Iterative Source-Channel Decoding using Short Block Codes

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Abstract
Iterative source-channel decoding (ISCD) exploits the residual redundancy of the source, e.g., codec parameters, for quality improvements. In contrast to the well-known convolutional coded ISCD systems, we propose in this paper an ISCD scheme which features a superior performance but is based solely on two short block codes. The block codes serve as highly redundant index assignment and rate-1 inner channel code respectively. The improved capabilities are confirmed by EXIT charts. Finally, the effects of imperfect knowledge of the receiver on the residual redundancy are analyzed. With the proposed flexible index assignment no feedback from the receiver to the transmitter is required and competitive results are obtained even for slightly erroneously estimated residual redundancy.

1. Introduction

To exploit the residual redundancy in source coded parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, *iterative source-channel decoding* (ISCD) has been presented, e.g., in [1]. So far, ISCD schemes usually use an index assignment with no or low redundancy for the source codec parameters. The artificial redundancy for error correction is added afterwards by a convolutional channel coding with low rates r_C, e.g., $r_C \leq 1/2$. The convolutional code is applied to the complete frame of parameters.

In this paper we propose a completely different approach. Instead of one convolutional code for the complete frame, we split the channel coding into two parts with short block codes and integrate the part which provides the major part of the artificial redundancy into the source (de)coder. This results in highly redundant index assignments [2] and rate-1 explicit (inner) channel coding. Furthermore, due to the flexibility of the highly redundant index assignments this scheme can be very easily extended to a versatile multi-mode system, which allows a flexible trade-off between error robustness for bad channels and high quality for good channels.

2. Conventional ISCD system model

In Fig. 1 the baseband model of conventional convolutional coded ISCD (CC-ISCD) is depicted. At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ of K_S source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1, \dots K_S$ denoting the position in the frame. The single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}_\kappa^{(\xi_\kappa)}$ with index $\xi_\kappa = 0, \dots Q_\kappa - 1$. The number of quantizer levels is usually assumed as $Q_\kappa = 2^{M_\kappa}$. To each quantizer level $\bar{u}_\kappa^{(\xi_\kappa)}$ selected at time instant τ a unique bit pattern $\mathbf{x}_{\kappa,\tau}$ of M_κ bits is assigned according to the index assignment Γ, $\mathbf{x}_{\kappa,\tau} = \Gamma(\xi_\kappa)$. For simplicity we assume $M_\kappa = M$ for all κ. The single bits of a bit pattern $\mathbf{x}_{\kappa,\tau}$ are indicated by $x_{\kappa,\tau}^{(m)}$, $m = 1, \dots M$. The frame of bit patterns is denoted by $\underline{\mathbf{x}}_\tau$. The bit interleaver

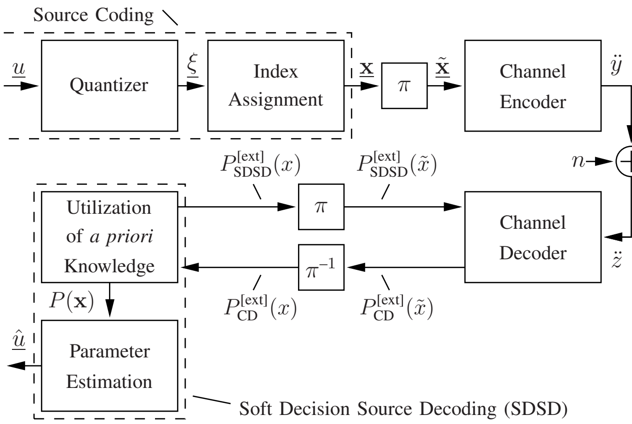

Fig. 1: Baseband model of iterative source-channel decoding

π scrambles the incoming frame $\underline{\mathbf{x}}_\tau$ of data bits to $\underline{\tilde{\mathbf{x}}}_\tau$ in a deterministic manner. We restrict the interleaving to a single time frame with index τ and omit the time frame index τ in the following where appropriate.

