A Comparison of OFDM with Cyclic Prefix and Unique Word Based on the Physical Layer of DVB-T

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Abstract. *Common OFDM systems guard individual symbols by means of Cyclic Prefix (CP). This does not only serve the purpose of preventing intersymbol interference (ISI), it also induces cyclicity which is necessary to equalize the received signal in the frequency domain. Recently unique word (UW) OFDM has been proposed as a new approach to create guard intervals which feature the same properties regarding ISI and cyclicity. In this paper we will shortly revisit both methods of creating a guard interval by means of cyclic prefix and unique word, respectively. In order to compare both OFDM concepts the physical layer of Digital Video Broadcasting for terrestrial television (DVB-T) was modified to support UW-OFDM. Extensive simulations have been performed to evaluate cyclic prefix und unique word in terms of bit error rate and spectral efficiency.*

Keywords

Cyclic Prefix, Unique Word, OFDM, DVB-T.

1. Introduction

Current communication systems like LTE [7], Wireless LAN IEEE802.11a [6] or DVB-T [4] all use *Orthogonal Frequency Divison Multiplexing* (OFDM). In order to protect the transmitted data against inter-symbol interference (ISI) a *Cyclic Prefix* (CP) is used as guard interval. Cyclic prefix is solely created in the time domain. It is implemented by copying the last part of the output of the *Inverse Discrete Fourier Transform* (IDFT) and using this copy as a prefix (cf. Fig. 1). The output and the prefix form the whole OFDM symbol. Hence, the guard interval is random as each output is random. Due to the resulting cyclicity of the OFDM symbol structure, straightforward Zero-forcing equalization can be performed at the receiver. Before demodulation the cyclic prefix is removed from the OFDM symbol and has no further use in the process of decoding or demodulation.

In [1], a new approach for creating a guard interval is proposed which is called *Unique Word* (UW). In contrast to cyclic prefix, the unique word is a known, unique sequence and thus can be used for synchronization and channel estimation. Therefore, dedicated pilot carriers are no longer needed compared to systems using cyclic prefix. The unique word is generated in frequency and time domain as a two-step approach [2]. First, redundancy induced in the frequency domain to create an output of the IDFT that ends with N_{uw} zeros. N_{uw} is the desired length of the guard interval. Second, the generated zeros are replaced with the known unique word sequence. In contrast to cyclic prefix, unique word introduces correlation within each OFDM symbol, which can be exploited at the receiver by means of a Wiener smoothing filter. Furthermore, the guard interval is part of the *Discrete Fourier Transform* (DFT) which is the main innovation of unique word.

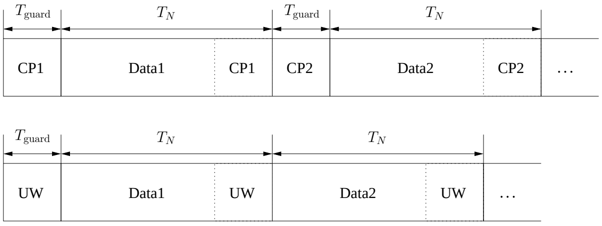

Fig. 1: OFDM transmit structure of CP (top) and UW (bottom).

In both systems the DFT and IDFT are implemented as *Fast Fourier Transform* (FFT) resp. *Inverse FFT* (IFFT).

2. Calculation of the Unique Word

Notation: Vectors are written as bold, lower case letters ($\mathbf{a}$). Matrices are written as bold, upper case letters ($\mathbf{A}$). In the frequency domain, vectors have a tilde ($\tilde{\mathbf{a}}$)

In the two-step approach, the output $\mathbf{x}$ created by the IDFT must end with N_{uw} zeros. This can be described as

$$\mathbf{F}_N^{-1}\tilde{\mathbf{x}} = \mathbf{x} = \begin{bmatrix} \mathbf{x}' \\ \mathbf{0} \end{bmatrix} . \tag{1}$$

