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Channel Codes for Softbit Source Decoding: Estimation of Correlated Parameters

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Abstract — **We present a new class of channel codes, which we call *Source Optimized Channel Codes* (SOCCs). These codes are designed by an optimization process, which takes into account the source and channel statistics as well as a quality measure. The quality measure is different from that of conventional channel coding, which usually aims at a minimum residual bit or sequence error rate. Instead, the code design is based on a quality measure in the domain of the continuous-valued source encoder symbols (e.g. speech parameters). At the receiver we exploit the code redundancy not for error correction, but to support parameter estimation in terms of *Soft Bit Source Decoding* (SBSD). Designing SOCCs with respect to the Signal to Noise Ratio (SNR) quality measure is considered in detail.**

To show the potential of SOCCs, we let them compete with a channel coding scheme including Unequal Error Protection (UEP) and Source Controlled Channel Decoding (SCCD). These experiments are performed using a common communication model to guarantee a fair comparison of the results.

I. INTRODUCTION

Source and channel coding play a major role in modern communication systems. While source coding removes redundancy and irrelevance from the source data to achieve a high bandwidth efficiency, channel coding, on the other hand, adds redundancy to protect the transmitted information against channel noise. An optimum performance of the overall system could be achieved if both source and channel coding would be optimal [1]. However, due to practical constraints, such as limited delay and computational power, optimality is usually not reached.

For this reason a jointly optimized source and channel coding scheme can possibly exhibit a better performance than one with separately designed components. In the past, several approaches to joint source channel coding have been proposed, such as Unequal Error Protection (UEP) [2] and Source Controlled Channel Decoding (SCCD) [3, 4]. Two approaches to exploit the a priori knowledge of an m-ary source in terms of channel decoding have been published in [5, 6].

Farvardin proposes in [7] *Channel Optimized (Vector) Quantization* and derives encoder/decoder pairs that minimize the mean square error of the decoded parameters specifically for a certain channel condition. In [8] an approach of joint source/channel coding utilizing optimized linear encoder mappings is investigated.

The new concept presented here makes use of source optimized quantizers in combination with a new class of non-linear channel codes, called *Source Optimized Channel Codes* (SOCCs). These codes are the result of an optimization process, which takes into account the source and channel statistics as well as a parameter-based quality measure. The code is derived by objective means directly from the given specification of the transmission system, unlike UEP design where often heuristic methods must be applied to classify and protect the transmitted bits according to their sensitivity. Thus the usage of SOCCs can be an alternative to UEP, especially when used in combination with conventional channel coding [9].

The usage of SOCCs influences not only the transmitter, but also the channel decoder design. While conventional channel decoders are designed such, that the residual bit or sequence error rate after decoding is minimized, our concept will make use of decoders that maximize a parameter-based quality measure. In case of the SNR quality measure, the Minimum Mean Square Error (MS) estimator is the optimal decoder. Thus the SOCC concept may also be seen as an extension of the SBSD-error concealment technique described in [10] and a generalization of the precoding approach presented in [11, 12, 13, 14].

An important issue for channel coding in general is the search for "good codes", which applies to SOCCs as well. In contrast to conventional codes, which are selected with respect to their Hamming distance properties, SOCCs must be designed such that they maximize the given quality measure. In fact, designing SOCCs turns out to be a very complex task, that can only be solved approximately by suboptimal optimization strategies.

Beside the discussion on SOCC design, this contribution presents results of a scientific competition between the Institute of Communication Systems and Data Processing at Aachen University of Technology (this paper) and the Institute for Communications Engineering at Munich University of Technology (see companion paper [4]). The task given to the competitors was to achieve a maximum parameter-SNR for a defined set of transmission model configurations, either with the methods described here or with the SCCD approach.

The paper is structured as follows: Section II describes the configuration of the applied communication model. Section III deals with the new *Source Optimized Channel Codes*, gives an example for a code search algorithm and generalizes the concept to Vector Source Optimized Channel Codes (VSOCCs). To explain the applied decoder structure, Section IV gives a brief review of *Soft Bit Source Decoding*. In Section V we present the simulation results achieved and compare the performance of our new approach with the results of [4]. The decoder complexity is estimated in Section VI.

II. Communication Model

To allow an objective comparison between our new channel coding strategy and the SCCD approach [3, 4], we first define a communication model that is mandatory for both. This model simulates a block-oriented speech transmission, as it is used in digital mobile communication systems, such as GSM. Figure 1 shows the basic structure.

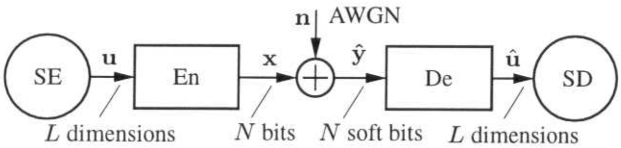

Fig. 1: Communication model

SE: parameter source, model of the source encoder
En: parameter encoder,
either quant. + SOCC
or quant. + UEP + conv. channel enc. [4]
De: parameter decoder,
either parameter estimation by SBSD
or SCCD + table lookup [4]
SD: parameter sink, model of the source decoder

Abbreviations

SOCC	Source Optimized Channel Code
SBSD	Soft Bit Source Decoding
UEP	Unequal Error Protection
SCCD	Source Controlled Channel Decoding

The source yields time discrete vectors $\mathbf{u}$ of L elements $u_i \in \mathbb{R}$ which are binary encoded to N-dimensional bit vectors $\mathbf{x} = (x_1, .., x_N)$ with $x_k \in \{-1, +1\}$. As channel model serves an AWGN channel with power spectral density $N_0/2$, BPSK modulation and matched filter receiver. The decoder receives the channel soft output vector $\hat{\mathbf{y}} \in \mathbb{R}^N$ to estimate sample vectors $\hat{\mathbf{u}}$ which are delivered to the sink.

A. Source Model

The source model is designed to approximate the characteristics of the source parameters generated by block based speech coding schemes. E.g. speech codecs utilizing Code Excited Linear Prediction (CELP) encode a speech segment of typically 5-20 ms by a set of speech parameters, such as LPC filter coefficients, gain factors and so on. In our model, such a set of parameters is represented by vector $\mathbf{u}$. Due to speech encoding the parameters representing one speech segment are mutually almost uncorrelated, while parameters of subsequent segments still have some residual correlation.

