E are obtained by a forward resp. backward recursion. For details we refer to the literature [14],[15].

3) Softbit Source Decoder (SBSD): The algorithm how to compute the $P_{\text{SBSD}}^{[\text{ext}]}(v)$ of SBSD is based on a fully connected trellis for the parameters and it has been derived in [8],[9],[10]. The *extrinsic* probability $P_{\text{SBSD}}^{[\text{ext}]}(v)$ is obtained by

$$P_{\text{SBSD}}^{[\text{ext}]}(v_{\kappa,\tau}^{(m)}=b) \sim \quad (4)$$
$$\sum_{\mathbf{v}_{\kappa,\tau}^{[\text{ext},m]}} \gamma(\mathbf{v}_{\kappa,\tau}^{[\text{ext},m]}) \sum_{\mathbf{v}_{\kappa,\tau-1}} P(\mathbf{v}_{\kappa,\tau}^{[\text{ext},m]}|\mathbf{v}_{\kappa,\tau-1}, v_{\kappa,\tau}^{(m)}{=}b)\cdot \alpha_{\tau\text{-}1}(\mathbf{v}_{\kappa,\tau-1}),$$

using the *a priori* knowledge $P(\mathbf{v}_{\kappa,\tau}|\mathbf{v}_{\kappa,\tau-1})$ and the present *extrinsic* edge transition probabilities $\gamma(\mathbf{v}_{\kappa,\tau}^{[\text{ext},m]})$, $\mathbf{v}_{\kappa,\tau} = \{v_{\kappa,\tau}^{(m)}, \mathbf{v}_{\kappa,\tau}^{[\text{ext},m]}\}$. The $\alpha_{\tau\text{-}1}(\mathbf{v}_{\kappa,\tau-1})$ are computed by a forward recursion. Again, we refer to the literature for details [8],[9],[10],[11].

4) Parameter Estimation: In the final iteration parameter-oriented *a posteriori* knowledge is determined by

$$P(\mathbf{v}_{\kappa,\tau}|z_1,\ldots z_\tau) = C\gamma(\mathbf{v}_{\kappa,\tau}) \sum_{\mathbf{v}_{\kappa,\tau-1}} P(\mathbf{v}_{\kappa,\tau}|\mathbf{v}_{\kappa,\tau\text{-}1})\alpha_{\tau\text{-}1}(\mathbf{v}_{\kappa,\tau\text{-}1}). \quad (5)$$

The constant factor C ensures that the *total probability theorem* is fulfilled. Thus, if the *minimum mean squared error* (MMSE) serves as fidelity criterion, the individual estimates are given by [11]

$$\hat{u}_{\kappa,\tau} = \sum_{\xi} \bar{u}_{\kappa}^{(\xi)} \cdot P(\mathbf{v}_{\kappa,\tau}\hat{=}\xi|z_1,\ldots z_\tau) \quad . \quad (6)$$

III. Simulation Results

The capabilities of the proposed *Turbo DeCodulation* scheme shall be demonstrated by simulation. Instead of using any specific speech, audio, or video encoder, we model $K_S=500$ statistically independent source codec parameters u by K_S independent 1^{st} order Gauss-Markov processes with auto-correlation $\rho=0.8$, a typical value for some parameters of source codecs. Each parameter $u_{\kappa,\tau}$ is scalarly quantized by a Lloyd-Max quantizer using $M=3$ bits/parameter. The

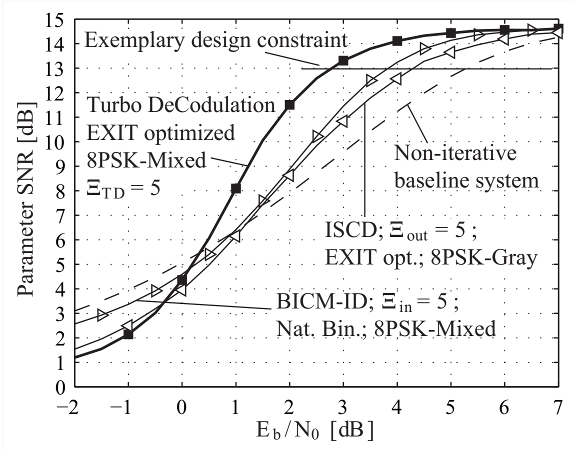

Fig. 2. Parameter SNR simulation results.