For channel encoding C of a frame $\underline{\tilde{\mathbf{x}}}$ of bits x we can use, e.g., a standard terminated *recursive (non-)systematic convolutional* code of constraint length $J+1$ and rate r_C. For the termination of the convolutional code, J tail bits are appended to $\underline{\tilde{\mathbf{x}}}$. The resulting codeword is denoted by $\underline{\mathbf{y}}$ with bits y, which are mapped to bipolar bits $\ddot{y} \in \{\pm 1\}$ for BPSK transmission with symbol energy $E_s = 1$.

On the channel additive white Gaussian noise (AWGN) n with a known power spectral density of $\sigma_n^2 = N_0/2$ is applied, i.e., $\ddot{z} = \ddot{y} + n$.

The received symbols z_k are evaluated in a Turbo process, which exchanges *extrinsic* reliabilities between the channel decoder (CD) and the *soft decision source decoder* (SDSD) [3].

The ISCD receiver is described in detail in [1, 4]. The SDSD determines *extrinsic* information mainly from the natural residual source redundancy which typically remains in the bit patterns $\mathbf{x}_{\kappa,\tau}$ after source encoding. Such residual redundancy appears on *parameter-level*, e.g., in terms of a non-uniform distribution $P(\mathbf{x}_{\kappa,\tau})$, in terms of correlation, or in any other possible mutual dependency in time τ. The latter terms of residual redundancy are usually approximated by a first order Markov chain, i.e., by the conditional probability distribution $P(\mathbf{x}_{\kappa,\tau}|\mathbf{x}_{\kappa,\tau-1})$. These source statistics can usually be measured once in advance. The technique how to combine this *a priori* knowledge $P(\mathbf{x}_{\kappa,\tau}|\mathbf{x}_{\kappa,\tau-1})$ on *parameter-level* with the soft-input values $P_{\text{CD}}^{[\text{ext}]}(x)$ on *bit-level* is also well-known in the literature. The algorithm how to compute the *extrinsic* $P_{\text{SDSD}}^{[\text{ext}]}(x)$ has been detailed, e.g., in [4].

3. Proposed ISCD with block codes

In [5] it was analytically shown that (in case of a binary erasure channel) the inner component(s) of a serially concatenated Turbo scheme should be of rate $r = 1$. ISCD with rate-1 convolutional channel coding (called Turbo *error concealment*) has been studied in [6]. An index assignment with $M = \log_2 Q$ bits was combined with a rate-1 convolutional code. In this paper we use a different approach. The usual Trellis-based convolutional channel coding is removed. Instead, the artificial redundancy is applied by two short block codes, one with $r < 1$ for a redundant index assignment and one with $r = 1$ as (inner) channel code. However, Fig. 1 also depicts the model of the block coded BC-ISCD scheme. Just the rates between ξ, $\mathbf{x}$, and $\ddot{y}$ change and the channel code is now a block code.

3.1. Redundant index assignments

Highly redundant index assignments for ISCD were proposed in [2]. In comparison to a conventional ISCD scheme with channel coding of rate r_C, the number of bits assigned to each quantizer level $\bar{u}_\kappa^{(\xi_\kappa)}$ can be increased by a factor of $1/r_C$, if we assume a constant gross bit rate. For example, if typical $r_C = 1/2$ channel coding and $M = 3$ bits per each of the Q quantizer levels were used for conventional ISCD , with the proposed scheme bit patterns $\mathbf{x}$ of size $M^\star = M/r_C = 6$ can be assigned to each quantizer level.