Therefore each component u_i of the source parameter vector $\mathbf{u} = (u_1, .., u_L)$ is modeled by an individual Gauss-Markov process of order one. The component sources u_i and u_j $(i \neq j)$ are statistically independent. Figure 2 depicts the generation of a single component u_i. White Gaussian noise w_i with variance 1 is processed by a first order recursive filter. In this way a well defined correlation is introduced into the vector components.

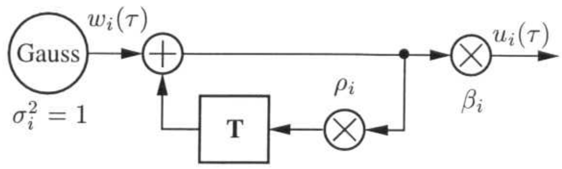

Fig. 2: AR model for source vector components

Letting $\sqrt{1-\rho_i^2} = \beta_i$, we get

$$\varphi_{u_i u_i}(0) = \sigma_i^2 = \sigma^2 = 1 \quad \text{and} \quad \varphi_{u_i u_i}(1) = \rho_i = \rho\,, \tag{1}$$

where $\varphi_{u_i u_i}(\cdot)$ represents the auto-correlation function of source component u_i.

The joint probability density function (pdf) of a tuple of two subsequent output values $u_i(\tau), u_i(\tau-1)$ is then given by (see Appendix A)

$$p(u_i(\tau), u_i(\tau-1)) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \cdot \exp\left(\frac{u_i^2(\tau)+u_i^2(\tau-1)-2\rho u_i(\tau)u_i(\tau-1)}{2\sigma^2(\rho^2-1)}\right). \tag{2}$$

B. System configuration

To cover a wide range of possible system configurations we vary the following parameters of our communication model:

- Correlation $\rho \in \{0, 0.5, 0.75, 0.9\}$
- Maximum allowed bit rate on the channel

  $N/L \in \{4, 6, 8\}$ bit/dim.
- Required SNR($\hat{\mathbf{u}}$) under noise-free conditions

TABLE I: SNR requirements

bit rate N/L	4	6	8
SNR($\hat{\mathbf{u}}$) [dB]	9	13	17

The number of bits N transmitted per vector $\mathbf{x}$ will be fixed to $N = 120$, thus the number of dimensions in $\mathbf{u}$ takes the values $L \in \{30, 20, 15\}$. For the described system configurations the mean SNR of the estimated vector $\hat{\mathbf{u}}$ has been determined as a function of E_S/N_0.

III. Source Optimized Channel Codes

The conventional way to adapt channel codes to the source properties and a given quality measure is to apply Unequal Error Protection with respect to the binary representation of the source symbols, i.e. sensitive bits are protected by strong channel codes and less sensitive by weak codes.

By contrast, *Source Optimized Channel Coding* is based on the idea to consider the quality measure in the parameter domain for the design of channel codes such, that the distance properties of the real valued source symbols remain preserved in the Hamming distance-domain of their assigned bit vectors.

A. Definition

We assume that the encoding block in Figure 1 is split into a quantization and a channel coding part, as shown by Figure 3. The quantizer maps the range of values of $\mathbf{u}$, i.e.

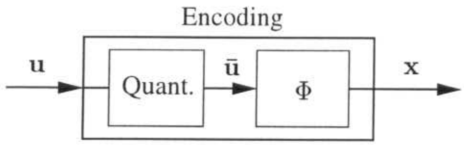

Fig. 3: Quantization and channel encoding

$\mathbb{R}^L$, to a set of M discrete vectors $\mathbb{U} = \{\bar{\mathbf{u}}^{(1)}, .., \bar{\mathbf{u}}^{(M)}\}$ of dimension L. In this way irrelevant information, that is not necessary to achieve the required accuracy at the receiver, is removed. A second, redundant mapping $\mathbf{x} = \Phi[\bar{\mathbf{u}}]$, which represents the channel coding part, assigns a unique channel symbol $\mathbf{x}$ consisting of $N > \log_2(M)$ bits to each of the discrete vectors $\bar{\mathbf{u}}$. Note that M is not necessarily a power of 2.

Our concept also differs from conventional channel coding in the way we use the redundancy introduced by Φ at the receiver: Instead for error correction of the binary representation of $\bar{\mathbf{u}}$, it is utilized to support the direct estimation of $\hat{\mathbf{u}}$. While conventional systems employ a channel decoder and subsequent table lookup, here these functions are replaced by a single parameter estimation unit (see Figure 4), which exploits

- code redundancy due to Φ,
- residual source redundancy $\Pr(\,\bar{\mathbf{u}}(\tau)\,|\,\bar{\mathbf{u}}(\tau-1)\,)$,
- channel soft output $\hat{\mathbf{y}}$ (reliability),
- channel quality information E_s/N_0.

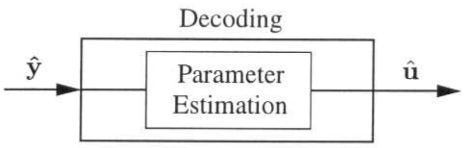

Fig. 4: Decoding by *Soft Bit Source Decoding*

By introducing a quality measure

$$\mathcal{D}(\bar{\mathbf{u}}, \hat{\mathbf{u}}) \; , \tag{3}$$

a statistical model of the transmission channel

$$\hat{\mathbf{y}} = t(\mathbf{x}) \quad (\text{described by } p_{\hat{\mathbf{y}}|\mathbf{x}}(\hat{\mathbf{y}}|\mathbf{x})\,) \tag{4}$$

and the decoder (estimator) which is optimal with respect to $\mathcal{D}$ and t

$$\hat{\mathbf{u}} = f_{\mathcal{D},t}(\hat{\mathbf{y}}) \; , \tag{5}$$

we can now define a SOCC as a code $\mathbb{C}$

$$\mathbb{C} = \{\,\mathbf{x}\,|\,\mathbf{x} = \Phi[\bar{\mathbf{u}}],\ \bar{\mathbf{u}} \in \mathbb{U}\,\} \; , \tag{6}$$

that is a solution to the optimization problem

$$\min_{\Phi} \mathbf{E}\,\{\mathcal{D}(\bar{\mathbf{u}}, f_{\mathcal{D},t}(\,t(\Phi[\bar{\mathbf{u}}])\,))\} \; . \tag{7}$$

Here, $\mathbf{E}\{\cdot\}$ denotes the expectation of the expression in braces.

