# Aspects of Random Linear Network Coding in Layered Networks

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## Abstract

Random linear network coding (RLNC) [1]–[5] is a method to maximize the information flow in a communication network by forming random linear combinations over some finite field $\mathbb{F}_q$ of the received information packets at each intermediate node. The network between one source node and one destination node acts as a linear map $\mathbb{F}_q^n \to \mathbb{F}_q^N$, which is represented by the *network channel matrix*. Since the linear factors, i.e., the coding coefficients at each intermediate node are drawn independently at random, there is no need of a central processor or of sharing side information between the nodes.

Currently, in RLNC there coexist different approaches for generating the linear combinations at intermediate nodes. Therefore, we classify and characterize the existing essentially two distinct variants and show their equivalence. In variant 1 each intermediate node calculates *one* linear combination and transmits it on its outgoing edges whereas in variant 2 the intermediate nodes compute *individual* linear combinations for each of their outgoing edges. Other variants can be seen as hybrids of these two variants. We show that each network which makes use of variant 2 (or of any hybrid variant) can be transformed into an equivalent network which applies variant 1, by splitting up intermediate nodes which transmit different messages on their outgoing edges into several single output edges.

Besides this classification into two encoding variants at node level, we introduce further structure in terms of layers into seemingly disparate and unstructured network topologies. By inserting redundant intermediate single input/single output nodes, arbitrary acyclic networks can be transformed into layered networks. Such layered networks constitute a special class of networks, where the intermediate nodes are arranged in $L$ layers. Nodes in layer $l$ only receive packets from nodes in layer $l-1$, i.e., there are no direct connections between non-adjacent layers. The three major advantages of the transformation of an arbitrary acyclic network into a layered network, which we denote *layering*, are:

I. The network channel matrix can be factorized into so called inter-layer matrices.
II. An inherent synchronization is provided, i.e., all paths that connect the source node and the destination node are equally long, i.e., each path has length $L$.
III. Layering simplifies or even enables an accurate analysis of RLNC systems.

We present two RLNC setups, which can be thoroughly analyzed because of their layered structure. At first, we examine the effects of joining or leaving nodes on the network channel matrix. This allows to derive a statistical network channel model for *slowly varying* networks which considers additive packet errors as well as changes in the network topology due to leaving or joining nodes [6]. In the second setup we derive an upper bound on the outage probability of two-layer network channel matrices, i.e., the probability that the network channel matrix does not have full column rank [7], [8]. This upper bound is particularly important for networks whose network channel matrix is sparse, i.e., in cases where the well known results for dense matrices do not apply. We numerically evaluate the proposed bound, compare it with corresponding Monte Carlo simulations and thereby show that this upper bound almost coincides with the simulations. Finally, we discuss the generalization of the bound to multi-layer networks.

## References

[1] R. Ahlswede, N. Cai, S.-Y. Li, R.W. Yeung, "Network Information Flow," *IEEE Transactions on Information Theory*, Vol. 46, No. 4, pp. 1204–1216, July 2000.
[2] S.-Y. Li, R. Yeung, N. Cai, "Linear Network Coding," *IEEE Transactions on Information Theory*, Vol. 49, No. 2, pp. 371–381, February 2003.
[3] R. Koetter, M. Médard, "An Algebraic Approach to Network Coding," *IEEE/ACM Transactions on Networking*, Vol. 11, No. 5, pp. 782–795, October 2003.
[4] T. Ho, M. Médard, R. Koetter, D. Karger, M. Effros, J. Shi, B. Leong. "A Random Linear Network Coding Approach to Multicast". *IEEE Transactions on Information Theory*, Vol. 52, No. 10, pp. 4413–4430, October 2006.
[5] D. Silva, F.R. Kschischang, R. Koetter. "Communication Over Finite-Field Matrix Channels". *IEEE Transactions on Information Theory*, Vol. 56, No. 3, pp. 1296–1305, March 2010.
[6] S. Puchinger, M. Cyran, R.F.H. Fischer, M. Bossert, J.B. Huber, "Error Correction for Differential Linear Network Coding in Slowly-Varying Networks," *accepted for presentation at ITG Conference on Systems, Communication and Coding (SCC) 2015*.
[7] B. Schotsch, M. Cyran, J.B. Huber, R.F.H. Fischer, P. Vary, "An Upper Bound on the Outage Probability of Random Linear Network Codes with Known Incidence Matrices," *accepted for presentation at ITG Conference on Systems, Communication and Coding (SCC) 2015*.
[8] B.E. Schotsch, "Rateless Coding in the Finite Length Regime," Ph.D. dissertation, IND, RWTH Aachen University, Aachen, Germany, 2014.