Parameter SNR Optimized Index Assignments and Quantizers based on First Order A Priori Knowledge for Iterative Source-Channel Decoding

Thorsten Clevorn, Peter Vary
Institute of Communication Systems and Data Processing
RWTH Aachen University, Germany
{clevorn,vary}@ind.rwth-aachen.de

Marc Adrat
Research Establishment for Applied Science (FGAN)
Wachtberg, Germany
adrat@fgan.de

Abstract— **Two key design aspects regarding the source coding part of iterative source-channel decoding (ISCD) are the index assignment and the quantizer. While the conventional index assignments are not suited for ISCD the so far presented optimized index assignments do only consider zeroth order a priori knowledge or optimize the parameter SNR only indirectly. In this paper we present a new cost function which directly optimizes the parameter SNR incorporating the first order a priori knowledge. With the same cost function we can also optimize the code book of the quantizer. Simulation results show the excellent performance the new parameter SNR optimized index assignments and quantizers exhibit.**

I. Introduction

With the discovery of Turbo codes channel coding close to the Shannon limit becomes possible with moderate computational complexity. In the past years the Turbo principle of exchanging extrinsic information between separate channel decoders has been adapted to other receiver components.

To exploit the residual redundancy in source codec parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, iterative source-channel decoding (ISCD)[1] has been presented in [1,2]. This residual redundancy occurs due to imperfect source encoding resulting, e.g., from the delay constraints. The a priori knowledge on the residual redundancy, e.g., non-uniform probability distribution, auto-correlation, or cross correlation, is utilized for error concealment by a derivative of a soft decision source decoder (SDSD) [3], which exchanges extrinsic reliabilities with a channel decoder.

One key design aspect for ISCD systems is the index assignment. The well-know conventional index assignments for the non-iterative case such as natural binary or Gray have only a limited suitability for ISCD since they do not consider the possible feedback available in the Turbo process. In [5,4] index assignments are presented which are optimized for ISCD with respect to a non-uniform probability distribution, i.e., zeroth order a priori knowledge. However, first order a priori knowledge, e.g., auto-correlation, is not incorporated in the optimization process. Such first order a priori knowledge is part of the design of the EXIT optimized index assignments introduced in [6]. Here, the optimization is based on the extrinsic information transfer (EXIT) characteristic [7,6] of the SDSD, which depends on the index assignment and the first order a priori knowledge. However, there is no direct relation between the extrinsic mutual information and the typical quality criterion, the parameter SNR. Thus, the latter is optimized only indirectly.

In this paper we present a novel cost function, which directly represents the parameter SNR and incorporates the first order a priori knowledge. Using this cost function we develop new index assignments. The simulation results show that these index assignments are noticeably superior to the ones in [5,4], which only consider zeroth order a priori knowledge, and even slightly outperform the EXIT optimized index assignments derived in [6].

Another element of the ISCD system, which has been usually ignored so far in the design, is the quantizer. Typically, a Lloyd-Max quantizer (LMQ) [8] was used [1,2,3,4,5,6], which is designed with respect to the non-uniform probability distribution. In [9] a quantizer optimization algorithm for ISCD is presented. Similarly to the EXIT optimized index assignments it is based on the EXIT characteristic of the SDSD. Using the underlying general design process in conjunction with our new cost function which represents the parameter SNR and includes first order a priori knowledge, we develop a parameter SNR optimized quantizer. Again, the simulation results show that it is slightly superior to the EXIT optimized quantizer in terms of parameter SNR.

II. Iterative Source-Channel Decoding

In Fig. 1 the baseband model of iterative source-channel decoding is depicted. At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ of K source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \ldots K$ denoting the position in the frame. The

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[1]Note, the term iterative source-channel decoding is used in the literature also with a different meaning, i.e., for the iterative evaluation of variable-length source codes and channel codes. Here ISCD serves for a proper segmentation of the reconstructed bit stream after channel decoding into bit patterns of specific length.