B. Code Search Strategy

It becomes obvious that solving the optimization problem (7) is a non-trivial task, when considering the enormous number of possible mappings Φ. In total these amount to $\frac{2^N!}{(2^N-M)!}$, which makes an exhaustive search prohibitive, as soon as the code vectors exceed a length of $N = 4$ bits.

Similar to the *Index Assignment* (IA) problem, the SOCC optimization problem can be classified as a so called *Non-Polynomial* (NP)-complete problem. For this class of problems there is no known algorithm in which the worst-case computational complexity is bounded by a polynomial in the size of the input. However, by suboptimal search algorithms [15, 16] at least local optima can be obtained in reasonable time.

To apply these algorithms, we have to introduce approximations that make the code search problem suited for computation without sacrificing too much of the code performance. As we will see later on, the code design algorithm requires the frequent evaluation of (7). Therefore, it is crucial to minimize the computational complexity of this expression. A major obstacle to do so is the continuous output of the AWGN channel, as the evaluation of the expectation in (7) would require an N-fold integration over all components of $\hat{\mathbf{y}}$.

To avoid this, we approximate the continuous AWGN channel by an AWGN channel combined with a subsequent one-bit quantizer for each vector component $\hat{y}_i$, as shown in Figure 5. It has to be stressed that this approximation is only used for the code design not for the decoding algorithm presented later.

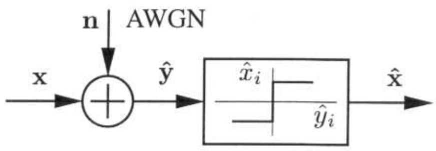

Fig. 5: AWGN channel with one-bit quantizer

The channel shown in Figure 5 can also be interpreted as a Binary Symmetric Channel (BSC). As the channel is assumed to be memoryless, its transition probability is given by

$$\Pr(\hat{\mathbf{x}}|\mathbf{x}) = (1-\epsilon)^{N-|\hat{\mathbf{x}}\oplus\mathbf{x}|} \cdot \epsilon^{|\hat{\mathbf{x}}\oplus\mathbf{x}|} \; , \tag{8}$$

with $|\cdot|$ denoting the Hamming weight of a bit vector and $\oplus$ the bitwise exclusive-or operation. Assuming identical distribution of the code bits

$$\Pr(x_k = -1) = \Pr(x_k = +1) = 0.5 \; ,$$

the bit error probability ϵ is given by

$$\epsilon = \sqrt{\frac{E_s/N_0}{\pi}} \int_{-\infty}^{0} e^{-\frac{E_s}{N_0}(\hat{y}-1)^2}\, d\hat{y} \; . \tag{9}$$

By quantizing the channel output the number of possible reception vectors $\hat{\mathbf{x}}$ becomes finite, as they stem from the discrete set $\mathbb{X} = \{\hat{\mathbf{x}}^{(1)}, .., \hat{\mathbf{x}}^{(2^N)}\}$ of all possible bit combinations consisting of N bits. Thereby we approximate the integrals, that would be necessary to compute for a channel with continuous output, by a sum over all possible reception vectors $\hat{\mathbf{x}}^{(j)}$.

In the sequel, we will focus our considerations on the SNR quality measure

$$\mathcal{D}(\bar{\mathbf{u}}, \hat{\mathbf{u}}) = \|\bar{\mathbf{u}} - \hat{\mathbf{u}}\|^2 \; . \tag{10}$$

In this case the optimal decoder is the Minimum Mean

Square (MS) estimator. Due to complexity aspects, the statistical dependence of corresponding elements u_i of subsequent vectors $\bar{\mathbf{u}}(\tau)$, $\bar{\mathbf{u}}(\tau-1)$ is not taken into consideration for the code design but for parameter estimation at the receiver. Thus the MS estimator is given by[1]

$$\hat{\mathbf{u}} = f_{\mathcal{D},t}(\hat{\mathbf{x}}) = \sum_{\bar{\mathbf{u}}\in\mathbb{U}} \bar{\mathbf{u}} \cdot \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}) \cdot \frac{\Pr(\bar{\mathbf{u}})}{\Pr(\hat{\mathbf{x}})} \,. \quad (11)$$

The probability of occurrence of the received channel symbols is obtained by $\Pr(\hat{\mathbf{x}}) = \sum_{\bar{\mathbf{u}}\in\mathbb{U}} \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}) \cdot \Pr(\bar{\mathbf{u}})$.

By inserting (11) and (10) into (7), we get after some algebraic operations (see Appendix B)

$$\mathbf{E}\{\mathcal{D}(\bar{\mathbf{u}}, \hat{\mathbf{u}})\} = \sum_{\bar{\mathbf{u}}\in\mathbb{U}} ||\bar{\mathbf{u}}||^2 \cdot \Pr(\bar{\mathbf{u}}) - \sum_{\hat{\mathbf{x}}\in\mathbb{X}} ||f_{\mathcal{D},t}(\hat{\mathbf{x}})||^2 \cdot \Pr(\hat{\mathbf{x}}) \,. \quad (12)$$

The first term in (12) is independent of Φ and therefore irrelevant for the optimization task. Thus it is sufficient to maximize the second term with respect to mapping function Φ. In general, the resulting SOCC $\mathbb{C}$ does not exhibit the linearity property of linear block codes. Hence SOCCs have to be considered as non-linear codes.

C. Scalar SOCCs

Utilizing the concept of SOCCs for the communication model described in section II, an obvious approach is to apply an individually optimized SOCC to each of the L vector components of $\bar{\mathbf{u}}$, i.e.

$$\Phi(\bar{\mathbf{u}}) = (\ \phi_1(u_1), .., \phi_L(u_L)\ ) \,. \quad (13)$$

By the separation of the mapping function Φ into such scalar SOCCs, it is possible to avoid long codes which are difficult to design and which cause a high decoding complexity. On the other hand it is a well known fact, that the performance of an error protection code increases with its length. To approach the theoretical performance bound, the code length must even tend to infinity [1]. Thus, scalar SOCCs are not the best choice to achieve high error robustness.

