Turbo DeCodulation using Highly Redundant Index Assignments and Multi-Dimensional Mappings

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Abstract

We propose a block coded Turbo DeCodulation scheme, combining iterative demodulation, channel decoding, and source decoding at the receiver. Turbo DeCodulation [1, 2] resembles a serial concatenation of bit-interleaved coded modulation (BICM) and iterative source-channel decoding (ISCD) where the two iterative loops are connected by a single convolutional code. In our contribution, explicit channel (de)coding by a convolutional code, used in most Turbo systems, is omitted. Instead, highly redundant index assignments and multi-dimensional mappings, both based on short block codes, introduce the artificial redundancy. Thus, the proposed receiver possesses only a single iterative loop. The simulations results show a competitive or even superior performance compared to previously known systems. Furthermore, we present an extension to a flexible multi-mode system and a novel, simple design method for the index assignments.

1 Introduction

With the discovery of Turbo codes channel coding close to the Shannon limit becomes possible with moderate computational complexity. In the past years the Turbo principle of exchanging *extrinsic* information between separate channel decoders has been adapted to other receiver components.

In [3, 4] *bit-interleaved coded modulation with iterative decoding* (BICM-ID) is presented, in which the channel decoder exchanges *extrinsic* reliabilities with the demodulator. To exploit the residual redundancy in source coded parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, *iterative source-channel decoding* (ISCD) has been presented in [5, 6]. This residual redundancy occurs due to imperfect source encoding resulting from the delay constraints.

The *Turbo DeCodulation* scheme [1, 2] is a multiple Turbo process, in which the *extrinsic* reliabilities are exchanged between all three receiver components: *demodulator*, *channel decoder*, and *(soft decision) source decoder*. The term *Turbo DeCodulation* is chosen in analogy according to the term *codulation* as introduced by Anderson and Lesh in [7]. In [8] Massey called the respective transmitter *SoCodulator*. Turbo DeCodulation can be interpreted as serial concatenation of BICM-ID and ISCD via a common channel code. The Turbo DeCodulation presented in [1, 2] uses a convolutional code for this task, resulting in two iterative loops, more or less corresponding to BICM-ID and ISCD.

In this paper we propose a different approach. Instead of one convolutional code for the complete frame, we split the channel coding into two parts with small block codes and integrate these into the source (de)coder and the (de)modulator. This results in highly redundant index assignments [9], multi-dimensional mappings [10, 11], and only a single Turbo loop. Furthermore, this scheme can be very easily extended to a versatile multi-mode system, which allows a flexible trade-off between error robustness for bad channels and high quality for good channels.

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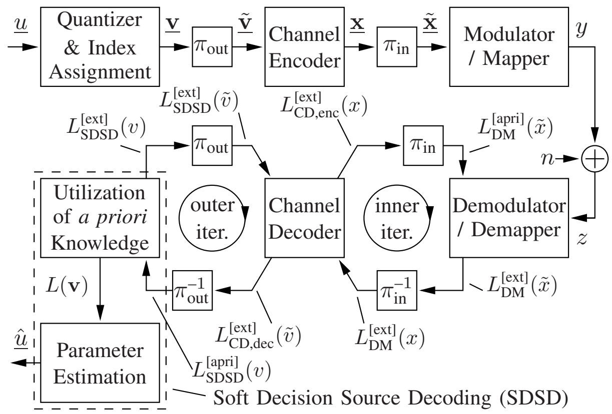

Fig. 1. Baseband Model of Turbo DeCodulation in [1, 2].

2 The Turbo DeCodulation Systems

At first we will briefly review the Turbo DeCodulation system in Section 2.1 and afterwards, in Section 2.2, present the modifications for the novel proposed Turbo DeCodulation system, which features only a single iterative loop.

2.1 Turbo DeCodulation

In Fig. 1 the baseband model of the *Turbo DeCodulation* system presented in [1, 2] is depicted. The *inner* iterative loop corresponds to a BICM-ID system, while the *outer* iterative loop is similar to an ISCD system.