The obvious choice now would be to use $Q^\star = 2^{M^\star} = 64$ quantizer levels to reduce the quantization noise. However, the only constraint for $Q^\star$ is $Q^\star \le 2^{M^\star}$. Thus, if, e.g., the quantization noise of $Q^\star = 8$ quantization levels is tolerable (as in the conventional case with $M = 3$ bits), we can apply a highly redundant index assignment with $M^\star = 6$ bits for each level. The benefits of the latter approach with $Q^\star < 2^{M^\star}$ over the obvious choice $Q^\star = 2^{M^\star}$ in case of channel errors have been analyzed in detail in [2]. Such an index assignment with $Q^\star < 2^{M^\star}$ can be considered as a (potentially non-linear) block code with the binary representation of the quantizer level index $\{\xi\}_2$, $\xi = 0, \ldots Q^\star - 1$, as input. In case of an index assignment by a linear block code with generator matrix $\mathbf{G}^\Gamma$ we get

$$\mathbf{x} = \Gamma(\xi) = (\{\xi\}_2) \cdot \mathbf{G}^\Gamma \quad . \tag{1}$$

For the exemplary $Q^\star = 8$ and $M^\star = 6$ we choose a simple (6,3) block code with

$$\mathbf{G}^\Gamma_{\text{BC(6,3)}} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \quad . \tag{2}$$

This $\mathbf{G}^{\Gamma}$ provides a minimum Hamming distances of $d_{\text{min}}=3$ between the valid bit patterns $\mathbf{x}$. With a minimum Hamming distance larger than 1, perfect a-priori information yields perfect extrinsic information. All bits except the one considered are known and the considered bit is then uniquely determined. Thus, the EXIT characteristic [4, 7] of the SDSD can reach the upper right corner, i.e., the point (1,1), of the EXIT Chart [7].

Furthermore, we do not need to restrict the number of quantization levels $Q^\star$ to powers of 2. Instead we can adjust $Q^\star \in \mathbb{N}$ gradually to achieve the tolerable quantization noise. This way we can build a flexible multi-mode system [2], which (if adapted) can provide always the best possible quality for a certain channel condition. The adaptation consists only of choosing the correct index assignment.

There exist several different methods for the optimization of the index assignment. The index assignments given in [2] are, e.g., optimized using EXIT charts [4] and represent a very non-linear block code. However, the proposed index assignments ($\text{BC}_{M^\star}^{Q^\star}$) based on linear block codes $\mathbf{G}^{\Gamma}$ can be easily derived in contrast to the EXIT optimized ones (EO_M^Q,$\text{EO}_{M^\star}^{Q^\star}$). In case $Q^\star$ is not a power of 2, the $\mathbf{G}^{\Gamma}$ for the next higher power of 2 is used as basis. Then some code words have to be excluded. If the removal starts with the bit patterns $\mathbf{x}$ for the largest ξ, the remaining bit patterns can still be generated using the linear block code in (1).

3.2. Rate-1 (inner) block code

For the inner component of the Turbo process we propose the usage of short block codes, each applied to a bit pattern $\tilde{\mathbf{x}}^\star$ of size $I^\star$ out of $\underline{\tilde{\mathbf{x}}}$. This can be also considered as a multi-dimensional mapping (MDM) for BPSK channel symbols [8] with the Turbo process then performing iterative demodulation. With MDMs an appropriate number of bits is mapped to a set (or vector) $\mathbf{y}$ of channel symbols rather than to a single channel symbol. For example, a bit pattern $\tilde{\mathbf{x}}^\star$ with $I^\star = 4$ bits can be mapped to four BPSK symbols. Using the construction method for optimal MDMs given in [8] this can be seen as linear block precoding the bit pattern $\tilde{\mathbf{x}}^\star$ of size $I^\star$ to $\mathbf{y}^\star$ and regular BPSK mapping afterwards. With the criterion given in [8] we can derive the following rate-1 generator matrix for the short block code:

$$\mathbf{G}_{\text{OPT4}}^{\mathsf{C}} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad . \tag{3}$$