D. Vector SOCCs

To overcome this shortcoming of scalar SOCCs, we propose so called Vector SOCCs. Similar to vector quantization, where vectors of parameters are commonly quantized, a VSOCC protects a group of parameters by a common code, e.g. with two-dimensional SOCCs the mapping looks like

$$\Phi(\bar{\mathbf{u}}) = (\ \phi_{1,2}(u_1, u_2), .., \phi_{L-1,L}(u_{L-1}, u_L)\ ) \,. \quad (14)$$

Of course, the usage of high-dimensional vector SOCCs is limited due to complexity aspects. But in section V we will show by simulation that a significant gain is already achievable by applying two-dimensional instead of scalar SOCCs.

A further enhancement could be obtained if correlated components $u_i(\tau-T)..u_i(\tau)$ were commonly vector-quantized and protected by SOCCs. However, with respect to the specified communication model, this would imply an illegal coding delay and was therefore not considered here.

[1] Here, we assume, that a source optimized quantizer is used, i.e. that $\bar{\mathbf{u}}^{(i)} = \mathbf{E}\{\mathbf{u}|\mathbf{u} \in S_i\}$, where S_i is the i-th quantizer partition.

E. Code Design Algorithm

For the optimization of (7), we employ a modified *Binary Switching Algorithm* (BSA) [15], which was originally proposed to optimize the index assignment of vector quantizers. The basic idea of this algorithms is, starting from an initial mapping Φ_{init}, to select a pair of reproduction levels $\bar{\mathbf{u}}^{(k)}, \bar{\mathbf{u}}^{(\ell)}$ and interchange their assigned codewords $\Phi[\bar{\mathbf{u}}^{(k)}] \leftrightarrow \Phi[\bar{\mathbf{u}}^{(\ell)}]$. If this modified mapping yields a reduced reduced distortion (12), it is accepted, otherwise the former is kept and another pair is checked. This procedure is repeated until a local minimum of the distortion is reached.

As the complexity caused by selection and tentative interchanging of codewords is negligible, the dominant part of the algorithmic complexity is due to the frequent evaluation of the sum

$$\mathcal{P}_f = \sum_{\hat{\mathbf{x}}\in\mathbb{X}} ||f_{\mathcal{D},t}(\hat{\mathbf{x}})||^2 \cdot \Pr(\hat{\mathbf{x}}) \,. \quad (15)$$

Unfortunately, in general all decoder output values $f_{\mathcal{D},t}(\hat{\mathbf{x}})$ as well as the probabilities of occurrence $\Pr(\hat{\mathbf{x}})$ vary if two codewords are swapped. But we show in Appendix C, that the re-computation of (15) can be performed using update rules with low complexity.

IV. Soft Bit Source Decoding

Soft Bit Source Decoding (SBSD) was recently proposed [10] to perform sophisticated error concealment for speech decoding. This technique can easily be generalized to derive a decoder for parameters that are protected by SOCCs.

Again, the decoder shall be optimal with respect to the SNR quality measure. But in contrast to the hard input decoder (11) which was utilized to solve the SOCC optimization problem, we will now consider a decoder that operates on the soft bits $\hat{\mathbf{y}}$ delivered by the channel. Besides, we take into account the statistical dependence of subsequent source samples to exploit the inherent redundancy as *a priori* knowledge for SBSD.

If no decoding delay is allowed, it can be shown that

$$\hat{\mathbf{u}} = \sum_{\bar{\mathbf{u}}(\tau)\in\mathbb{U}} \bar{\mathbf{u}}(\tau) \cdot \Pr(\,\bar{\mathbf{u}}(\tau)\,|\,\hat{\mathbf{y}}(\tau), .., \hat{\mathbf{y}}(1)\,) \quad (16)$$

is the optimal estimator for the parameter at time instant τ. In (16), the sequence $\hat{\mathbf{y}}(\tau), .., \hat{\mathbf{y}}(1)$ represents the complete history of received soft bit vectors. The *a posteriori* probabilities in (16) can be computed by a recursion formula [17, 10]. Denoting the *a posteriori* probabilities by $P(\bar{\mathbf{u}}(\tau)) = \Pr(\,\bar{\mathbf{u}}(\tau)\,|\,\hat{\mathbf{y}}(\tau), .., \hat{\mathbf{y}}(1)\,)$ we obtain (see Appendix D)

$$P(\bar{\mathbf{u}}(\tau)) = \frac{p(\,\hat{\mathbf{y}}(\tau)\,|\,\bar{\mathbf{u}}(\tau)\,)}{p(\,\hat{\mathbf{y}}(\tau)\,|\,\hat{\mathbf{y}}(\tau-1), .., \hat{\mathbf{y}}(1)\,)} \cdot \sum_{\bar{\mathbf{u}}(\tau-1)\in\mathbb{U}} \Pr(\bar{\mathbf{u}}(\tau)\,|\,\bar{\mathbf{u}}(\tau-1)) \cdot P(\bar{\mathbf{u}}(\tau-1)) \,. \quad (17)$$

Explicit computation of the conditional pdf in the denominator of equation (17) is not necessary as it is determined by the condition $\sum_{\bar{\mathbf{u}}(\tau)\in\mathbb{U}} P(\bar{\mathbf{u}}(\tau)) = 1$. The transition probabilities of the quantized source samples $\Pr(\bar{\mathbf{u}}(\tau) \mid \bar{\mathbf{u}}(\tau-1))$ can be obtained by integrating the joint pdf (2) over the quantizer partitions or by measurement.

V. SOCC Performance

Before we present the performance results of SOCCs for the various system configurations described in section II.B, the adjustment of basic design parameters, such as SOCC dimensionality, BER ϵ and quantizer SNR is discussed.

A. Design Parameters: Example

To investigate the influence of dimensionality on the SOCC performance, we performed simulations with a one-, two- and three-dimensional SOCC. In all cases the parameter correlation was $\rho = 0$ and the gross rate $N/L = 4$ bits per dimension. The employed Lloyd-Max quantizer had 6 reproduction levels. As the curves in Figure 6 show, a significant gain of parameter SNR (about 0.8 dB at $E_s/N_0 = 0$ dB) can be achieved by the two-dimensional compared to the one-dimensional SOCC, whereas using the three-dimensional SOCC only yields a negligible enhancement in the range from $E_s/N_0 \in [0, 6]$ dB. Thus, a dimension of two is a good trade-off between code performance and decoder complexity.