$\mathbf{F}_N$ is the DFT matrix containing the element $[\mathbf{F}_N]_{m,n} = \frac{1}{\sqrt{N}}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}mn}$ in the m-th row and n-th column ($m, n = 0, 1, 2, \ldots, N-1$). It computes the DFT defined as

$$\tilde{x}_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}mn} . \tag{2}$$

Equation (1) can only be solved if at least N_{uw} values of $\tilde{\mathbf{x}}$ contain so-called redundant symbols $\tilde{\mathbf{r}}$ which must be determined based on the mapped symbols $\tilde{\mathbf{d}}$. We only consider the case in which the quantity N_r of redundant symbols $\tilde{\mathbf{r}}$ equals N_{uw}. If we write $\tilde{\mathbf{x}}$ as a permutation matrix $\mathbf{P}$ multiplied by a vector composed of the mapped symbols $\tilde{\mathbf{d}}$ and the induced redundancy $\tilde{\mathbf{r}}$, we can rewrite (1):

$$\mathbf{F}_N^{-1}\tilde{\mathbf{x}} = \mathbf{F}_N^{-1}\mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} = \mathbf{M}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} . \tag{3}$$

By dividing $\mathbf{M}$ into sub matrices we can find an expression for $\tilde{\mathbf{r}}$

$$\begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12}\\ \mathbf{M}_{21} & \mathbf{M}_{22}\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} = \begin{bmatrix}\mathbf{x}'\\ \mathbf{0}\end{bmatrix} \tag{4}$$

$$\tilde{\mathbf{r}} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\tilde{\mathbf{d}} = \mathbf{T}\tilde{\mathbf{d}} . \tag{5}$$

Now, $\tilde{\mathbf{x}}$ can be calculated in one step with the help of a generator matrix $\mathbf{G}$. $\mathbf{I}$ denotes the identity matrix:

$$\tilde{\mathbf{x}} = \mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{I}\\ \mathbf{T}\end{bmatrix}\tilde{\mathbf{d}} = \mathbf{G}\tilde{\mathbf{d}} . \tag{6}$$

After the IFFT, the unique word $\mathbf{x}_{\text{uw}}$ can be added in the time domain

$$\mathbf{x}_{\text{tot}} = \mathbf{x} + \mathbf{x}_{\text{uw}} = \begin{bmatrix}\mathbf{x}'\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{x}'_{\text{uw}}\end{bmatrix} . \tag{7}$$

At the receiver, we apply the FFT to the received signal $\mathbf{y}$, which is $\mathbf{x}_{\text{tot}}$ distorted by the impulse response and the additive noise $\mathbf{n}$ of the channel. In the frequency domain, we can describe $\mathbf{y}$ as

$$\tilde{\mathbf{y}} = \mathbf{H}\tilde{\mathbf{x}}_{\text{tot}} + \tilde{\mathbf{n}} \tag{8}$$

where $\mathbf{H}$ is a diagonal matrix that represents the frequency response of the channel. By means of $\mathbf{H}$, we can subtract the unique word in frequency domain right after the FFT

$$\tilde{\mathbf{z}} = (\mathbf{H}\tilde{\mathbf{x}}_{\text{tot}} + \tilde{\mathbf{n}}) - \mathbf{H}\tilde{\mathbf{x}}_{\text{uw}} = \tilde{\mathbf{y}} - \mathbf{H}\tilde{\mathbf{x}}_{\text{uw}} . \tag{9}$$

In order to reduce the influence of noise, we apply the Wiener smoothing filter $\mathbf{W}$ adapted from [5] which is a filter achieving the *Linear Minimum Mean Square Error* (LMMSE). It exploits the correlation within $\tilde{\mathbf{x}}$ induced by $\mathbf{T}$. For UW-OFDM the estimated data vector $\hat{\tilde{\mathbf{d}}}$ is

$$\hat{\tilde{\mathbf{d}}} = \mathbf{W}\mathbf{H}^{-1}\tilde{\mathbf{z}} = \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H + \frac{\sigma_n^2}{\sigma_d^2}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right)^{-1}\mathbf{H}^{-1}\tilde{\mathbf{z}} . \tag{10}$$