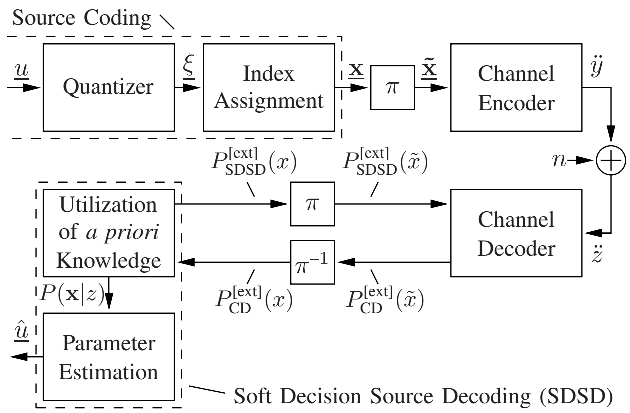

Fig. 1. Baseband model of iterative source-channel decoding

single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}^{(\xi_\kappa)}_{\kappa,\tau}$ with index $\xi_\kappa = 0, \ldots Q_\kappa - 1$. The number of quantizer levels is $Q_\kappa = 2^{M_\kappa}$. To each quantizer level $\bar{u}^{(\xi_\kappa)}_{\kappa,\tau}$ a unique bit pattern $\mathbf{x}_{\kappa,\tau}$ of M_κ bits is assigned according to the index assignment Γ, $\mathbf{x}_{\kappa,\tau} = \Gamma(\xi_\kappa)$. For simplicity we assume $M_\kappa = M$ for all κ. The single bits of a bit pattern $\mathbf{x}_{\kappa,\tau}$ are indicated by $x^{(m)}_{\kappa,\tau}$, $m = 1, \ldots M$. The frame of bit patterns is denoted by $\underline{\mathbf{x}}_\tau$. The bit interleaver π scrambles the incoming frame $\underline{\mathbf{x}}_\tau$ of data bits to $\underline{\tilde{\mathbf{x}}}_\tau$ in a deterministic manner. To simplify the notation, we restrict the interleaving to a single time frame with index τ and omit the time frame index τ in the following where appropriate.

For channel encoding C of a frame $\underline{\tilde{\mathbf{x}}}$ of bits x we can use, e.g., a standard terminated recursive (non-)systematic convolutional code of constraint length $J+1$ and rate r_{C}. In general, any channel code can be used as long as the respective decoder is able to provide the required extrinsic reliabilities. For the termination of the convolutional code, J tail bits are appended to $\underline{\tilde{\mathbf{x}}}$. The resulting codeword is denoted by $\underline{y}$ with bits y, which are mapped to bipolar bits $\ddot{y} \in \{\pm 1\}$ for BPSK transmission with symbol energy $E_s = 1$.

On the channel additive white Gaussian noise (AWGN) n with a known power spectral density of $\sigma_n^2 = N_0/2$ is applied, i.e., $\ddot{z} = \ddot{y} + n$.

The received symbols z are evaluated in a Turbo process, which exchanges extrinsic reliabilities between the channel decoder (CD) and the soft decision source decoder (SDSD). Such reliability information can either be evaluated in terms of probabilities $P(\cdot)$ or log-likelihood ratios (L-values) $L(\cdot)$.

The ISCD receiver is described in detail in [1,2,6]. For convolutional codes the equations for the computation of the extrinsic probabilities or their respective L-values are well known [10,11]. The SDSD determines extrinsic information mainly from the natural residual source redundancy which typically remains in the bit patterns $\mathbf{x}_{\kappa,\tau}$ after source encoding. Such residual redundancy appears on parameter-level, e.g., in terms of a non-uniform distribution $P(\bar{u}_{\kappa,\tau})$, in terms of correlation, or in any other possible mutual dependency in time τ. The latter terms of residual redundancy are usually approximated by a first order Markov chain, i.e., by the conditional probability distribution $P(\bar{u}_{\kappa,\tau}|\bar{u}_{\kappa,\tau-1})$. These source statistics can usually be measured once in advance for a representative signal data base. Furthermore, we can likewise use $P(\mathbf{x}_{\kappa,\tau})$ and $P(\mathbf{x}_{\kappa,\tau}|\mathbf{x}_{\kappa,\tau-1})$ instead of $P(\bar{u}_{\kappa,\tau})$ and $P(\bar{u}_{\kappa,\tau}|\bar{u}_{\kappa,\tau-1})$ due to the one-to-one index assignment. The technique how to combine this a priori knowledge $P(\mathbf{x}_{\kappa,\tau}|\mathbf{x}_{\kappa,\tau-1})$ on parameter-level with the soft-input values $P^{\text{[ext]}}_{\text{CD}}(x)$ on bit-level is also well-known in the literature. The algorithm how to compute the extrinsic $P^{\text{[ext]}}_{\text{SDSD}}(x)$ has been detailed, e.g., in [1,2,6]. For quality evaluation we consider the parameter signal-to-noise ratio (SNR)

$$\mathcal{P} = 10 \cdot \log_{10}(\mathrm{E}\{|u|^2\} / \mathrm{E}\{|u - \hat{u}|^2\}) \quad . \tag{1}$$

