

# On Reverse Waterfilling in Closed-Loop LPC with Noise Shaping

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## Abstract

A new criterion is derived for the design of the noise weighting filter in linear predictive coding (LPC) that accounts for the issue of noise propagation. It will be shown that the new filter design constitutes an approximation of the reverse waterfilling procedure known from rate-distortion theory and audio transform coding. It improves the overall coding SNR of closed-loop LPC with noise shaping. A practical application within the 3GPP AMR-WB codec is presented. Objective test results indicate notable quality improvements.

## 1 Introduction

For a long time linear predictive coding (LPC) [1] has been (and still is) the method of choice for low-rate speech coding with either scalar (e.g. ADPCM codecs) or vector quantization (CELP codecs [2]). For all approaches, to actually turn the LPC prediction gain (at least partially) into an overall SNR gain, the (filtered) quantization error  $e_f(k)$  must be fed back and subtracted from the (unquantized) prediction residual  $d(k)$ , cf. Figure 1. However, so far, the surprisingly intricate interaction between the quantizer  $\mathcal{Q}$  and the error feedback has not been adequately addressed in codec design.

### 1.1 Noise Propagation in Closed-Loop LPC

In a previous paper [3], we have proposed a new quantization noise production and propagation model for open and closed-loop LPC to generalize the conventional high rate theory of LPC [4] towards lower bit rates. In LPC, quantization noise is effectively processed by the cascade of an error weighting filter  $1 - F(z)$  and autoregressive synthesis with  $H_A^{-1}(z) = (1 - A(z))^{-1}$ . In [3], we used a *noise propagation network* to model the effect of this processing on the quantization noise. In this model, the signal  $x(k)$  is generated by an autoregressive process (all-pole filter  $H_0(e^{j\Omega})$ ) driven by a spectrally white excitation signal  $d_0(k)$ . The quantization noise  $\Delta(k) = \tilde{d}(k) - d'(k)$  is assumed to be generated by a *power-controlled additive noise source*:

$$E\{\Delta^2(k)\} = \frac{E\{d'^2(k)\}}{\text{SNR}_0} \Rightarrow G_{d',\Delta} \doteq \frac{E\{\Delta^2(k)\}}{E\{d'^2(k)\}} = \text{SNR}_0^{-1}, \quad (1)$$

motivated by the fact that practical quantizers for LPC are operated with a nearly constant quantization SNR<sub>0</sub> [5]. Based on this model, the overall coding SNR of LPC was derived:

$$\begin{aligned} \text{SNR}_{\text{LPC}} &= E\{x^2(k)\} / E\{(\tilde{x}(k) - x(k))^2\} \\ &= \frac{G_{d_0,x}}{G_{\Delta,\tilde{x}-x} \cdot G_{d_0,d}} \left(1 - \frac{G_{\Delta,e_f}}{\text{SNR}_0}\right) \cdot \text{SNR}_0 \end{aligned} \quad (2)$$

comprising the following set of filter gains:

$$G_{\Delta,\tilde{x}-x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{1 - F(e^{j\Omega})}{H_A(e^{j\Omega})} \right|^2 d\Omega \quad (3)$$

$$G_{\Delta,e_f} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |F(e^{j\Omega})|^2 d\Omega \quad (4)$$

$$G_{d_0,d} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{H_A(e^{j\Omega})}{H_0(e^{j\Omega})} \right|^2 d\Omega \quad (5)$$

$$G_{d_0,x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{1}{H_0(e^{j\Omega})} \right|^2 d\Omega \quad (6)$$

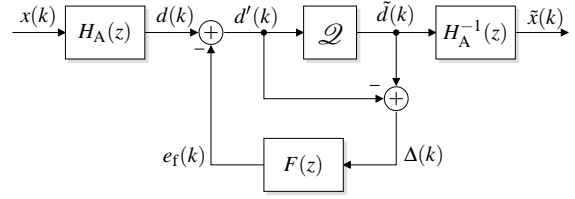


Figure 1: Closed-Loop LPC with noise feedback.

with the “noise gain”  $G_{\Delta,\tilde{x}-x}$  and the “feedback gain”  $G_{\Delta,e_f}$ .