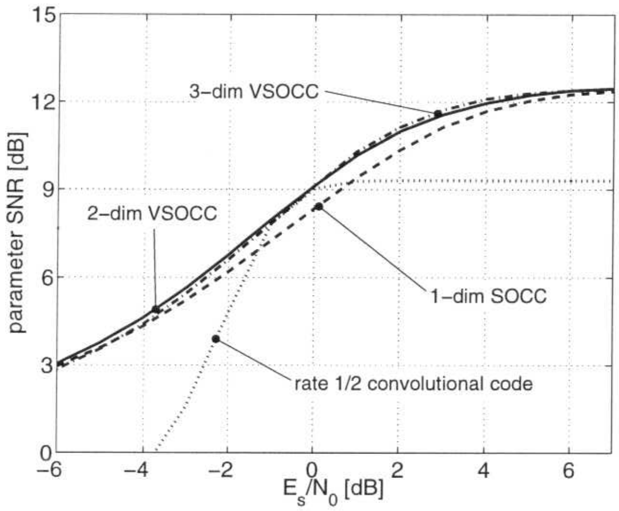

Fig. 6: SOCC dimensionality and performance

Comparing our approach with a memory 3, rate 1/2 convolutional code and soft input Maximum Likelihood decoding, as shown in Figure 6, we identified the interval from $[-1, +1]$ dB channel SNR as the most critical. If we optimize the SOCC for $E_s/N_0 = 0$ dB, then it outperforms the convolutional code for all channel conditions.

TABLE II: LMQ reproduction levels

required SNR [dB]	9	13	17
reproduction levels per dimension	6	12	20

The number of quantizer reproduction levels was selected such that the SNR requirements for noise-free conditions given in Table I are fulfilled and at the same time a maximum SNR at $E_s/N_0 = 0$ dB is achieved. Table II shows the applied settings.

B. Transmission with 4 Bits per Dimension

For the transmission with 4 Bits per Dimension we applied a SOCC with $M = 36$ code vectors of dimension $D = 2$ out of 256 possible bit combinations, i.e. the code rate was $\log_2(M)/(4D) = \log_2(36)/8 \approx 0.65$. Figure 7 depicts the simulation results for the given correlation factors ρ. Due to *Soft Bit Source Decoding* (SBSD) a remarkable enhancement in terms of parameter SNR can be achieved if $\rho > 0$.

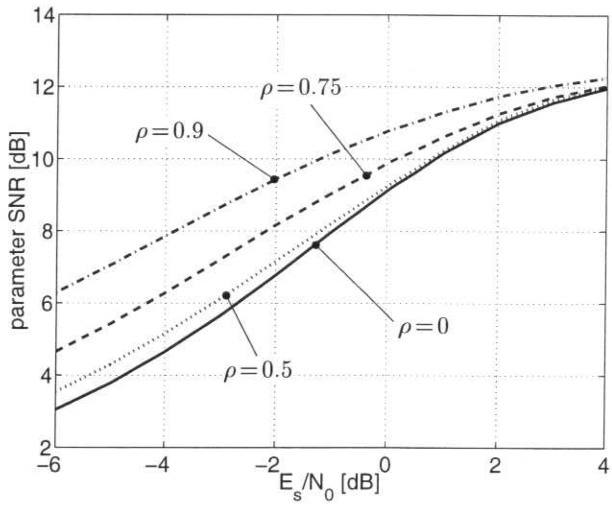

Fig. 7: SOCC/SBSD, 4 bits per dimension and $D = 2$

The UEP/SCCD approach works with a memory 6 *Recursive Systematic Convolutional* (RSC) coder and a two-step decoder structure [4]. In the first decoding step one set of parameter *a posteriori* probabilities are computed with respect to a received block $\hat{\mathbf{y}}$. With a formalism similar to (17) a second set of parameter *a posteriori* probabilities is derived from previously decoded parameter values by exploiting the time correlation of the source. The combination of both probability sets is used as *a priori* information for the second decoding step. As both decoding steps involve convolutional decoding of a received block, the complexity of the UEP/SCCD system becomes very high.

In Figure 8 our SOCC approach is compared to UEP/SCCD for $\rho = 0$, $\rho = 0.75$ and $\rho = 0.9$. While for correlation $\rho = 0$ there is a small SNR-range from $E_s/N_0 \in [-1, 0]$ dB where the SOCC/SBSD performs slightly worse than the UEP/SCCD scheme, SOCC/SBSD outperforms UEP/SCCD for all channel conditions if $\rho \geq 0.75$. For $\rho = 0.9$ a gain in parameter SNR of at least 1 dB and up to 3 dB can be observed.

In addition, the SOCC/SBSD system exhibits a graceful analogue-type degradation, whereas the system operating with UEP/SCCD suffers from a threshold effect known from conventional channel coding.

C. Transmission with 6 Bits per Dimension

Figure 9 shows the simulation results for transmission with 6 bit per dimension and $D = 2$. The rate of the ap-

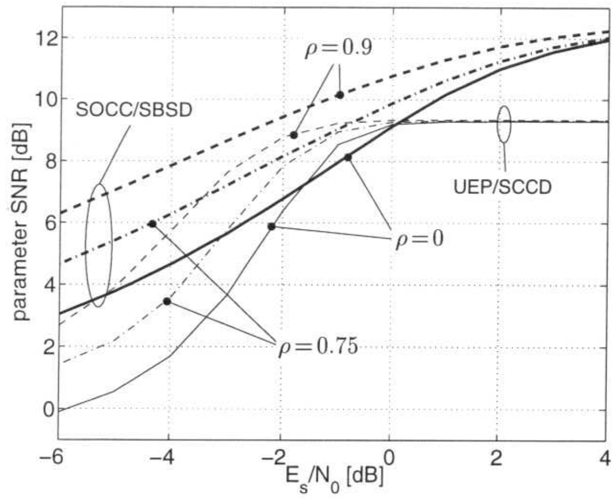

Fig. 8: SOCC/SBSD ($D = 2$) vs. UEP/SCCD, transmission with 4 bits per dimension

plied codes was $\log_2(M)/(6D) = \log_2(36)/12 = 0.43$. Compared to 4 bit transmission the parameter SNR at $E_s/N_0 = 0$ dB is increased by about $3 - 4$ dB.