III. Index Assignment Optimization

The index assignment is a key factor regarding the performance of ISCD systems. There exist numerous well-known index assignments such as natural binary (NB), folded binary (FB), and Gray (GR) [8,12]. However, these index assignments are suited only for the non-iterative case [6,4,5]. In Table I all the index assignments considered in this paper are listed. We concentrate for the results on the case of bit patterns with $M = 3$ bits being assigned to $Q = 2^M = 8$ quantizer levels. Nevertheless, the algorithms are directly applicable to other values of M and $Q \leq 2^M$. Similar to [6] we give as shorthand notation for the index assignment $\Gamma : \xi \mapsto \mathbf{x}$ the decimal representations $\{\mathbf{x}\}_{10}$ for the bit patterns $\mathbf{x}$ for an increasing level ξ, e.g., for FB

$$\begin{aligned} \xi = 0 &\mapsto (\{\mathbf{x}\}_{10} = 7 \Rightarrow \{\mathbf{x}\}_2 = 111) \\ \xi = 1 &\mapsto (\{\mathbf{x}\}_{10} = 6 \Rightarrow \{\mathbf{x}\}_2 = 110) \\ &\vdots \\ \xi = 7 &\mapsto (\{\mathbf{x}\}_{10} = 3 \Rightarrow \{\mathbf{x}\}_2 = 011) \quad . \end{aligned}$$

A. Optimization with zeroth order a priori knowledge (AK0)

In [5,4] index assignments for ISCD are presented, which are optimized with respect to a large Euclidean distance between quantizer levels whose bit patterns differ by only a single bit. The optimization process includes the non-uniform quantizer level (or parameter) distribution by weighting each squared Euclidean distance with its probability of occurrence $P(\bar{u})$. Since this method aims at optimizing the parameter SNR and uses zeroth order a priori knowledge (AK0), we denote these index assignments as SNR optimized (SOAK0). The cost function to be maximized can be written as [5,4]

$$D^{\text{[SOAK0]}} = \frac{1}{QM} \sum_{\bar{u} \in \mathbb{U}} \sum_{m=1}^{M} P(\bar{u}) \|\bar{u} - \breve{\bar{u}}\|^2 \rightarrow \max \tag{2}$$

By $\breve{\bar{u}}$ we refer to the quantizer level with the same bit pattern as $\bar{u}$, except for an inverted bit at position m. The bit patterns of the compared channel symbols $\bar{u}$ and $\breve{\bar{u}}$ differ in a single bit, because the optimization is aimed at good channels with very few bit errors. For a numerical approach with reasonable complexity the optimization (to a possibly local optimum) can be performed for large values of Q and M by the binary switching algorithm (BSA) [13,5,6]. For small values, e.g., $M \leq 4$, an exhaustive search of all possible index assignments is possible, yielding a global optimum. The SOAK0 index assignment for $Q = 8$ quantizer levels and $M = 3$ bits is given in Table. I. Nevertheless, a major drawback is the fact that the first order residual redundancy $P(\bar{u}_\tau|\bar{u}_{\tau-1})$ in terms of, e.g., an auto-correlation ρ, is still not incorporated in the optimization of the index assignments.

B. New optimization with first order a priori knowledge (AK1)

In the following we will derive novel index assignments which also take into account the exploitation of first order a priori knowledge in the SDSD. Similar to (2) we consider the squared Euclidean distance (or noise energy) $\|\bar{u}_\tau - \check{\bar{u}}_\tau\|^2$ between the correct quantizer level $\bar{u}_\tau$ at time instance τ and the erroneous quantizer level $\check{\bar{u}}_\tau$, whose assigned bit pattern differs only at bit position m. Furthermore, we assume that the quantizer level $\bar{u}_{\tau-1}$ of the previous time instance has been correctly estimated. This is a valid assumption because the optimization shall take place for good parameter SNRs, i.e., a high channel quality. Based on the probability $P(\bar{u}_{\tau-1})$ we can incorporate the first order a priori knowledge by the conditional probabilities $P(\bar{u}_\tau|\bar{u}_{\tau-1})$ and $P(\check{\bar{u}}_\tau|\bar{u}_{\tau-1})$ and weight the squared Euclidean distance $\|\bar{u}_\tau - \check{\bar{u}}_\tau\|^2$ according to its probability of occurrence, i.e., the joint probability $P(\check{\bar{u}}_\tau, \bar{u}_\tau, \bar{u}_{\tau-1})$. Note, $\bar{u}_\tau$ and $\check{\bar{u}}_\tau$ are statistically independent. The resulting cost function

$$D^{\text{[SOAK1]}} = \qquad (3)$$

$$\frac{1}{QM} \sum_{\bar{u}_\tau \in \mathbb{U}} \sum_{m=1}^{M} \sum_{\bar{u}_{\tau-1} \in \mathbb{U}} P(\check{\bar{u}}_\tau, \bar{u}_\tau, \bar{u}_{\tau-1}) \|\bar{u}_\tau - \check{\bar{u}}_\tau\|^2 \rightarrow \min$$

$$\text{with } P(\check{\bar{u}}_\tau, \bar{u}_\tau, \bar{u}_{\tau-1}) = P(\check{\bar{u}}_\tau|\bar{u}_{\tau-1}) P(\bar{u}_\tau|\bar{u}_{\tau-1}) P(\bar{u}_{\tau-1})$$