At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ of K_S source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \dots K_S$ denoting the position in the frame. The single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$

is individually mapped to a quantizer reproduction level $\bar{u}_\kappa^{(\xi_\kappa)}$ with index $\xi_\kappa = 0, \dots Q_\kappa - 1$. The number of quantizer levels is usually assumed as $Q_\kappa = 2^{M_\kappa}$. To each quantizer level $\bar{u}_\kappa^{(\xi_\kappa)}$ selected at time instant τ a unique bit pattern $\mathbf{v}_{\kappa,\tau}$ of M_κ bits is assigned according to the index assignment Γ, $\mathbf{v}_{\kappa,\tau} = \Gamma(\xi_\kappa)$. For simplicity we assume $M_\kappa = M$ for all κ and omit the time index τ in the following. The single bits of a bit pattern $\mathbf{v}_\kappa$ are indicated by $v_\kappa^{(m)}$, $m = 1, \dots M$. The frame of bit patterns is denoted by $\underline{\mathbf{v}}$. The first and outer bit interleaver π_{out} scrambles the incoming frame $\underline{\mathbf{v}}$ of data bits to $\underline{\tilde{\mathbf{v}}}$ in a deterministic manner.

For channel encoding of a frame $\underline{\tilde{\mathbf{v}}}$ of bits v we can use, e.g., a standard terminated *recursive systematic convolutional* (RSC) code of constraint length $J+1$ and rate r_C. In general, any channel code can be used as long as the respective decoder is able to provide the required *extrinsic reliabilities*. For the termination of the RSC code, J tail bits are appended to $\underline{\tilde{\mathbf{v}}}$. The resulting codeword is denoted by $\underline{\mathbf{x}}$. The second, inner bit-interleaver π_{in} permutes this codeword $\underline{\mathbf{x}}$ to $\underline{\tilde{\mathbf{x}}}$.

The interleaved codeword $\underline{\tilde{\mathbf{x}}}$ is divided into bit patterns $\tilde{\mathbf{x}}_k$, $k = 1, \dots K_C$, with I single bits $\tilde{x}_k^{(i)}$, $i = 1, \dots I$. In case the last bit pattern $\tilde{\mathbf{x}}_{K_C}$ is not completely filled the remaining $K_C I - (K_S M + J)/r_C$ positions are padded by zeros. The modulator maps each pattern $\tilde{\mathbf{x}}_k$ according to a mapping rule μ to a complex modulated symbol y_k of the *signal constellation set* (SCS) $\mathcal{Y}$, $y_k = \mu(\tilde{\mathbf{x}}_k)$. The modulated symbols are normalized to an average energy of $\mathrm{E}\{\|y_k\|^2\} = 1$.

On the channel complex additive white Gaussian noise (AWGN) $n_k = n'_k + j n''_k$ with a known power spectral density of $\sigma_n^2 = N_0$ ($\sigma_{n'}^2 = \sigma_{n''}^2 = N_0/2$) is applied, i.e., $z_k = y_k + n_k$.

The received symbols z_k are evaluated in a multiple Turbo process, which exchanges *extrinsic* reliabilities between demodulator (DM) and channel decoder (CD) in the *inner* iterations, and between channel decoder and *soft decision source decoder* (SDSD) in the *outer* iterations. Such reliability information can either be evaluated in terms of probabilities $P(\cdot)$ or in *log-likelihood* ratios, or short L-values, $L(\cdot)$.

The Turbo DeCodulation receiver is described in detail in [1]. For the equations for the computation of the *extrinsic* probabilities or their respective L-values in each component we refer to the literature, i.e., for the demodulator to [3, 4], for the channel decoder to [12, 13], and for the soft decision source decoder (including parameter estimation with the minimum mean squared error serving as fidelity criterion) to [5, 6, 14]. Note, the channel decoder in a Turbo DeCodulation system needs to compute extrinsic reliabilities for decoded bits x as well as for encoded bits v. In case of a systematic channel code, the channel decoder shall implicitly forward the extrinsic reliabilities for the systematic bits from the demodulator and the SDSD.

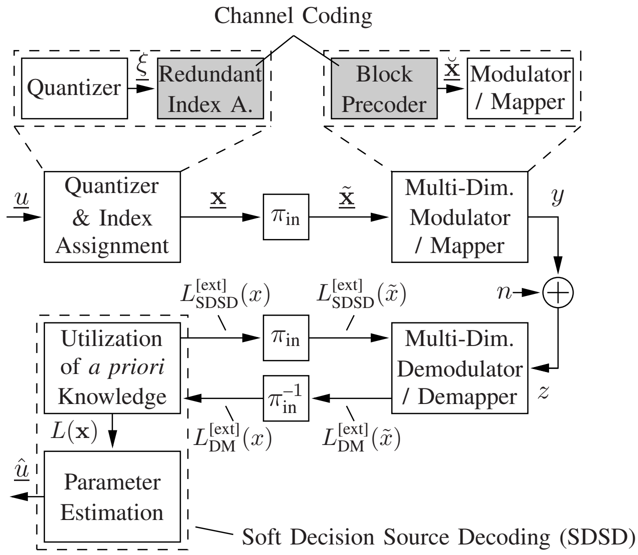

Fig. 2. Baseband model of the new proposed Turbo DeCodulation system with a redundant index assignment and multi-dimensional mapping.