The encoded bit pattern $\mathbf{y}^\star$ can be obtained by

$$\mathbf{y}^\star = \tilde{\mathbf{x}}^\star \cdot \mathbf{G}^{\mathsf{C}} \quad . \tag{4}$$

The complete multi-dimensional mapping can be easily stored in a lookup-table. At the receiver the *extrinsic* probabilities for the i-th bit $\tilde{x}_k^{\star(i)}$ of the k-th bit pattern $\tilde{\mathbf{x}}_k^\star$ are given by [8]

$$P_{\text{CD}}^{[\text{ext}]}\Big(\tilde{x}_k^{\star(i)}{=}b\Big) \sim \sum_{\mathbf{y}_k \in \mathcal{Y}_b^i} P(\mathbf{y}_k|\mathbf{z}_k) \prod_{j\neq i} P_{\text{SDSD}}^{[\text{ext}]}\Big(\tilde{x}_k^{\star(j)} {\in} \mathbf{y}_k (\mathbf{G}^{\mathsf{C}})^{\text{-}1}\Big) \tag{5}$$

where $\mathbf{z}_k$ is the received vector for $\mathbf{y}_k$. The set $\mathcal{Y}_b^i$ comprises all $\mathbf{y}_k$ for which the i-th bit of the respective $\tilde{\mathbf{x}}_k^\star = \mathbf{y}_k \cdot (\mathbf{G}^{\mathsf{C}})^{\text{-}1}$ is b, $b \in \{0, 1\}$. The product is taken for the $P_{\text{SDSD}}^{[\text{ext}]}(\tilde{x}_k^{\star(j)})$ of this $\tilde{\mathbf{x}}_k^\star$.

4. Simulation Results

The capabilities of the proposed BC-ISCD scheme compared to CC-ISCD shall be demonstrated by simulation. The *parameter SNR* between the original parameters u_κ and the reconstructed estimates $\hat{u}_\kappa$ is used for quality evaluation. We model $K_S = 250$ statistically independent source codec parameters u by K_S independent Gauss processes with $\sigma_u^2 = 1$.

Each parameter u_κ is scalarly quantized by a $Q = Q^\star = 8$ level Lloyd-Max quantizer using $M = 3$ or $M^\star = 6$ bits/parameter. As index assignment $\Gamma : \xi \mapsto \mathbf{x}$ for the iterative schemes serves either the EXIT optimized (EO_M^Q, $\text{EO}_{M^\star}^{Q^\star}$) index assignments presented in [2, 4] or the proposed block code based index assignment $\text{BC}_{M^\star}^{Q^\star}$.

For channel coding we assume as reference $r_{\mathsf{C}} = 1/2$, resulting in a bit interleaver of size 1500. As block code for BC-ISCD we use $\mathbf{G}_{\text{OPT4}}^{\mathsf{C}}$ and for CC-ISCD we consider memory 3 recursive non-systematic convolutional codes with $\mathbf{G}_{r_{\mathsf{C}}=1/2}^{\mathsf{C}} = \left(\frac{1+D+D^3}{1+D+D^2+D^3}, \frac{1+D^2+D^3}{1+D+D^2+D^3}\right)$ for $r_{\mathsf{C}} = 1/2$ and $\mathbf{G}_{r_{\mathsf{C}}=1}^{\mathsf{C}} = \left(\frac{1}{1+D+D^2+D^3}\right)$ for $r_{\mathsf{C}} = 1$.

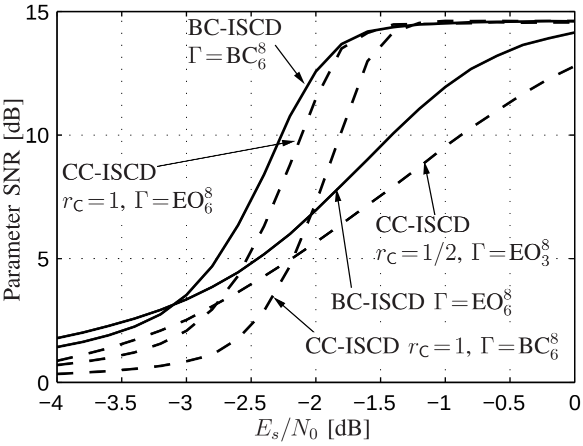

Fig. 2: Parameter SNR for BC- and CC-ISCD.