includes an additional sum over the possible previous quantizer levels $\bar{u}_{\tau-1}$. Note that in contrast to (2) this cost function has to be minimized. In the derivation of (2) it was assumed that the SDSD requires a large Euclidean distance to be able to distinguish between the quantizer levels $\bar{u}_\tau$ and $\check{\bar{u}}_\tau$ using the first order a priori knowledge [5,4]. However, for (3) this exploitation of the residual redundancy is included in the index assignment and the noise $\|\bar{u}_\tau - \check{\bar{u}}_\tau\|^2$ generated by an erroneous decision for $\check{\bar{u}}_\tau$ shall be as small as possible. Thus, (3) must be minimized, e.g., by an exhaustive search or the BSA. The emerging index assignments are denoted by SOAK1 and vary in dependence of the residual redundancy in terms of, e.g., an auto-correlation ρ. For some values of ρ the SOAK1 index assignments are given in Table I for $Q = 8$ quantizer levels and $M = 3$ bits.

C. EXIT optimized index assignments

Another optimization method for index assignments which considers the first order residual redundancy $P(\bar{u}_\tau|\bar{u}_{\tau-1})$ is presented in [6]. This algorithm is based on EXIT charts [7]. EXIT charts visualize and analyze the exchange of extrinsic mutual information in a Turbo process. Each component is characterized by an EXIT characteristic, which describes the generated extrinsic mutual information in dependence on the input a priori mutual information, but independent of the other component. The actual convergence in the iterative processing is depicted by a decoding trajectory. For more details we refer to the literature, e.g., [7,6].

TABLE I
INDEX ASSIGNMENTS $\Gamma : \xi \mapsto \mathbf{x}$ FROM $Q = 8$ QUANTIZER LEVELS $\bar{u}^{(\xi)}$, $\xi = 0, \ldots Q-1$, TO BIT PATTERNS $\mathbf{x}$ WITH $M = 3$ BITS.

index assignment Γ		$\{\mathbf{x}\}_{10} = \Gamma(\xi)$
natural binary (NB)		0,1,2,3,4,5,6,7
Gray, (GR)		0,1,3,2,6,7,5,4
folded binary (FB)		7,6,5,4,0,1,2,3
parameter SNR optimized (SOAK0)		7,1,2,4,6,5,3,0
parameter SNR optimized (SOAK1)	$\rho = 0.4$	0,1,3,2,6,4,5,7
	$\rho = 0.7$	0,1,3,2,6,4,5,7
	$\rho = 0.8$	0,5,7,3,2,6,4,1
	$\rho = 0.9$	0,6,5,1,3,2,4,7
re-optimized for SOQ (SOAK1Q)	$\rho = 0.9$	0,6,7,1,2,4,5,3
EXIT optimized (EO)	$\rho = 0.4$	7,1,2,4,3,5,6,0
	$\rho = 0.7$	7,4,2,1,5,6,3,0
	$\rho = 0.8$	5,6,3,0,7,4,1,2
	$\rho = 0.9$	4,7,1,2,5,6,0,3

The optimization algorithm for the index assignments makes use of a special property of the EXIT characteristics of SDSD. Accordingly, we denote the outcome of the optimization as EXIT optimized (EO) index assignment. The maximum extrinsic mutual information $\mathcal{I}_{\text{SDSD}}^{\text{[ext,max]}}$ at $\mathcal{I}_{\text{SDSD}}^{\text{[apri]}} \approx 1$ bit depends on the index assignment and the auto-correlation. For a good iterative performance the $\mathcal{I}_{\text{SDSD}}^{\text{[ext,max]}}$ should be as large as possible. To be exact, actually the intersection with the EXIT characteristic of the second component defines the performance. But with an appropriately designed second component this stopping intersection will reside usually at values close to $\mathcal{I}_{\text{SDSD}}^{\text{[apri]}} \approx 1$ bit. Thus, the value

$$D^{\text{[EO]}} = \mathcal{I}_{\text{SDSD}}^{\text{[ext,max]}} = \mathcal{I}_{\text{SDSD}}^{\text{[ext]}}(\mathcal{I}_{\text{SDSD}}^{\text{[apri]}} \approx 1 \text{ bit}) \rightarrow \max \qquad (4)$$

is a quite suitable indicator for the possible performance of the respective ISCD system [6] and serves as cost function for a maximization.

Furthermore, the value of $\mathcal{I}_{\text{SDSD}}^{\text{[ext,max]}}$ can be calculated numerically without extensive simulations, because with $\mathcal{I}_{\text{SDSD}}^{\text{[apri]}} \approx 1$ bit the a priori information is perfectly reliable and not defined by a random process [14,6]. Similar to the parameter SNR optimized index assignments in the previous section a full search for small M and the BSA for large M can be used to obtain a (for the BSA local) maximum for $\mathcal{I}_{\text{SDSD}}^{\text{[ext,max]}}$ [6]. For $Q = 8$ quantizer levels and $M = 3$ bits the resulting EXIT optimized index assignments EO for four different values of the auto-correlation ρ are given in Table I.