Compared to the well-known high rate approximation of (2), i.e.  $\text{SNR}_{\text{LPC,hr}} = G_{d_0,x} \cdot \text{SNR}_0$  for  $F(z) = A(z)$ , it can be observed that  $\text{SNR}_{\text{LPC}}$  is significantly smaller at low bit rates which is consistent with measurement results. As another important finding, the noise feedback in closed-loop LPC can lead to encoder instabilities and overall performance losses. The formal condition for stable encoder operation is given as

$$G_{\Delta,e_f} \stackrel{!}{<} \text{SNR}_0. \quad (7)$$

### 1.2 Paper Overview

In this paper, we propose a new design for the noise feedback filter  $F(z)$  based on a revised LPC optimization criterion. This design constitutes a time-domain approximation of the *reverse waterfilling* principle known from rate-distortion theory and audio transform coding. In accord with rate-distortion theory, the new design leads to a higher coding SNR than the conventional error weighting filter  $F_{\text{conv}}(z) = A(z/\gamma)$  [6]. We will then briefly discuss the applicability of the new findings to CELP speech codecs. An example implementation of the proposed noise feedback filter within the AMR-WB encoder [7, 8] is presented and objective evaluation results are given. In the final discussion, we will address in how far several techniques encountered in modern speech codecs (although they have initially been introduced for other reasons) already help to mitigate the observed adverse effects to a certain extent.

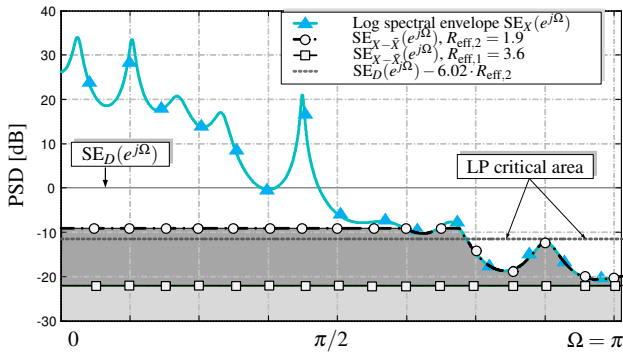
## 2 New LPC Optimization Criterion

An important observation is that  $d(k)$  and  $d'(k)$  in Figure 1 can significantly differ. The quantizer input is given as

$$d'(k) = x(k) - \underbrace{\sum_{i=1}^{N_{\text{LPC}}} a_i \cdot x(k-i)}_{d(k)} - \sum_{i=1}^{N_{\text{F}}} b_i \cdot \Delta(k-i), \quad (8)$$

whereby, in the new filter design, we consider separate coefficient sets for the LP analysis filter  $H_A(z) = 1 - \sum_{i=1}^{N_{\text{LPC}}} a_i \cdot z^{-i}$  and for the error weighting filter  $F(z) = \sum_{i=1}^{N_{\text{F}}} b_i \cdot z^{-i}$ . Assuming a constant SNR<sub>0</sub>, the noise power in the decoded output is always proportional to the power of  $d'(k)$ , i.e.

$$\begin{aligned} E\{(\tilde{x}(k) - x(k))^2\} &= G_{\Delta,\tilde{x}-x} \cdot E\{\Delta^2(k)\} \\ &\stackrel{(1)}{=} \frac{G_{\Delta,\tilde{x}-x}}{\text{SNR}_0} \cdot E\{d'^2(k)\}. \end{aligned} \quad (9)$$



**Figure 2:** Reverse waterfilling for  $R_{\text{eff},1} = 3.6$  bits/sample and  $R_{\text{eff},2} = 1.9$  bits/sample. The spectral envelopes  $\text{SE}_{(\cdot)}(e^{j\Omega})$  are based on an exemplary set of AR coefficients.

Taking this fact into account, a new optimization criterion for the optimal LP coefficients in closed-loop quantization arises:

$$E\{d^2(k)\} \rightarrow \min \quad \wedge \quad G_{\Delta, \tilde{x}-x} \rightarrow \min \quad (10)$$

which, obviously, significantly differs from the conventional criterion  $E\{d^2(k)\} \rightarrow \min$ . Unfortunately, as  $H_A^{-1}(z)$  represents an IIR filter,  $G_{\Delta, \tilde{x}-x}$  as defined in (3) is difficult to minimize in the time-domain. Hence, instead of a computationally complex frequency domain design, we propose the following two-step procedure which leads to a simple and efficient solution that is still consistent with the optimum reverse waterfilling procedure known from rate-distortion theory.