In Fig. 2 the parameter SNR after 25 iterations for BC-ISCD (solid curves) and CC-ISCD (dashed curves) is depicted. Fig. 3 contains the respective EXIT charts [7] for $E_s/N_0 = -2.5$ dB to demonstrate the convergence behavior. As visible, the proposed BC-ISCD with BC_6^8 exhibits the best performance. It preserves the maximum parameter SNR longer than the other schemes and possesses a relatively smooth degradation towards low E_s/N_0, too. The corresponding EXIT chart in Fig. 3a illustrates that the slightly S-shaped EXIT characteristic of BC_6^8 matches very well to the one of $\mathbf{G}_{\mathrm{OPT4}}^{\mathsf{C}}$. In contrast, the almost linear EXIT characteristic of EO_6^8, which also reaches the point (1,1), yields a stopping intersection at low mutual information $\mathcal{I}$ (see Fig. 3b). Due to the relatively linear EXIT characteristic of $\mathbf{G}_{r_\mathsf{C}=1}^{\mathsf{C}}$, the situation is vice versa for CC-ISCD with $r_\mathsf{C} = 1$. Here, the EXIT characteristic of EO_6^8 (see Fig. 3d) matches better than the one of BC_6^8 (see Fig. 3c). Consequently, CC-ISCD with EO_6^8 surpasses CC-ISCD with BC_6^8 as visible in Fig. 2. At high parameter SNRs both $r_\mathsf{C} = 1$ schemes, BC-ISCD as well as CC-ISCD, significantly outperform classic CC-ISCD with $r_\mathsf{C} = 1/2$. BC-ISCD is superior in the whole E_s/N_0-range.

5. Conclusion

In this paper we presented a novel ISCD scheme, which is based on two very short linear block codes rather than on convolutional codes. One block code provides a highly redundant index assignment and the other acts as rate-1 inner

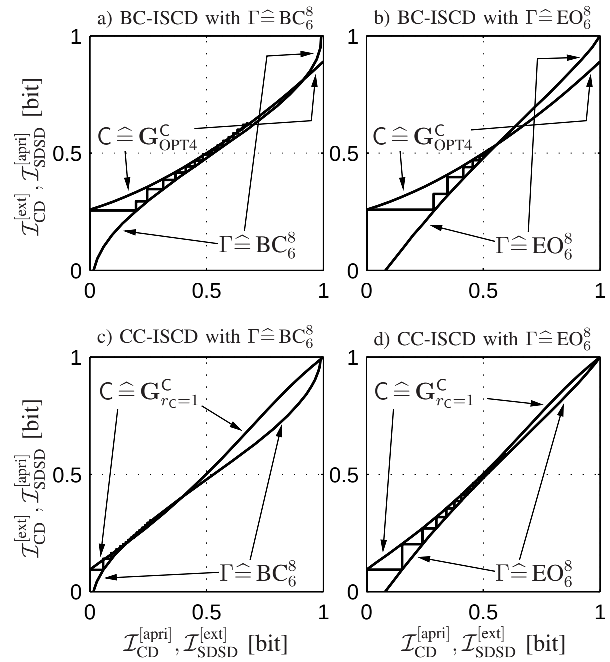

Fig. 3: EXIT Charts at $E_s/N_0 = -2.5$ dB.

channel code, resembling a multi-dimensional mapping. The simulation results indicate that with the proposed flexible index assignment previously known convolutional coded ISCD systems with a similar computational complexity are outperformed by the presented block code based ISCD.

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