2.2 New proposed Turbo DeCodulation

In [15] it was analytically shown that (in case of a binary erasure channel) the inner component(s) of a serially concatenated Turbo scheme should be of rate $r = 1$. In accordance, the (de)modulator can be considered as a rate-1 code. ISCD with rate-1 (convolutional) channel coding (called Turbo *error concealment*) has been studied in [16]. The application of this concept to the Turbo DeCodulation system presented in the previous section is straightforward.

In this paper we use a different approach. The usual Trellis-based convolutional channel coding is removed. Instead, as shown in Fig. 2, the artificial redundancy is applied by two small block codes, one with $r < 1$ for a redundant index assignment and one with $r = 1$ as a precoder for the modulator resulting in a multi-dimensional mapping. As a consequence the resulting receiver consists only of a single iterative loop. Since, e.g., in terms of bits per frame this loop is similar to the inner loop of the former system we use the respective notation.

2.2.1 Redundant index assignments

Highly redundant index assignments for ISCD were proposed in [9]. A related concept for the non-iterative case has been presented in [17]. In comparison to a transmission scheme with explicit (outer) channel coding of rate r_C, the number of bits assigned in Fig. 2 to each quantizer level $\bar{u}_\kappa^{(\xi_\kappa)}$ can be increased by a factor of $1/r_C$, if we assume a constant gross bit rate. For example, if typical $r_C = 1/2$ channel coding and $M = 3$ bits per each of the Q quantizer levels were used in Fig. 1, with the proposed scheme of Fig. 2 bit patterns $\mathbf{x}$ of size $M^\star = M/r_C = 6$ can be assigned to each quantizer level.

The obvious choice now would be to use $Q^\star = 2^{M^\star} = 64$ quantizer levels to reduce the quanti-

TABLE I

BLOCK CODE GENERATOR MATRICES $\mathbf{G}^{\Gamma}$ FOR REDUNDANT INDEX ASSIGNMENTS WITH $M^{\star}=6$ BITS.

$\mathbf{G}^{\Gamma}_{\text{BC(6,2)}}$ $(Q^{\star}=2^2=4)$	$\mathbf{G}^{\Gamma}_{\text{BC(6,3)}}$ $(Q^{\star}=2^3=8)$	$\mathbf{G}^{\Gamma}_{\text{BC(6,4)}}$ $(Q^{\star}=2^4=16)$	$\mathbf{G}^{\Gamma}_{\text{BC(6,5)}}$ $(Q^{\star}=2^5=32)$
$\begin{pmatrix} 1&0&1&1&0&1 \\ 0&1&1&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 1&0&0&1&0&1 \\ 0&1&0&1&1&0 \\ 0&0&1&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 1&1&1&1&1&1 \\ 1&0&0&1&0&1 \\ 0&1&0&1&1&0 \\ 0&0&1&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 1&0&0&0&0&1 \\ 0&1&0&0&0&1 \\ 0&0&1&0&0&1 \\ 0&0&0&1&0&1 \\ 0&0&0&0&1&1 \end{pmatrix}$

zation noise. However, the only constraint for $Q^{\star}$ is $Q^{\star} \leq 2^{M^{\star}}$. Thus, if, e.g., the quantization noise for $Q^{\star}=8$ quantization levels is specified as target, we can apply a highly redundant index assignment with $M^{\star}=6$ bits for each level[1]. Such an index assignment can be considered as a (potentially non-linear) block code with the binary representation $\{\xi\}_2$ of the quantizer level index ξ, $\xi=0,\dots Q^{\star}-1$, as input. In case of a linear block code with a generator matrix $\mathbf{G}^{\Gamma}$ we get

$$\mathbf{x} = \Gamma(\xi) = (\{\xi\}_2) \cdot \mathbf{G}^{\Gamma} \tag{1}$$

In Table I some generator matrices $\mathbf{G}^{\Gamma}$ for redundant index assignments with $M^{\star}=6$ bits are given[2]. These $\mathbf{G}^{\Gamma}$ have been chosen such that the Hamming distances between the valid bit patterns $\mathbf{x}$ are large[3]. With a minimum Hamming distance of at least 2, certain bit patterns of length $M^{\star}$ are not allowed. In the special case of perfect a-priori information $L^{[\text{ext}]}_{\text{DM}}(x)$ all bits except the one considered are known and the considered bit is then uniquely determined and then perfect extrinsic information $L^{[\text{ext}]}_{\text{SDSD}}(x)$ can be fed back to the demodulator. Thus, the EXIT characteristic [18, 19] of the SDSD can reach the upper right corner, i.e., the point (1,1), of the EXIT Chart [18].