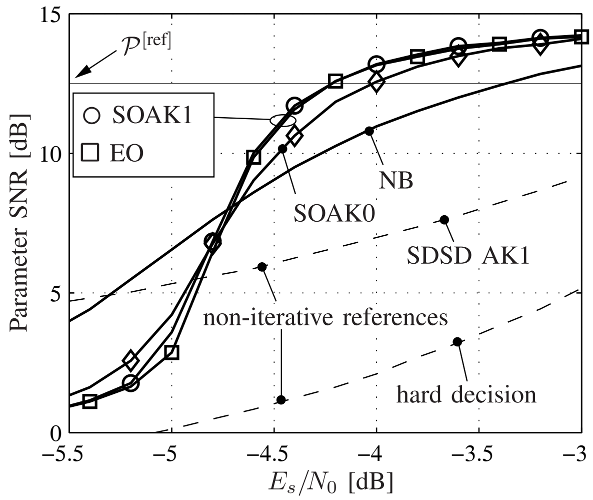

Fig. 2. Parameter SNR performance of optimized index assignments.

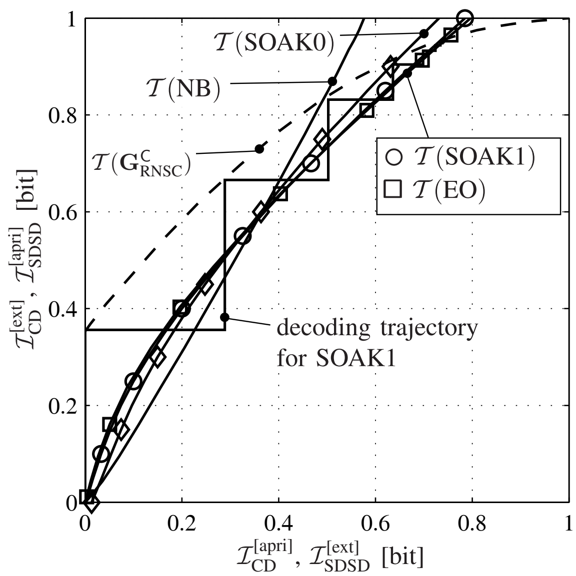

Fig. 3. EXIT chart for optimized index assignments at $E_s/N_0 = -4$ dB.

D. Simulation results

In Fig. 2 the parameter SNR performance of ISCD with the optimized index assignments is compared to the one with the conventional natural binary index assignment. Instead of using any specific speech, audio, or video encoder, we model $K = 250$ statistically independent source codec parameters u by K independent Gauss-Markov processes with $\sigma_u^2 = 1$ and auto-correlation $\rho = 0.9$. Even such high values of auto-correlation occur for certain source codec parameters in today's communication systems such as GSM or UMTS. Each parameter u_κ is scalarly quantized by a $Q = 8$ level Lloyd-Max quantizer (LMQ) using $M = 3$ bits/parameter, resulting in $X = 750$ uncoded bits per frame. For channel encoding C a memory $J = 3$, $r_{\mathsf{C}} = 1/2$ recursive non-systematic convolutional (RNSC) code with generator polynomials $\mathbf{G}^{\mathsf{C}}_{\mathrm{RNSC}} = \{13_8/17_8, 15_8/17_8\}$ is applied. In [6] is has been shown that for ISCD RNSC codes are superior to their systematic counterparts. 10 iterations are executed at the receiver. An EXIT chart analysis reveals that more iterations are not beneficial. For comparison we assume a reference parameter SNR of $\mathcal{P}^{[\mathrm{ref}]} = 12.5$ dB.

All ISCD systems are clearly superior to the non-iterative reference, for which the residual redundancy is explored only once in the SDSD. Already the system with the SOAK0 index assignment significantly outperforms the conventional NB index assignment despite the SOAK0 index assignment being optimized only for zeroth order a priori knowledge. Nevertheless, with the index assignments optimized with first order a priori knowledge, SOAK1 and EO, further noticeable improvements can be realized. The performance of the ISCD systems with SOAK1 and EO index assignment is almost identical with the SOAK1 index assignment being negligible better. However, the optimization for SOAK1 is very simple and does not require the computation or measurements of extrinsic mutual information as the optimization for the EO index assignments does.