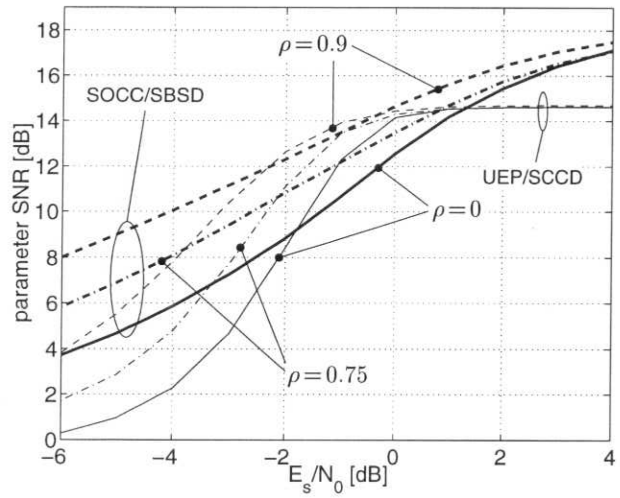

Fig. 9: SOCC/SBSD ($D = 2$) vs. UEP/SCCD, transmission with 6 bits per dimension

D. Transmission with 8 Bits per Dimension

Due to the enormous optimization complexity, a two-dimensional SOCC design was not feasible with the modified BSA described above. Hence, we investigated some approaches to constrain the set of potential codewords, such as selecting SOCC codewords only from the codeword set given by a linear code, or to align the SOCC to a tree structure. However, experiments showed that the performance loss due to these constraints almost consumes the dimensionality gain. Therefore we applied a one-dimensional SOCC with $M = 20$ levels for transmission at 8 bits per sample.

Alternatively, we considered a hybrid solution consisting of a concatenation of SOCCs and linear convolutional codes to protect one parameter block. By utilizing a *systematic* code (generated by a *Recursive Systematic Convolutional* (RSC) coder), the distance properties of the SOCC are preserved to some extend, as the systematic part of the code contains the unchanged SOCC codewords. Hence the resulting code can be interpreted as a SOCC given additional constraints.

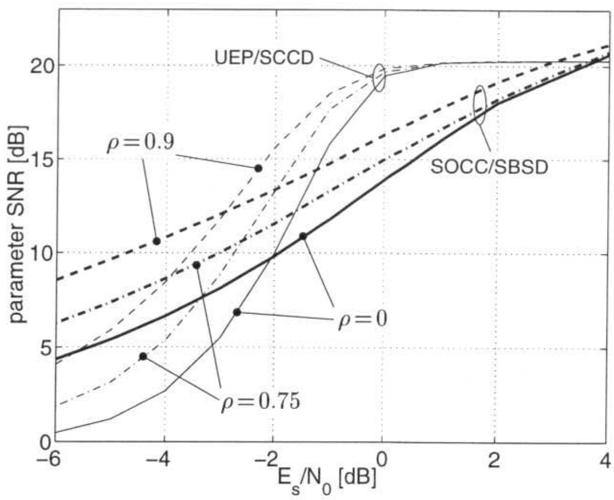

Fig. 10: SOCC/SBSD ($D = 1$) vs. UEP/SCCD, transmission with 8 bits per dimension

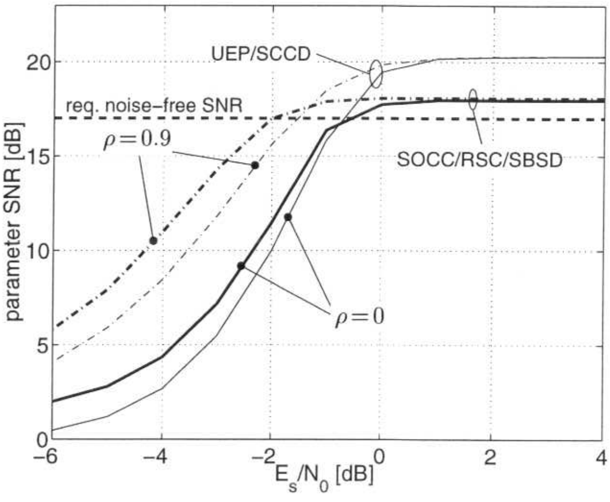

Fig. 11: SOCC/RSC/SBSD ($D = 1$, $M = 12$) vs. UEP/SCCD, transmission with 8 bits per dimension

To decode such a SOCC/RSC combination, we employ an MSE-optimal parameter decoder based on a combination of the forward-backward algorithm [17] and the recursion (17): By exploiting the time correlation of the transmitted parameters in terms of (17), parameter probabilities for the current block are estimated considering the history of received parameter blocks. These probability estimates serve as *a priori* information for a parameter-based forward-backward recursion running over the current SOCC/RSC-encoded block. As result we get a set of *a posteriori* probabilities for each transmitted parameter in that block. These probabilities allow to determine the parameter estimates $\hat{\mathbf{u}}$ and simultaneously are the basis to proceed with recursion (17).

Figure 11 depicts the simulation results yielded by the alternative approach in comparison to the UEP/SCCD approach. Under noise-free conditions an SNR of 17 dB (horizontal line) is required (see Table I) which is achieved by a $M = 12$-level LMQ. Thus we employ a rate $\log_2(12)/4$ SOCC and concatenate it with a rate 1/2 memory 6 RSC (same as used for UEP/SCCD). Although our scheme exhibits a lower quality under good channel conditions, the 17 dB-requirement is fulfilled and under bad channel conditions we gain up to 1.5 dB in parameter SNR.

VI. Decoder Complexity

To determine the decoder complexity we consider equations (16) and (17). Applying a scalar quantizer with R reproduction levels and D-dimensional SOCCs (hence $R^D = M$), a straightforward implementation of the decoder has a complexity of

$$C = \frac{R^D L}{D}\left[\left(\frac{ND}{L} + R^D + 2\right)\mu + (R^D + 1)\,\alpha\right], \quad (18)$$

where α denotes additions and μ multiplications. In Table III the decoder complexities for the three considered coding rates are listed. The MOPS column is calculated under the assumption that a parameter block has to be transmitted within 20 ms.

TABLE III: Decoding complexities for one block $\hat{\mathbf{u}}$

bit rate	add.	mult.	total op's	MOPS [a]
4	19980	24840	44820	2.24
6	208800	227520	436320	21
8[b]	6300	9000	15300	0.765

[a] MOPS are given for a 20 ms block rate.
[b] For rate 8 one-dimensional SOCC were applied.