index assignment Γ is either *natural binary* (if SBSD standalone) or *EXIT optimized* [16] (if SBSD in the loop). The bit-interleaved frame $\underline{\tilde{\mathbf{v}}}_\tau$ is channel encoded using a memory $J = 2$, terminated RSC code with generator polynomial $\mathbf{G} = (1, \frac{1}{1+D+D^2})$. The modulator maps $I = 3$ bits to one channel symbol by either 8PSK-Gray (if demodulator standalone) or 8PSK-Mixed mapping [4] (if demodulator in the loop). In [5] other mappings are presented, which exhibit a better asymptotic performance than 8PSK-Mixed. But with the SBSD providing good *extrinsic* information to the channel decoder already at relatively low E_b/N_0, the 8PSK-Mixed mapping is the better choice for the given settings, due to its waterfall region being at a likewise low E_b/N_0 [4],[5].

At the *Turbo DeCodulation* receiver Ξ_{outer} *outer* iterations and Ξ_{in} *inner* iterations are performed. For the order of inner and outer iterations there exist lots of possibilities. For simplicity we set $\Xi_{\text{TD}} = \Xi_{\text{in}} = \Xi_{\text{out}}$ and restrict ourselves to the case in which demodulator, channel decoder and SBSD are executed sequentially in this order. This results in single *inner* and *outer* iterations being performed alternately. Each *inner* iteration is followed by one *outer* iteration. Except for the first and the last iteration, this can be seen as both iterative loops being executed in parallel, i.e., the channel decoder feeding both loops simultaneously.

The *parameter signal-to-noise ratio* (SNR) between the originally generated parameters $u_{\kappa,\tau}$ and the reconstructed estimates $\hat{u}_{\kappa,\tau}$ is used for quality evaluation. The simulation results are depicted in Fig. 2.

The dashed line is the result for a non-iterative ($\Xi_{\text{TD}} = 1$) baseline system which uses *natural binary* index assignment Γ and standard 8PSK-Gray mapping μ. The solid line marked "◁" uses ISCD with $\Xi_{\text{out}} = 5$ iterations and *EXIT optimized* index assignment. Since the demodulator is not part of the iterative process ($\Xi_{\text{in}} = 1$), still the, for this case, optimum 8PSK-Gray mapping is applied. Due to the relatively weak memory 2 channel code the performance gain of ISCD is only $\Delta_{E_b/N_0} = 1.0$ dB if a reference parameter SNR of 13 dB is assumed as exemplary design constraint. At this parameter SNR BICM-ID with 8PSK-Mixed mapping (solid curve "▷") allows an improvement of $\Delta_{E_b/N_0} = 1.6$ dB. In the *inner* loop $\Xi_{\text{in}} = 5$ iterations are executed ($\Xi_{\text{out}} = 1$) and the standard *natural binary* index assignment is used.

At the baseline parameter SNR the performance gain of the combined *Turbo DeCodulation* system (solid curve "■") compared to the non-iterative system is approximately (for the given settings) with $\Delta_{E_b/N_0} = 2.5$ dB the sum of the gains by the ISCD and BICM-ID systems used for comparison. For *Turbo DeCodulation* 8PSK-Mixed mapping and *EXIT optimized* index assignment are applied and $\Xi_{\text{TD}} = 5$ iterations are carried out.

IV. Conclusion

We presented a novel iterative system denoted as *Turbo DeCodulation*, which uses iterative processing for all three components of the considered receiver. This combination of iterative source-channel decoding and iterative demodulation results in a multiple Turbo process with exchanging *extrinsic* reliabilities in two separate loops. In comparison to a non-iterative system as well as previously developed systems with a single iterative loop, i.e., ISCD and BICM-ID, simulations demonstrated significant quality gains by *Turbo DeCodulation*.

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