Furthermore, we do not need to restrict the number of quantization levels $Q^{\star}$ to powers of 2. Instead we can adjust $Q^{\star} \in \mathbb{N}$ gradually to achieve the tolerable quantization noise. This way we can build a flexible multi-mode system [9], which when adapted can provide always the best possible quality for a certain channel condition. The adaptation consists only of choosing the correct index assignment, e.g., from a lookup-table.

There exist several different methods for the optimization of the index assignment. The index assignments given in [9] are, e.g., optimized using EXIT charts [19]. Table II lists the novel index assignments $\Gamma : \xi \mapsto \mathbf{x}$ according to (1) which are based on the block code (BC) generator matrices $\mathbf{G}^{\Gamma}$ given in Table I.

Example: With $Q^{\star}=4$, $M^{\star}=6$, and $\mathbf{G}^{\Gamma}_{\text{BC(6,2)}}$ follows: $\xi=0 \mapsto \{\mathbf{x}\}_{10}=0 \quad \dots \quad \xi=3 \mapsto \{\mathbf{x}\}_{10}=45$.

[1]The computational complexity of the Trellis-based utilization of a priori knowledge in the SDSD does not increase significantly when increasing M to $M^{\star}$ as the shape of the fully connected Trellis solely depends on the number of quantization levels (states) Q or $Q^{\star}$. Only the M bits per Trellis edge are mapped to a pattern with $M^{\star}$ bits.

[2]$\mathbf{G}^{\Gamma}_{\text{BC(6,4)}}$ is not given in systematic form, but rather as an extension of $\mathbf{G}^{\Gamma}_{\text{BC(6,3)}}$. Thus, $\mathbf{G}^{\Gamma}_{\text{BC(6,4)}}$ and $\mathbf{G}^{\Gamma}_{\text{BC(6,3)}}$ yield the same results for $\xi<8$ in (1). $\mathbf{G}^{\Gamma}_{\text{BC(6,5)}}$ represents a simple single parity check code.

[3]The Hamming distance may not be the optimum criterion, but as the simulation results show it yields good results. It is also a simple criterion especially when considering the block coding of (1).

In case $Q^{\star}$ is not a power of 2, the $\mathbf{G}^{\Gamma}$ for the next higher power of 2 is used as basis. Then the appropriate number of code words is removed. The removal starts with the bit patterns for the largest ξ. Thus, the remaining bit patterns can still be generated using the linear block code in (1). We denote these index assignments by $\text{BC}^{Q^{\star}}_{M^{\star}}$.

However, if the Hamming distance spectrum of the index assignment shall be optimized, sometimes other bit patterns must be removed. This can easily be achieved by detailed analysis of the Hamming distances of the bit patterns of the block code. In this case a table lookup replaces (1). The resulting index assignments are labeled by $\text{BCNL}^{Q^{\star}}_{M^{\star}}$. Thus, the proposed block code based index assignments do not require complex optimization in contrast to EXIT optimized (EO) ones.

TABLE II

INDEX ASSIGNMENTS Γ BASED ON $\mathbf{G}^{\Gamma}$ IN TABLE I

Γ, $(\cdot)^{Q^{\star}}_{M^{\star}}$	$\{\mathbf{x}\}_{10}=\Gamma(\xi)$, $\xi=0,\dots Q^{\star}-1$
BC^{3}_{6}	0,27,45
BC^{4}_{6}	0,27,45,54
BC^{5}_{6}	0,11,22,29,37
BC^{6}_{6}	0,11,22,29,37,46
BC^{7}_{6}	0,11,22,29,37,46,51
BC^{8}_{6}	0,11,22,29,37,46,51,56
BC^{12}_{6}	0,11,22,29,37,46,51,56,63,52,41,34
BC^{16}_{6}	0,11,22,29,37,46,51,56,63,52,41,34,26,17,12,7
BC^{24}_{6}	0,3,5,6,9,10,12,15,17,18,20,23,24,27,29,30,33, 34,36,39,40,43,45,46
BC^{32}_{6}	0,3,5,6,9,10,12,15,17,18,20,23,24,27,29,30,33, 34,36,39,40,43,45,46,48,51,53,54,57,58,60,63
BCNL^{5}_{6}	0,11,29,46,51
BCNL^{6}_{6}	0,11,22,29,46,51
BCNL^{12}_{6}	0,11,22,29,37,46,51,56,63,52,41,7