VII. Conclusion

We presented a new class of channel codes which we call *Source Optimized Channel Codes* (SOCCs). These codes are designed according to an optimization procedure, which takes into account a *parameter*-based quality measure. The redundancy introduced by SOCCs is not used to detect/correct bit errors at the receiver, but to support parameter estimation in terms of *Soft Bit Source Decoding* (SBSD).

We performed detailed simulations in order to compare the new approach with known error protection schemes, such as convolutional codes in combination with *Unequal Error Protection* (UEP) and *Source Controlled Channel Decoding* (SCCD). It turned out that SOCC/SBSD is able to outperform UEP/SCCD under most channel conditions. Especially, at low gross rates and if the transmitted parameters have time correlation, SOCC/SBSD is superior. Gains in parameter SNR of at least 1 dB and up to 3 dB were observed compared to UEP/SCCD.

Combining SOCCs and conventional linear systematic channel codes offers new possibilities to the system design concerning the trade-off between source coding and error protection. By a close alignment to the system requirements we could achieve further performance gains under bad channel conditions.

Appendix

A. Joint PDF of AR Model

According to Figure 2 the current output value $u_i(\tau)$ is composed of

$$u_i(\tau) = \rho \cdot u_i(\tau-1) + \beta \cdot w_i(\tau)\,. \quad (A.1)$$

Due to the central limit theorem, $u_i(\tau)$ and consequently also $u_i(\tau-1)$ are Gaussian distributed with variance σ^2 and zero mean. Hence $u_i(\tau-1)$ has the pdf

$$p(u_i(\tau-1)) = \frac{1}{\sqrt{2\pi}\sigma}\cdot\exp\left(-\frac{u_i^2(\tau-1)}{2\sigma^2}\right). \quad (A.2)$$

The conditional pdf of $u_i(\tau)$ given $u_i(\tau-1)$ is also Gaussian distributed, but because of (A.1) has variance $\sigma^2\beta^2$ and mean $\rho u_i(\tau-1)$, i.e.

$$p(u_i(\tau)|u_i(\tau-1)) = \frac{1}{\sqrt{2\pi}\sigma\beta} \cdot\exp\left(-\frac{(u_i(\tau)-\rho u_i(\tau-1))^2}{2\sigma^2\beta^2}\right). \quad (A.3)$$

Multiplying (A.2) and (A.3) and inserting $\beta = \sqrt{1-\rho^2}$ yields equation (2).

B. Derivation of the Optimization Criterion for the BSC

The BSC channel model according to Figure 5 is described by the equation

$$\hat{\mathbf{x}} = t(\mathbf{x}) = \mathbf{x} \oplus \mathbf{e}, \quad (B.4)$$

where $\mathbf{e}$ is a binary error vector. Inserting (B.4) into (7) yields

$$\min_{\Phi} \mathbf{E}_{\bar{\mathbf{u}},\mathbf{e}}\left\{\mathcal{D}(\bar{\mathbf{u}}, f_{\mathcal{D},t}(\,\Phi[\bar{\mathbf{u}}]\oplus\mathbf{e}\,))\right\}, \quad (B.5)$$

and due to $\mathbf{x} = \Phi[\bar{\mathbf{u}}]$, the criterion (B.5) transforms to

$$\min_{\Phi} \mathbf{E}_{\bar{\mathbf{u}},\hat{\mathbf{x}}}\left\{\mathcal{D}(\bar{\mathbf{u}}, f_{\mathcal{D},t}(\hat{\mathbf{x}}))\right\}. \quad (B.6)$$

Now we can evaluate the expectation and we get

$$\min_{\Phi} \sum_{\bar{\mathbf{u}}\in\mathbb{U}}\sum_{\hat{\mathbf{x}}\in\mathbb{X}} \mathcal{D}(\bar{\mathbf{u}}, f_{\mathcal{D},t}(\hat{\mathbf{x}}))\cdot\Pr(\bar{\mathbf{u}},\hat{\mathbf{x}})\,. \quad (B.7)$$

With the abbreviation $\hat{\mathbf{u}}(\hat{\mathbf{x}}) = f_{\mathcal{D},t}(\hat{\mathbf{x}})$, the SNR quality measure according to (10) expands to

$$\begin{aligned}||\bar{\mathbf{u}}-\hat{\mathbf{u}}(\hat{\mathbf{x}})||^2 &= (\bar{\mathbf{u}}-\hat{\mathbf{u}}(\hat{\mathbf{x}}))^{\mathrm{T}}(\bar{\mathbf{u}}-\hat{\mathbf{u}}(\hat{\mathbf{x}}))\\ &= ||\bar{\mathbf{u}}||^2 + ||\hat{\mathbf{u}}(\hat{\mathbf{x}})||^2 - \bar{\mathbf{u}}^{\mathrm{T}}\hat{\mathbf{u}}(\hat{\mathbf{x}}) - \hat{\mathbf{u}}(\hat{\mathbf{x}})^{\mathrm{T}}\bar{\mathbf{u}}\,. \quad (B.8)\end{aligned}$$

By noting that the expectation of the first mixed term (and equivalently of the second) in (B.8) is

$$\begin{aligned}\sum_{\bar{\mathbf{u}}\in\mathbb{U}}\sum_{\hat{\mathbf{x}}\in\mathbb{X}} &\bar{\mathbf{u}}^{\mathrm{T}}\hat{\mathbf{u}}(\hat{\mathbf{x}})\cdot\Pr(\bar{\mathbf{u}},\hat{\mathbf{x}})\\ &= \sum_{\hat{\mathbf{x}}\in\mathbb{X}}\left(\sum_{\bar{\mathbf{u}}\in\mathbb{U}}\bar{\mathbf{u}}^{\mathrm{T}}\cdot\Pr(\bar{\mathbf{u}}|\hat{\mathbf{x}})\right)\hat{\mathbf{u}}(\hat{\mathbf{x}})\cdot\Pr(\hat{\mathbf{x}})\\ &= \sum_{\hat{\mathbf{x}}\in\mathbb{X}}||\hat{\mathbf{u}}(\hat{\mathbf{x}})||^2\cdot\Pr(\hat{\mathbf{x}})\,, \quad (B.9)\end{aligned}$$

equation (12) results.