In (10), σ_d^2 and σ_n^2 are the variances of $\tilde{\mathbf{d}}$ resp. $\mathbf{n}$ as we assume zero mean data. If a receiver does not apply the Wiener filter, it always performs worse than a system using cyclic prefix because it uses more energy without benefitting from it.

3. Energy Calculation

In order to compare the performance of the simulated systems, the energy of the created symbols needs to be determined. We assume the data symbols to be uncorrelated with zero mean and variance σ_d^2. In both systems the total number of subcarriers is N and the length of the guard interval is N_{cp} resp. N_{uw}. Furthermore, Parseval's theorem states $E\{\mathbf{x}^H\mathbf{x}\} = E\{\tilde{\mathbf{x}}^H\tilde{\mathbf{x}}\}$ based on (2).

Cyclic prefix The energy of a symbol consists of the energy E_d of the data, the energy E_p of the pilots and the energy of the cyclic prefix. Since cyclic prefix is a partial copy of IFFT output, the total energy of the OFDM symbol can be calculated as

$$E_{\text{tot,cp}} = (E_d + E_p)\,\frac{N + N_{\text{cp}}}{N} \tag{11}$$

$$= \left(N_d\sigma_d^2 + E_p\right)\frac{N + N_{\text{cp}}}{N} . \tag{12}$$

Unique word In this case, the total energy $E_{\text{tot,uw}}$ is the sum of the energy E_d of the data, the energy E_r of the induced redundancy and the energy E_{uw} of the unique word itself

$$E_{\text{tot,uw}} = E_d + E_r + E_{\text{uw}} . \tag{13}$$

It can be shown that E_r is the trace of $(\mathbf{T}\mathbf{T}^H)$ multiplied by σ_d^2 [3]. E_{uw} can be chosen freely, however, it is set equal to the energy of the pilots in the cyclic prefix case because the unique word should serve the same purpose as the pilots. This yields

$$E_{\text{tot,uw}} = N_d\sigma_d^2 + \text{tr}\left(\mathbf{T}\mathbf{T}^H\right)\sigma_d^2 + E_{\text{uw}} \tag{14}$$

$$= \left[N_d + \text{tr}\left(\mathbf{T}\mathbf{T}^H\right)\right]\sigma_d^2 + E_{\text{uw}} . \tag{15}$$

Because $\mathbf{T}$ depends solely on $\mathbf{P}$, the right choice of $\mathbf{P}$ is crucial to the symbol energy $E_{\text{tot,uw}}$. A bad choice of $\mathbf{P}$ can cause an extraordinary high energy E_r.

4. DVB-T with Unique Word OFDM

In [1], UW-OFDM has been studied for the Wireless LAN IEEE 802.11a physical layer, which uses a half-rate convolutional encoder [6]. The FFT length of IEEE 802.11a is $N = 64$. In this paper, UW-OFDM is investigated for the *Terrestrial Digital Video Broadcasting* (DVB-T) physical layer [4] as a more complex system. DVB-T uses Reed-Solomon encoding in combination with convolutional encoding with puncturing (see Fig. 2 and Tab. 1 for details). Here, the 2K-Mode of DVB-T (FFT length $N = 2048$) with a guard interval length of $N/4$ is analyzed. Although DVB-T has a choice of multiple modulation schemes, here only QPSK ($\sigma_d^2 = 1$) is used.