2.2.2 Multi-dimensional mappings for modulation

The decisive design parameter for the performance of iterative demodulation by the BICM-ID scheme is the mapping of the bit patterns of $\tilde{\mathbf{x}}$ to the complex channel symbols [4]. While for the non-iterative case Gray mappings are optimal, in the iterative case the best results are obtained by non-Gray mappings as shown, e.g., in [4, 20, 21]. These mappings maximize the (average) Euclidean decision distance, i.e., improve the reliability of the decision, for each bit assuming error-free feedback (EFF) by the adjacent decoder(s) for the remaining bits of the respective symbol.

Another strategy for improvement are multi-dimensional mappings (MDM), first considered for Trellis coded modulation [22, 23] and applied to

BICM-ID in, e.g., [10, 11]. With MDMs an appropriate number of bits is mapped to a set (or vector) of channel symbols rather than to a single channel symbol. For example, a bit pattern $\tilde{\mathbf{x}}^\star$ with $I^\star = 6$ bits can be mapped to two 8PSK symbols or six BPSK symbols. Using the construction method for optimal MDMs given in [10] this can be considered as linear block precoding the bit pattern $\tilde{\mathbf{x}}^\star$ of size $I^\star$ to $\breve{\mathbf{x}}^\star$ and regular mapping afterwards. With the criterion in [10] we can derive, e.g., the rate-1 generator matrices

$$\mathbf{G}^{\text{MDM}}_{\text{OPT6}} = \begin{pmatrix} 0&1&1&1&1&1\\ 1&0&1&1&1&1\\ 1&1&0&1&1&1\\ 1&1&1&0&1&1\\ 1&1&1&1&0&1\\ 1&1&1&1&1&1 \end{pmatrix} \text{ and } \mathbf{G}^{\text{MDM}}_{\text{OPT4}} = \begin{pmatrix} 0&1&1&1\\ 1&0&1&1\\ 1&1&0&1\\ 1&1&1&1 \end{pmatrix}.$$

The precoded bit pattern $\breve{\mathbf{x}}^\star$ can be obtained by

$$\breve{\mathbf{x}}^\star = \tilde{\mathbf{x}}^\star \cdot \mathbf{G}^{\text{MDM}} \quad . \tag{2}$$

Then, $\breve{\mathbf{x}}^\star$ is split up into smaller bit patterns appropriate for the regular mapping μ. The complete multi-dimensional mapping μ^{MDM} can be easily stored in a lookup-table. The demodulator needs to be only slightly modified to operate with this lookup-table on $\tilde{\mathbf{x}}^\star$ instead of $\tilde{\mathbf{x}}$ [10, 11]. As the MDMs comprise several symbols iterative demodulation can yield significant improvements in conjunction with BPSK, too [10].

3 Simulation Results

The capabilities of the proposed block coded Turbo DeCodulation (BC-TDeC) scheme shall be demonstrated by simulation. The *parameter signal-to-noise ratio* (SNR) between the originally generated parameters u_κ and the reconstructed estimates $\hat{u}_\kappa$ is used for quality evaluation. Instead of using any specific speech, audio, or video encoder, we model $K_S = 250$ statistically independent source codec parameters u by K_S independent 1^{st} order Gauss-Markov processes with auto-correlation ρ.

Each parameter u_κ is scalarly quantized by a Q or $Q^\star$ level Lloyd-Max quantizer using M or $M^\star$ bits/parameter. As index assignment $\Gamma : \xi \mapsto \mathbf{x}$ for the iterative schemes serves either the EXIT optimized (EO^Q_M, $\text{EO}^{Q^\star}_{M^\star}$) index assignments presented in [9, 19] or the block code based index assignments $\text{BC}^{Q^\star}_{M^\star}$ and $\text{BCNL}^{Q^\star}_{M^\star}$ proposed in Section 2.2.1. Note that the EXIT optimized index assignments always need to be optimized for the present auto-correlation ρ.