### Step I: Modified LP Analysis

In the first step we couple  $A(z)$  and  $F(z)$  as closely as possible, a restriction which will be relaxed later, i.e. in the second step. Hence, we let  $a_i = b_i$  in this step and determine the coefficients  $b_i$  of the filter  $F(z)$  by minimizing  $E\{d^2(k)\}$ . We therefore have  $F(z) = 1 - H_A(z)$  and it is important to note that, at this point, also the second constraint in (10) is perfectly fulfilled ( $G_{\Delta, \tilde{x}-x} = 1$ ). Assuming that  $x(k)$  and  $\Delta(k)$  are independent and that  $\Delta(k)$  is uncorrelated, the optimum coefficients  $b_i$  are implicitly given by

$$\begin{pmatrix} \varphi_{xx}(1) \\ \vdots \\ \varphi_{xx}(N_F) \end{pmatrix} = (\Phi_x + \Phi_\Delta) \cdot \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N_F} \end{pmatrix} \quad (11)$$

with

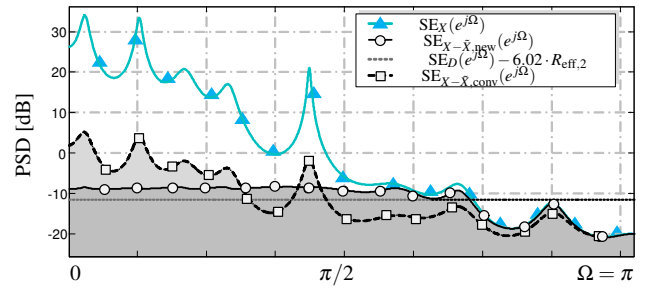
$$\Phi_x = \begin{pmatrix} \varphi_{xx}(0) & \cdots & \varphi_{xx}(1 - N_F) \\ \vdots & \ddots & \vdots \\ \varphi_{xx}(N_F - 1) & \cdots & \varphi_{xx}(0) \end{pmatrix} \quad (12)$$

and

$$\Phi_\Delta = \varphi_{\Delta\Delta}(0) \cdot \mathbf{I} \quad (13)$$

where  $\varphi_{xx}(i)$  and  $\varphi_{\Delta\Delta}(i)$  denote the autocorrelation functions of the respective signals and  $\mathbf{I}$  is the  $N_F \times N_F$  unity matrix. The matrix  $\Phi_x$  also appears in the conventional optimization approach, while the second matrix  $\Phi_\Delta$  accounts for the feedback of the quantization error via the weighting filter  $F(z)$ . According to (1),  $\varphi_{\Delta\Delta}(0) = E\{\Delta^2(k)\}$  depends on the performance of the quantizer ( $\text{SNR}_0$ ) and on the power of the signal  $d'(k)$  which again depends on the coefficients  $b_i$ . Therefore, (11) is not a linear equation system which inhibits a straight forward solution.

A reasonable approximation for  $\varphi_{\Delta\Delta}(0)$  is to set it to a constant value  $\varphi_{\Delta\Delta}(0) = C_\Delta \cdot \varphi_{xx}(0)$ . This result is well-known as *white-noise-correction* which, however, was originally introduced to avoid ill-conditioned autocorrelation matrices in LP



**Figure 3:** Reverse waterfilling in LPC with noise feedback based on the noise shaping filter  $F_{\text{new}}(z)$  (circle markers) and conventional noise weighting with  $F_{\text{conv}}(z) = A(z/\gamma)$  (square markers). In this example,  $F_{\text{new}}(z)$  improves the overall SNR by  $\approx 6$  dB.

analysis, e.g. [9]. The constant  $C_\Delta$  is a parameter which should be adapted to the bit rate. In practice, it can be determined with offline simulations. The resulting error weighting filter is denoted as  $F_{\text{new}}(z)$ . Note that  $F_{\text{new}}(z)$  is not required to be of order  $N_F = N_{\text{LPC}}$ .

### Step II: Minimization of $d(k)$

According to [3],  $E\{d^2(k)\}$  can be separated as follows:

$$E\{d^2(k)\} = \underbrace{\left(1 - \frac{G_{\Delta, e_f}}{\text{SNR}_0}\right)^{-1}}_{f(b_i)} \cdot \underbrace{E\{d^2(k)\}}_{g(a_i)}. \quad (14)$$

Therefore, with fixed (optimum) coefficients  $b_i$  we can, in a second step, minimize  $E\{d^2(k)\}$  by minimizing  $E\{d^2(k)\}$ . The obtained coefficients  $a_i$  correspond to the conventional LP analysis result, thus approximating  $H_0(z)$  by  $H_A(z)$ .