C. Update Rules for Codeword Swapping

Simplification of (15) yields

$$\mathcal{P}_f = \sum_{\hat{\mathbf{x}} \in \mathbb{X}} \frac{\left\| \sum_{\bar{\mathbf{u}} \in \mathbb{U}} \bar{\mathbf{u}} \cdot \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}) \Pr(\bar{\mathbf{u}}) \right\|^2}{\Pr(\hat{\mathbf{x}})} . \qquad \text{(C.10)}$$

For both, numerator and denominator we will now derive update rules with low computational complexity.

The probability of occurrence of the received bit vectors $\hat{\mathbf{x}}$ in the denominator is is given by the marginal distribution

$$\Pr(\hat{\mathbf{x}}) = \sum_{\bar{\mathbf{u}} \in \mathbb{U}} \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}) \cdot \Pr(\bar{\mathbf{u}}) . \qquad \text{(C.11)}$$

If we interchange the code vectors assigned to $\bar{\mathbf{u}}^{(k)}$ and $\bar{\mathbf{u}}^{(\ell)}$ only the conditional probabilities $\Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(k)})$ and $\Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(\ell)})$ vary, actually their values are swapped:

$$\Pr'(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(k)}) = \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(\ell)}),$$
$$\Pr'(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(\ell)}) = \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(k)}) .$$

Hence, the probability of occurrence can be updated according to

$$\Pr'(\hat{\mathbf{x}}) = \Pr(\hat{\mathbf{x}}) + \left[\Pr(\bar{\mathbf{u}}^{(k)}) - \Pr(\bar{\mathbf{u}}^{(\ell)})\right] \cdot \left[\Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(\ell)}) - \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(k)})\right] . \qquad \text{(C.12)}$$

By introducing the abbreviation $||\mathcal{N}(\hat{\mathbf{x}})||^2$ for the numerator, we get similarly

$$\mathcal{N}'(\hat{\mathbf{x}}_j) = \mathcal{N}(\hat{\mathbf{x}}) + \left[\bar{\mathbf{u}}^{(k)} \Pr(\bar{\mathbf{u}}^{(k)}) - \bar{\mathbf{u}}^{(\ell)} \Pr(\bar{\mathbf{u}}^{(\ell)})\right] \cdot \left[\Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(\ell)}) - \Pr(\hat{\mathbf{x}}|\bar{\mathbf{u}}^{(k)})\right] . \qquad \text{(C.13)}$$

Finally, the combination of (C.12) and (C.13) allows a fast update of the criterion (C.10):

$$\mathcal{P}'_f = \sum_{\hat{\mathbf{x}} \in \mathbb{X}} \frac{||\mathcal{N}'(\hat{\mathbf{x}})||^2}{\Pr'(\hat{\mathbf{x}})} . \qquad \text{(C.14)}$$

D. Recursive Computation of *A Posteriori* Probabilities

For notational simplicity we abbreviate a vector $\hat{\mathbf{y}}(\tau)$ received at time instant τ by $\hat{\mathbf{y}}_\tau$ and a sequence by $\hat{\mathbf{y}}_1^\tau = \hat{\mathbf{y}}(\tau), .., \hat{\mathbf{y}}(1)$. Accordingly a sequence of quantized source symbols will be denoted by $\bar{\mathbf{u}}_1^\tau$.

The *a posteriori* probability $\Pr(\bar{\mathbf{u}}_\tau \,|\, \hat{\mathbf{y}}_1^\tau)$ can be interpreted as the marginal distribution

$$\Pr(\bar{\mathbf{u}}_\tau \,|\, \hat{\mathbf{y}}_1^\tau) = \sum_{\forall \bar{\mathbf{u}}_1^{\tau-1}} \Pr(\bar{\mathbf{u}}_\tau, \bar{\mathbf{u}}_1^{\tau-1} \,|\, \hat{\mathbf{y}}_1^\tau) , \qquad \text{(D.15)}$$

where the sum runs over all possible $M^{\tau-1}$ variations of the sequence $\bar{\mathbf{u}}_1^{\tau-1}$. Repeatedly applying Bayes' theorem on the term under the sum yields

$$\Pr(\bar{\mathbf{u}}_\tau, \bar{\mathbf{u}}_1^{\tau-1} \,|\, \hat{\mathbf{y}}_1^\tau) = \frac{p(\hat{\mathbf{y}}_1^{\tau-1})}{p(\hat{\mathbf{y}}_1^\tau)} \cdot p(\hat{\mathbf{y}}_\tau \,|\, \hat{\mathbf{y}}_1^{\tau-1}, \bar{\mathbf{u}}_\tau, \bar{\mathbf{u}}_1^{\tau-1}) \cdot \Pr(\bar{\mathbf{u}}_\tau \,|\, \bar{\mathbf{u}}_1^{\tau-1}, \hat{\mathbf{y}}_1^{\tau-1}) \cdot \Pr(\bar{\mathbf{u}}_1^{\tau-1} \,|\, \hat{\mathbf{y}}_1^{\tau-1}) . \qquad \text{(D.16)}$$

By exploiting the memorylessness of the transmission channel and the Markov property of the source

$$\Pr(\bar{\mathbf{u}}_\tau \,|\, \bar{\mathbf{u}}_1^{\tau-1}) = \Pr(\bar{\mathbf{u}}_\tau \,|\, \bar{\mathbf{u}}_{\tau-1}) , \qquad \text{(D.17)}$$

we obtain

$$\Pr(\bar{\mathbf{u}}_\tau, \bar{\mathbf{u}}_1^{\tau-1} \,|\, \hat{\mathbf{y}}_1^\tau) = \frac{p(\hat{\mathbf{y}}_1^{\tau-1})}{p(\hat{\mathbf{y}}_1^\tau)} \cdot p(\hat{\mathbf{y}}_\tau \,|\, \bar{\mathbf{u}}_\tau) \cdot \Pr(\bar{\mathbf{u}}_\tau \,|\, \bar{\mathbf{u}}_{\tau-1}) \cdot \Pr(\bar{\mathbf{u}}_1^{\tau-1} \,|\, \hat{\mathbf{y}}_1^{\tau-1}) . \qquad \text{(D.18)}$$

Inserting (D.18) into (D.15) yields equation (17).

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