We consider different non-iterative and iterative reference configurations with 3-bit quantization of the parameters u, $r_C = 1/2$ channel coding and a random bit interleaver π_{in} of size $MK_S/r_C = 1500$ (or $1500{+}J/r_C$ in case of terminated convolutional channel coding with memory J). This corresponds to a gross bit rate of $M^\star = 6$ bits/parameter. Note, for a Turbo process the frame size of 1500 bits is relatively small. The number of iterations involving the interleaver π_{in} will be denoted as Ξ_{in}.

The non-multi-dimensional mappings $\mu : \tilde{\mathbf{x}} \mapsto y$ considered for the iterative demodulation are the 8PSK-SSP (semi-set partitioning) mapping [4], which is the optimum regular 8PSK mapping with respect to the bit-error floor, and the 8PSK-Mixed mapping [3]. Despite its inferior bit error floor the latter mapping is sometimes more suited in the context of Turbo processes using SDSD because it provides an earlier convergence to its bit error floor and the parameter SNR is limited for good channels by the quantization noise and not the residual bit errors. The specific labelings can be found in the respective literature [3, 4]. For the multi-dimensional mapping $\mu^{\text{MDM}} : \tilde{\mathbf{x}}^\star \mapsto y$ we use the precoder generator matrix $\mathbf{G}^{\text{MDM}}_{\text{OPT6}}$ given in Section 2.2.2 in combination with 8PSK-Gray mapping[4], i.e., two 8PSK SCSs are precoded together. A fair comparison is achieved by plotting the results with respect to the channel quality measure E_x/N_0, where E_x is the energy per bit x.

3.1 BC-TDeC with 8PSK

In Fig. 3 we compare the performance of the proposed BC-TDeC scheme (solid lines) with BC^8_6 and $\mathbf{G}^{\text{MDM}}_{\text{OPT6}}$ to the convolutional coded Turbo DeCodulation (CC-TDeC) system (dashed lines) presented in [1, 2]. The auto-correlation is assumed as $\rho = 0.95$, a value which can occur, e.g., for the pitch period of the GSM enhanced full-rate codec. The CC-TDeC system uses EO^8_3 index assignment, a memory $J = 2$ recursive systematic convolutional code with generator polynomials $\mathbf{G} = (1, \frac{1}{1+D+D^2})$, and 8PSK-Mixed or 8PSK-SSP mapping.

At the BC-TDeC receiver $\Xi_{\text{in}} = 10$ iterations are performed, and $\Xi_{\text{out}} = 10$ outer iterations and $\Xi_{\text{in}} = 10$ inner iterations for the CC-TDeC receiver. More iterations do not improve the performance any further. For the order of inner and outer iterations there exist lots of possibilities. For simplicity we restrict ourselves to the case in which demodulator, channel decoder and SDSD are executed sequentially in this order. This results in single inner and outer iterations being performed alternately. Except for the first and last iteration, this can be seen as both iterative loops being executed in parallel, i.e., the channel decoder feeding both loops simultaneously.

The non-iterative reference systems (marked "o") with $\Xi_{\text{in}} = \Xi_{\text{out}} = 1$ use both 8PSK-Gray mapping. Natural binary index assignment is applied for the CC-TDeC reference system and BC^8_6 acts as $r_C = 1/2$ block code and index assignment for BC-TDeC.

As visible in Fig. 3 the proposed BC-TDeC outperforms the iterative CC-TDeC in the whole depicted E_x/N_0-range, despite featuring only a single iterative loop connecting two components instead of two loops connecting three components. Up to $\Delta_{E_x/N_0} \approx 1$ dB

[4] As the optimization of $\mathbf{G}^{\text{MDM}}$ in [10] is based on BPSK and $\mathbf{G}^{\text{MDM}}$ by itself ensures the large euclidean decision distances for the EFF case, 8PSK-Gray provides very good results.

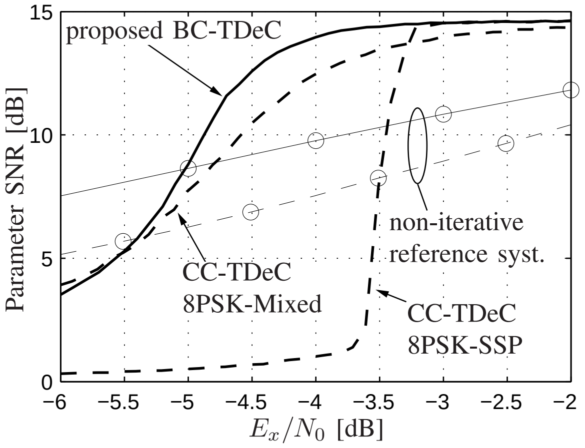

Fig. 3. Parameter SNR for CC-TDeC and the proposed BC-TDeC.

gain (at a reference parameter SNR of 14 dB) can be achieved compared to CC-TDeC with 8PSK-Mixed mapping, which features, similar to BC-TDeC, a relatively smooth degradation towards lower E_x/N_0. CC-TDeC with 8PSK-SSP mapping can retain the maximum quality onto lower E_x/N_0 than with 8PSK-Mixed mapping, but at $E_x/N_0 \approx -3.2$ dB it exhibits a very abrupt impairment.