Fig. 3 contains the corresponding EXIT chart at $E_s/N_0 = -4$ dB. The indicator regarding the convergence behavior is the first intersection between the EXIT characteristics of the channel code $\mathcal{T}(\mathbf{G}^{\mathsf{C}}_{\mathrm{RNSC}})$ and the SDSD. $\mathcal{T}(\Gamma)$. NB index assignments yields the stopping intersection at the lowest mutual information followed by SOAK0 index assignment. Again, SOAK1 and EO index assignment exhibit an almost identical behavior. However, despite the difference being negligible in terms of optimization, the situation is vice versa with respect to the parameter SNR performance of Fig. 2. The EO index assignment provides a slightly higher mutual information at the intersections as well as for $\mathcal{I}^{[\mathrm{ext,max}]}_{\mathrm{SDSD}} = \mathcal{I}^{[\mathrm{ext}]}_{\mathrm{SDSD}}(\mathcal{I}^{[\mathrm{apri}]}_{\mathrm{SDSD}} \approx 1\text{ bit})$. This shows that the parameter SNR and the extrinsic mutual information are only indirectly related, because a mapping of extrinsic mutual information to parameter SNR would have to take the index assignment into account. Nevertheless, the differences are negligible.

The depicted decoding trajectory for SOAK1 index assignments confirms that 10 iterations are sufficient. The decoding trajectory exceeding the EXIT characteristic of the SDSD $\mathcal{T}(\Gamma)$ is typical for ISCD. The reason is the additional knowledge, refined by the iterations, from the previous frames in the SDSD. But the intersection of the EXIT characteristics is still the limiting factor.

IV. Quantizer Optimization

The design of the conventional Lloyd-Max Quantizer (LMQ) [8] assumes error-free channel conditions and yields a quantizer code book $\mathbb{U}$ with the minimum mean squared error (MMSE) $\mathrm{E}\{|u - \bar{u}|^2\}$ between the original source signal u and its quantized representation $\bar{u}$. However, similar to the non-optimized index assignments, an LMQ does not reflect the specifics of the iterative processing in ISCD, e.g., a residual

TABLE II

QUANTIZER CODE BOOKS $\mathbb{U}$ AND $\mathbb{U}^{\star}$ WITH $Q\!=\!8$ QUANTIZER LEVELS, $\mathcal{P}^{[\text{TARGET}]}\!=\!13.5$ dB, AUTO-CORRELATION $\rho\!=\!0.9$.

LMQ	$\bar{u} \in \{\pm 2.1519, \pm 1.3439, \pm 0.7560, \pm 0.2450\}$
SOQ ($\Gamma \widehat{=}$SOAK1)	$\bar{u} \in \{\pm 2.7127, \pm 1.7656, \pm 1.0141, \pm 0.3348\}$
EOQ ($\Gamma \widehat{=}$EO)	$\bar{u} \in \{\pm 2.7127, \pm 1.7656, \pm 1.0141, \pm 0.3348\}$

redundancy in terms of an auto-correlation and its exploitation at the ISCD receiver is not considered in the design process.

An alternative for a quantizer design which incorporates ISCD is presented in [9]. The improved error robustness in bad channel conditions is achieved at the cost of a slightly higher mean squared error (MSE) $\mathrm{E}\{|u-\bar{u}|^2\}$ and thus, a weaker baseline quality in almost error-free channel situations.

The switch from a quantizer code book $\mathbb{U}$ to an optimized quantizer code book $\mathbb{U}^{\star}$ affects several components of the ISCD transmitter and receiver. Beside the quantizer itself and its counterpart at the receiver, the parameter estimation, the utilization of source statistics at the receiver is also affected. With a new quantizer code book $\mathbb{U}^{\star}$ the statistics $P(\bar{u})$ and $P(\bar{u}_\tau|\bar{u}_{\tau-1})$ of the residual source redundancy change. Via the index assignment $\mathbf{x} = \Gamma(\bar{u})$ these are directly related to the probabilities $P(\mathbf{x})$ and $P(\mathbf{x}_\tau|\mathbf{x}_{\tau-1})$. Thus, the index assignment heavily influences the design process. Therefore, in many situations it will be beneficial to re-optimize the index assignment Γ as well.

One design criterion for the new optimized quantizer (OQ) shall be that the parameter SNR $\mathcal{P}^{[\max]}$ for an error-free channel does not drop below a target parameter SNR $\mathcal{P}^{[\text{target}]}$, $\mathcal{P}^{[\max]} \geq \mathcal{P}^{[\text{target}]}$, which, e.g., may be slightly higher than the reference parameter SNR $\mathcal{P}^{[\text{ref}]}$. Using (1) and

$$\mathrm{E}\{|u-\hat{u}|^2\} \approx \mathrm{E}\{|u-\bar{u}|^2\} + \mathrm{E}\{|\bar{u}-\hat{u}|^2\} \quad , \qquad (5)$$

with equality for quantizers minimizing $\mathrm{E}\{|u-\bar{u}|^2\}$ [15], this relates for the error-free channel $\mathrm{E}\{|\bar{u}-\hat{u}|^2\}\approx 0$ to an upper bound on the quantization noise $N_{\mathsf{Q}} = \mathrm{E}\{|u-\bar{u}|^2\}$ denoted by $N_{\mathsf{Q}}^{[\max]}$.