The integration of unique word to DVB-T poses one main problem. It can be seen in Tab. 1 that not all OFDM

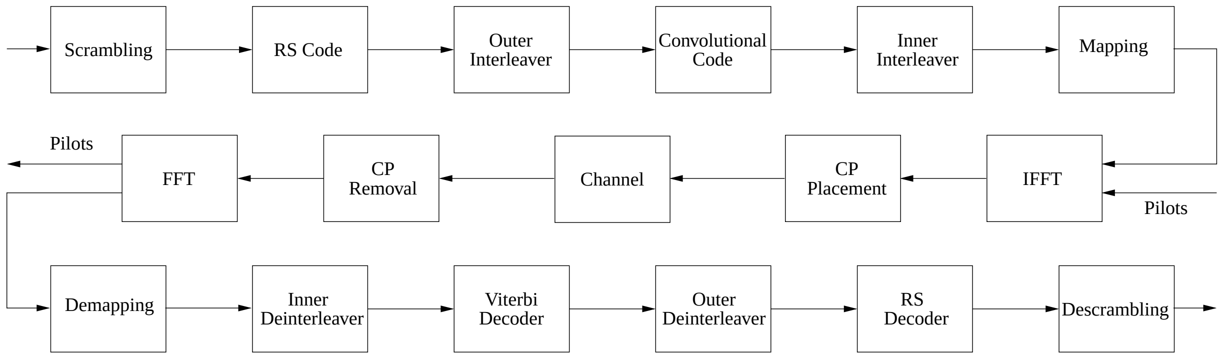

Fig. 2: Block diagram of the DVB-T physical layer according to [4].

subcarriers are used in DVB-T. The last 343 carriers of each OFDM symbol are unused. Therefore, equation (3) has to be adapted by replacing matrix $\mathbf{P}$ with matrix $\mathbf{B}$. Matrix $\mathbf{B}$ consists of a permutation matrix $\mathbf{P}'$ and 343 empty rows below

$$\mathbf{B} = \begin{bmatrix} \mathbf{P}' \\ \mathbf{0} \end{bmatrix} . \tag{16}$$

However, a direct substitution of cyclic prefix by unqiue word is still impossible. Although several permutations $\mathbf{P}'$ have been tried, all resulted in a redundant energy E_r of at least 195 dB which makes the new system neither comparable to the case of cyclic prefix nor usable in reality.

Therefore, the DVB-T physical layer has to be adjusted slightly. In the case of CP-OFDM, the unused subcarriers are replaced with data and pilot carriers, however, the ratio of pilots and data carriers are kept similar. In the case of UW-OFDM, the 512 redundant carriers are distributed uniformly among all 2048 carriers because this minimizes E_r according to [3]. Thus, in the case of the optimal distribution of redundant carriers, E_r equals E_d. The remaining carriers are used as data carriers. All other parts of the DVB-T physical layer (i.e., scrambling, coding and modulation) are not modified. The new set-up for both systems is stated in Tab. 2. The block diagram of the DVB-T physical layer with UW-OFDM is shown in Fig. 3.

It is obvious that both new systems differ in spectral efficiency η. The bandwidth of a single carrier of DVB-T in 2k-Mode is $\Delta f = 4.464\,\mathrm{kHz}$ and the duration of one OFDM symbol without cyclic prefix is $T_N = 224\,\mu\mathrm{s}$ [4]. Therefore, in case of QPSK mapping without coding, the spectral efficiencies of both systems can be calculated:

$$\eta_{\mathrm{cp}} = \frac{2N_d}{T_N\left(1+\frac{N_{\mathrm{cp}}}{N}\right)} \cdot \frac{1}{\Delta f N} = 1.42\,[\mathrm{bit/s/Hz}] \tag{17}$$

$$\eta_{\mathrm{uw}} = \frac{2N_d}{T_N} \cdot \frac{1}{\Delta f N} = 1.50\,[\mathrm{bit/s/Hz}] \tag{18}$$

The spectral efficiency of the system using UW is slightly higher. However, Tab. 2 shows that the energy needed for transmission of one UW symbol is also higher compared to the system using CP.

number of carriers N	**2048**
number of data carriers N_d	**1512**
number of pilot carriers N_p	**193**
length of the guard interval N_{cp}	**512**
energy of data symbols E_d	**31.80 dB**
energy of pilot symbols E_p	**24.01 dB**
total energy $E_{\mathrm{tot,cp}}$	**33.43 dB**
coderate of Reed-Solomon code r_{rs}	**188/204**
coderate of convolutional code r_{cc}	**1/2,2/3,3/4,5/6,7/8**

Tab. 1: DVB-T properties.