Simulation results for the extension of the BC-TDeC scheme to a multi-mode system are depicted in Fig. 4. This multi-mode system is based on the fact that several realizations of $Q^\star \le 2^{M^\star}$ are possible for a given $M^\star$. The lower the number of $Q^\star$ is, the higher the error robustness can be. Of course, at the same time higher quantization noise has to be accepted in good channel conditions. Thus, adaptive mode-switching according to the channel condition will be beneficial. Notice, in contrast to prior multi-mode joint source-channel coding standards like the GSM-AMR (adaptive multi-rate), the second component, the (multi-dimensional) mapping with $r = 1$, needs not to be adapted in case of a dynamic dimensioning of $Q^\star$. The number $M^\star \cdot K_S$ of bits x is the same for all realizations of $Q^\star$.

When for every E_x/N_0 always the best choice of the considered $\mathrm{BC}_{M^\star}^{Q^\star}$ and $\mathrm{BCNL}_{M^\star}^{Q^\star}$ index assignments is selected, we obtain the multi-mode envelope (thick solid line). This multi-mode scheme gives superior performance in all cases. It features large quality gains in parameter SNR for good channels as well as robust transmission for bad channel conditions. Note that especially with the used block code based index assignments the adaptation of the index assignment is very simple.

In Fig. 5 the convergence behavior of BC-TDeC at $E_x/N_0 = -4$ dB is analyzed using EXIT charts [18]. Mainly due to the improved EXIT characteristic of the now multi-dimensional mapping the number of reasonable iterations increases with the proposed components (left plot) and the decoding trajectory (step curve) gets closer to the upper right corner. The right plot shows the behavior for BC-TDeC with known components (see also dashed curve in Fig. 4). The EXIT characteristics

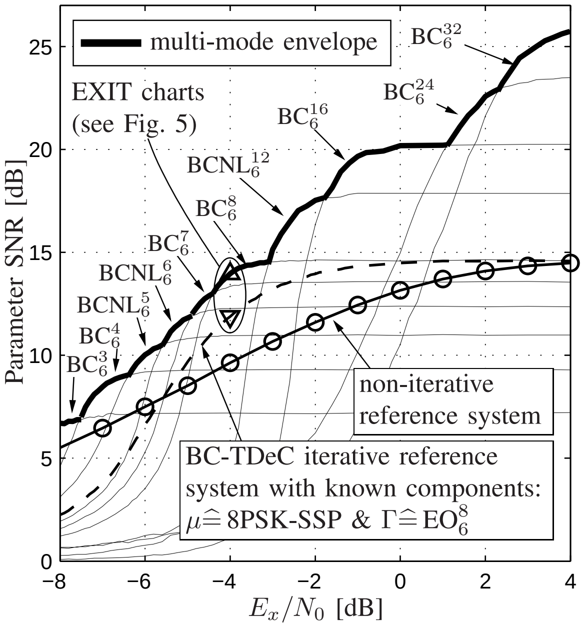

Fig. 4. Performance of multi-mode BC-TDeC with $M^\star = 6$.

of the BC and the EO index assignment are very similar. However, two advantages of the BC index assignments are their straightforward derivation and their independence of the auto-correlation. For the analysis of the convergence behavior of CC-TDeC systems, requiring 3-dimensional EXIT Charts, we refer to [2].

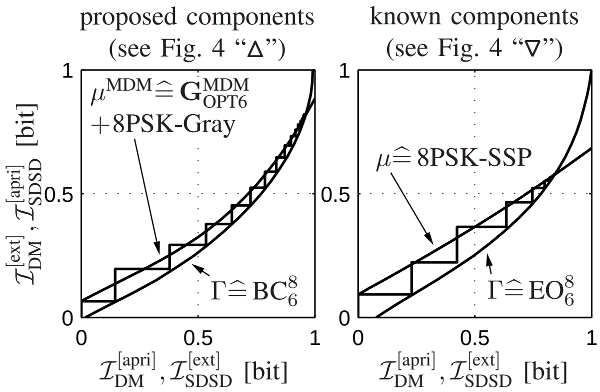

Fig. 5. EXIT charts for BC-TDeC at $E_x/N_0 = -4$ dB (cmp. Fig. 4).