Unfortunately, a closed form determination of the optimized quantizer is an impractically complex task as all blocks of the source encoder and the SDSD have an impact on the optimization process. Additionally the search for a proper index assignment has to be incorporated. Thus, in [9] a step-wise optimization is proposed. First the index assignment is optimized with the initial code book $\mathbb{U}$, e.g., as described in Section III.

Afterwards, the optimized quantizer code book $\mathbb{U}^{\star}$ is searched by an iterative algorithm. For the details of the quantizer optimization algorithm we refer to [9] and briefly review it in the following. It was observed that the magnitude of the quantizer reproduction levels $\bar{u}$ is increased, i.e., the probability $P(\bar{u})$ of unlikely outer quantizer reproduction levels is further reduced. In turn, the probability $P(\bar{u})$ of the center quantizer reproduction levels, which have also been more probable beforehand, rises. For the sake of simplicity a symmetric probability density function of the source codec parameters which increases from the margins to the center is assumed.

Initialization:
Define $N_{\mathsf{Q}}^{[\max]}$ by $\mathcal{P}^{[\text{target}]}$ and choose the index assignment and the starting quantizer code book $\mathbb{U}$, e.g, SOAK1 and LMQ. Compute the source statistics $P(\bar{u})$ and $P(\bar{u}_\tau|\bar{u}_{\tau-1})$ and the cost function D, e.g., (3) or (4).

Iterative Processing:

a) Set quantizer interval counter to $q\!=\!1$.
b) If $q \geq 2^{M-1}$ stop optimization process.
c) Initialize step-size to, e.g., $\Delta P\!=\!1/2^M$.
d) Compute temporary $\tilde{P}(\bar{u})\!=\!P(\bar{u})\!-\!\Delta P$ for the q-th and the $(2^M\!-\!q\!+\!1)$-th interval (symmetry assumption).
e) If these $\tilde{P}(\bar{u}) < 0$ undo and repeat step d) with smaller step-size, e.g., $\Delta P\!=\!\Delta P/2$. If the step-size drops below a limit, e.g., $\Delta P > 10^{-9}$, undo step d), increase the interval counter $q\!=\!q\!+\!1$, and return to step b) to start with the next inner interval. Otherwise proceed to step f).
f) Add the spare $2\Delta P$ of d) equally distributed to the $P(\bar{u})$ of the inner $2^M\!-\!2q$ intervals, i.e., $\tilde{P}(\bar{u})\!=\!P(\bar{u})\!+\!\frac{2\Delta P}{2^M-2q}$.
g) Recompute the quantizer code book $\bar{u} \in \mathbb{U}$ from $\tilde{P}(\bar{u})$. Compute $P(\bar{u}_\tau|\bar{u}_{\tau-1})$, the cost D, and $\mathrm{E}\{|u-\bar{u}|^2\}$.
h) If $\mathrm{E}\{|u-\bar{u}|^2\} < N_{\mathsf{Q}}^{[\max]}$ check if the cost D is "better" than any previous cost. If so, save the $\bar{u}$ as optimized code book $\mathbb{U}^{\star}$. In any case return to step d).

The resulting optimized code books $\mathbb{U}^{\star}$ with $Q\!=\!8$ quantizer levels and a target parameter SNR of $\mathcal{P}^{[\text{target}]} = 13.5$ dB are given in Table II. As residual source redundancy an auto-correlation of $\rho = 0.9$ was assumed. The parameter SNR optimized quantizer (SOQ) is obtain by applying the cost function $D^{[\text{SOAK1}]}$ of (3). As initial index assignment served SOAK1. For an EXIT optimized quantizer (EOQ) [9] which maximizes $\mathcal{I}_{\text{SDSD}}^{[\text{ext}]}(\mathcal{I}_{\text{SDSD}}^{[\text{apri}]} \approx 1 \text{ bit})$ (4) is used as cost function and EO as initial index assignment. The quantizer optimization yields identical results for both cases, SOQ and EOQ.

In a next step, the index assignment can be re-optimized. For the considered example, the index optimization according to (4) in Section III-C results again in EO. With the cost function $D^{[\text{SOAK1}]}$ of Section III-B a new index assignment is obtained, which will be denoted as SOAK1Q and is listed in Table I. In the following another iteration for the optimization could be performed by optimizing the quantizer again. However, for the given example this yields no further improvements of the parameter SNR.