	CP	**UW**
total number carriers N	**2048**	**2048**
number of data carriers N_d	**1816**	**1536**
number of pilot carriers N_p	**232**	**—**
number of redundant carriers N_r	**—**	**512**
length of guard interval N_{cp} or N_{uw}	**512**	**512**
energy of data symbols E_d	**32.59 dB**	**31.86 dB**
energy of pilots E_p	**24.81 dB**	**—**
energy of redundant symbols E_r	**—**	**31.86 dB**
energy of unique word E_{uw}	**—**	**24.81 dB**
total energy $E_{\mathrm{tot,cp}}$ / $E_{\mathrm{tot,uw}}$	**33.26 dB**	**35.28 dB**

Tab. 2: Properties of the adapted CP-OFDM system and its UW-OFDM counterpart.

5. Simulation Results

In order to compare the two adapted DVB-T physical layer implementations, both systems have been simulated for transmission over an AWGN channel for five different code rates $r_{\mathrm{cc}} \in \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}\}$. Without loss of generality, QPSK has been employed as modulation scheme. Both systems have been compared in terms of their residual bit error rate (BER). In Fig. 4 the simulation results are depicted for both CP-OFDM (—) and UW-OFDM (- - -). It can be seen that CP-OFDM performs up to 0.25 dB better

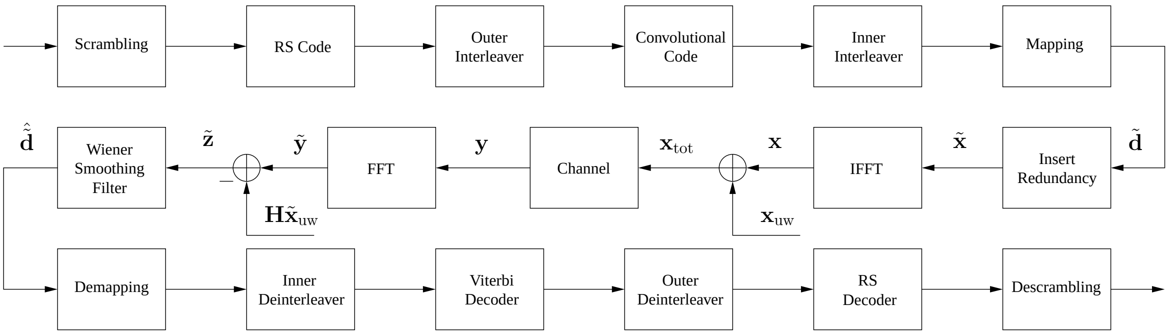

Fig. 3: Block diagram of DVB-T physical layer with UW-OFDM.