3.2 BC-TDeC with BPSK

The technique of multi-dimensional mappings also enables iterative demodulation in conjunction with BPSK modulation. In Fig. 6 the simulation result for BC-TDeC is depicted when $\mathbf{G}_{\mathrm{OPT4}}^{\mathrm{MDM}}$ is applied to four BPSK SCSs (curve "□"). In this example the auto-correlation is now $\rho = 0.2$ and $\Xi_{\mathrm{in}} = 25$ iterations are executed. Since BC-TDeC with BPSK could be considered also as block coded ISCD [24], results of two convolutional coded ISCD schemes using BPSK are depicted for comparison. The "conventional" ISCD system with $r_C = 1/2$ (dashed curve) uses a terminated memory $J = 3$ recursive *non-systematic* convolutional (RNSC) code with generator polynomials $\mathbf{G} = \left(\frac{1+D+D^3}{1+D+D^2+D^3}, \frac{1+D^2+D^3}{1+D+D^2+D^3}\right)$ and EO_3^8 index assignment. The ISCD system with $r_C = 1$ (curve "◇")

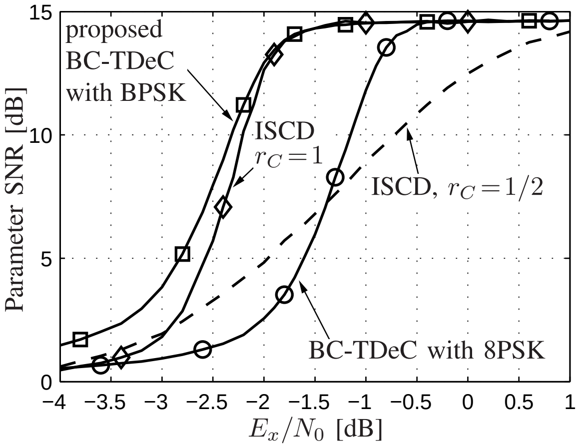

Fig. 6. Parameter SNR for BC-TDeC and ISCD with BPSK.

and redundant index assignment EO_6^8 proposed in [9] applies a rate-1, memory $J = 3$ RNSC code with $\mathbf{G} = (\frac{1}{1+D+D^2+D^3})$.

The proposed BC-TDeC with BPSK shows a similar performance at high parameter SNRs compared to $r_C = 1$ ISCD with convolutional coding, but exhibits a smoother degradation towards lower E_x/N_0. Even BC-TDeC with MDM-8PSK-Gray mapping with $\mathbf{G}_{\text{OPT6}}^{\text{MDM}}$ (curve "o") approaches the performance of $r_C = 1$ ISCD by $\Delta_{E_x/N_0} < 1.2$ dB. However, with its 8PSK mapping BC-TDeC requires only one third of the channel symbol rate or bandwidth of $r_C = 1$ ISCD using BPSK. The "conventional" $r_C = 1/2$ ISCD system is outperformed at high parameter SNR by the other three depicted systems, which all feature an $r \approx 1$ inner component of the Turbo process. The BC-TDeC using BPSK is superior to $r_C = 1/2$ ISCD in the whole E_x/N_0-range.

4 Conclusion

In this paper we propose a Turbo DeCodulation system, which features iterative demodulation, channel decoding, and source decoding in a single iterative loop. No explicit channel coding is performed, but instead the artificial redundancy is added by redundant index assignment and multi-dimensional mapping for modulation. The inner component is of rate-1 in accordance to the recent research findings for serially concatenated systems. Furthermore, we present a novel method for the flexible development and implementation of the redundant index assignments based on simple block codes without, e.g., an EXIT chart optimization. Simulation results show a competitive or superior performance of the proposed block coded Turbo DeCodulation compared to convolutional coded Turbo DeCodulation or ISCD systems with relatively similar computational complexity. This might be due to the combination of residual and artificial redundancy in a single block, the redundant index assignment, allowing an efficient exploitation, additionally supported by the increased diversity of the multi-dimensional mappings. By varying the redundant index assignment an easy extension to a multi-mode system is possible, which provides on optimal trade-off between quality and error robustness for different channel conditions.

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