A. Simulation results

The parameter SNR performance of ISCD with the new optimized quantizers EOQ and SOQ is compared to a conventional LMQ in Fig. 4. The simulation settings are identical to those used in Fig. 2. For high E_s/N_0 the parameter SNR with EOQ and SOQ approaches $\mathcal{P}^{[\text{target}]}\!=\!13.5$ dB as expected by the design constraints. This is less than the LMQ enables, but nevertheless surpasses the assumed required reference parameter SNR $\mathcal{P}^{[\text{ref}]}\!=\!12.5$ dB. Towards lower E_s/N_0 the SOQ can preserve $\mathcal{P}^{[\text{ref}]}$ for about $\Delta_{E_s/N_0}\!\approx\!0.7$ dB longer and achieve

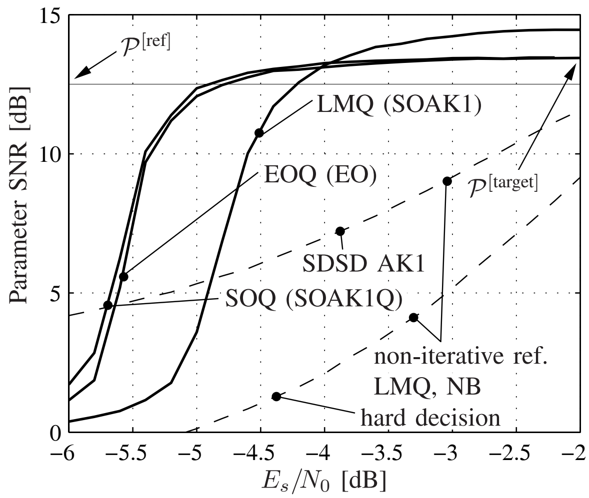

Fig. 4. Parameter SNR performance of optimized quantizers.

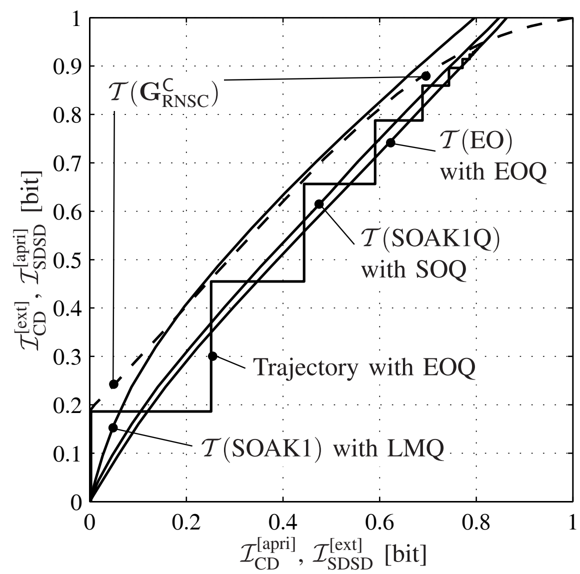

Fig. 5. EXIT chart for optimized quantizers at $E_s/N_0 = -5$ dB.

a gain of $\Delta_{E_s/N_0} \approx 0.8$ dB in the waterfall region. The EOQ retains $\mathcal{P}^{[\mathrm{ref}]}$ approximately $\Delta_{E_s/N_0} \approx 0.5$ dB towards worse channel conditions compared to the LMQ. The difference in performance between EOQ and SOQ is very small, similar to the difference between the index assignments EO and SOAK1. But again, the system optimized with respect to the parameter SNR, the SOQ, yields the slightly better results.

The EXIT chart in Fig. 5 visualizes the advantages in the convergence behavior of the new optimized quantizers in comparison to the conventional LMQ. At $E_s/N_0 = -5$ dB the EXIT characteristic $\mathcal{T}(\mathrm{SOAK1})$ with the LMQ yields an early stopping intersection with the EXIT characteristic $\mathcal{T}(\mathbf{G}_{\mathrm{RNSC}}^{\mathrm{C}})$ of the channel code. In contrast the stopping intersections defined by the EXIT characteristics $\mathcal{T}(\mathrm{SOAK1Q})$ with the SOQ and $\mathcal{T}(\mathrm{EO})$ with the EOQ is at high extrinsic mutual information $\mathcal{I}^{[\mathrm{ext}]}$. A long tunnel between the EXIT characteristics can be explored by ISCD, which is confirmed by the depicted trajectory for the EOQ case. The situation regarding the order of parameter SNR optimized and EXIT optimized systems in the EXIT charts is similar to the one for index assignment optimization in Section III-D. Despite performing better in terms of parameter SNR, the parameter SNR optimized SOQ system exhibits a slightly worse EXIT characteristic than the EXIT optimized EOQ system.

V. Conclusion

In this paper we presented new parameter SNR optimized index assignments and quantizers for ISCD, which are based on a novel cost function. The new cost function directly represents the usual quality criterion, the parameter SNR, and incorporates the first order a priori knowledge in contrast to previous optimizations. These were either limited to zeroth order a priori knowledge or optimized the parameter SNR indirectly via the EXIT characteristic. Simulation results confirm the superior performance of the proposed index assignments and quantizers. An EXIT chart analysis reveals the slight mismatch of the indirect optimization with the EXIT characteristics.

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