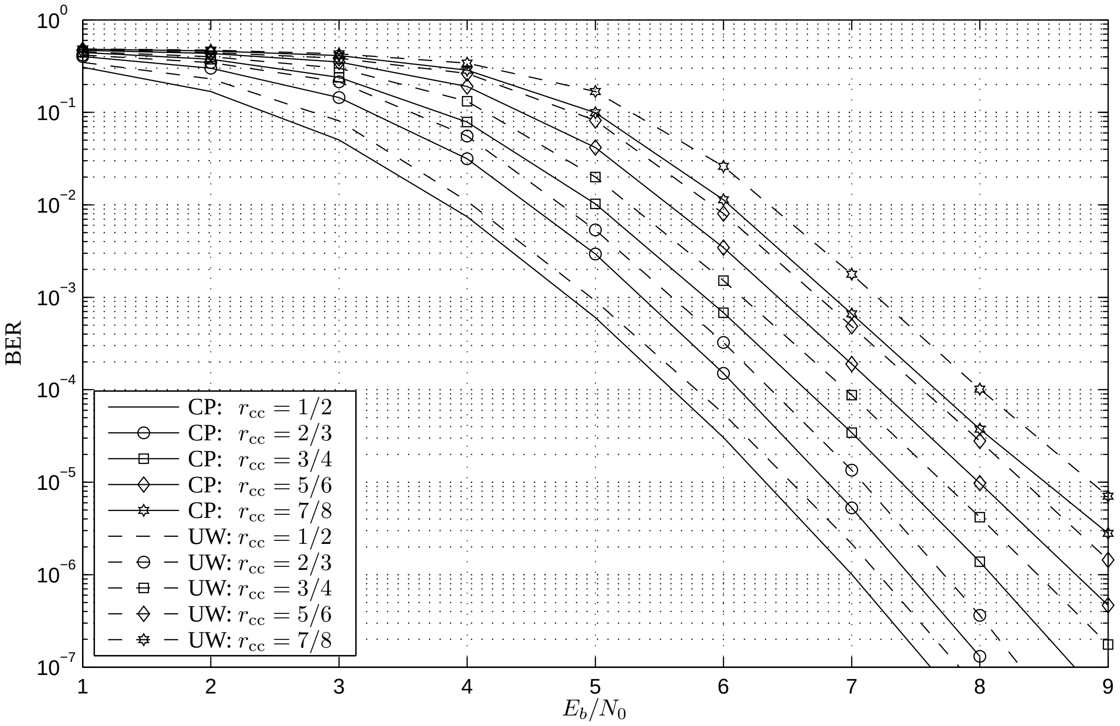

Fig. 4: Bit Error Rates of adapted DVB-T with CP-OFDM (—) and UW-OFDM (- - -)

for all cases compared to UW-OFDM even though the total energy $E_{\mathrm{tot,uw}}$ of UW-OFDM is slightly higher than the total energy $E_{\mathrm{tot,cp}}$.

6. Conclusion

In this paper, a novel OFDM signaling concept that was proposed in [1], is investigated, where the guard intervals are built by unique words instead of cyclic prefixes. Since the unique word concept has been studied only for systems with short FFT length so far [1], in this contribution the DVB-T physical layer was chosen as base for the simulation due to its higher FFT length of $N = 2048$. In order to allow for a performance comparison of the novel UW-OFDM with the common CP-OFDM, the DVB-T physical layer was adapted slightly to ease the integration of unique word and to support a fair comparison of both OFDM concepts. It has been shown that the creation of the guard interval with a cyclic prefix cannot simply be replaced by a unique word. The distribution of subcarriers which are used for data (N_d) and redundancy (N_r), respectively, has to be chosen carefully. Especially, in case some of the subcarriers are unused (as in DVB-T), the redundant subcarries needed for the generation of the unique word can cause a very high and thus infeasible total energy of one OFDM symbol.

Extensive simulations have been carried out to evaluate the performance of both OFDM concepts. It was shown that even though the spectral efficiency of UW-OFDM is slightly higher the novel concept performs 0.25 dB worse than CP-

OFDM in terms of bit error rate. Although an analysis for more realistic (i.e. frequency selective) channel models has yet to be conducted, it can be concluded that it is unlikely that UW-OFDM will be a suitable future alternative for the current CP-OFDM. Despite the fact that the unique word slightly increases the spectral efficiency by using the guard interval for pilot transmission, the overall performance in terms of bit error rate is reduced. Furthermore, more computational complexity is needed for the generation of the unique word, especially for high FFT lengths ($N \geq 2048$).

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