In practice, the described two-step-procedure can be easily realized, e.g. by computing the two coefficient sets with two separate runs of the Levinson-Durbin algorithm.

## 3 Relation to Reverse Waterfilling

For a better comprehension of the impact of the proposed noise weighting filter, a frequency domain analysis for an exemplary set of AR coefficients is conducted, see Figures 2 and 3. Therefore, the log spectral envelope of the AR synthesis filter  $H_0^{-1}(z)$  is considered:

$$\text{SE}_X(e^{j\Omega}) = 10 \cdot \log |H_0^{-1}(z = e^{j\Omega})|^2. \quad (15)$$

The example exhibits a low pass characteristic which is typical for audio signals. The AR filter also has zero-mean property, i.e.  $\int_{-\pi}^{\pi} \ln(|H_s(e^{j\Omega})|) / (2\pi) d\Omega = 0$  which entails two consequences:  $\text{SE}_X(e^{j\Omega})$  is located around the 0-dB-line and a decorrelation of the time-domain signal, e.g. by means of LP analysis filtering, produces the log spectral envelope of the LP residual, i.e.  $\text{SE}_D(e^{j\Omega}) \equiv 0$  dB in Figure 2.

### 3.1 Theory of Reverse Waterfilling

To recall the reverse waterfilling principle, Figure 2 shows a high and a low effective bit rate example whereby the quantizer performance is assumed to follow the 6-dB-per-bit rule.

Reverse waterfilling prescribes that the quantization noise level cannot exceed the signal level at any frequency, e.g. [10]. Therefore, at the high bit rate, the quantization noise remains white (square markers).

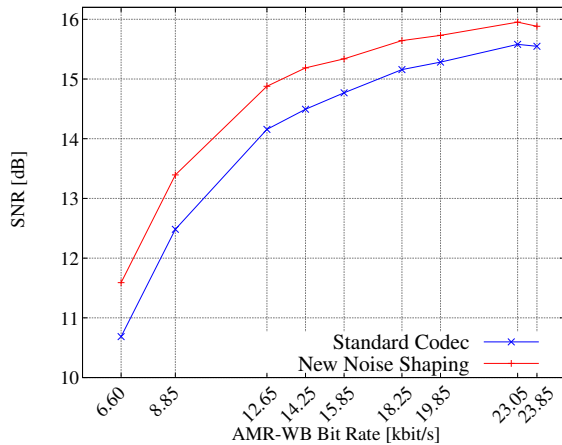


Figure 4: SNR-evaluation of the modified AMR-WB codec

For the low bit rate of  $R_{\text{eff},2} = 1.9$  bits/sample, the noise envelope deviates from the spectrally white distribution (dotted line): It is reduced in the areas denoted as the *LP critical areas* to follow the spectral envelope  $\text{SE}_X(e^{j\Omega})$ , at other frequencies, its level is slightly raised. The resulting envelope is shown with circle markers.

### 3.2 Application in Closed-Loop Quantization

In LPC with noise feedback (see Figure 1), the spectral envelope of the processed quantization noise in the decoder output is influenced by the noise weighting filter  $F(z)$  according to  $\tilde{\Delta}(z) = (1 - F(z))/(1 - A(z)) \cdot \Delta(z)$  as shown in Figure 3 for different configurations of  $F(z)$ .

For the high bit rate example from the previous section, the configuration  $F(z) = A(z)$  is well possible because feedback problems do not occur (i.e. (7) is fulfilled). The spectral envelope of the quantization noise is constant for all  $\Omega$  and located at the same position as in Figure 2. Note that this case is not shown in Figure 3.

For the more interesting low bit rate example, setting  $A(z) = F(z)$  would lead to coding artifacts since the stability constraint is not fulfilled anymore. Instead,  $F_{\text{new}}(z) \neq A(z)$  is computed according to the new optimization criterion from Section 2. The log spectral envelope of the processed quantization noise in the decoder output is shown in Figure 3 with circle markers. Obviously, the theoretical reverse waterfilling curve from the low bit rate example in Figure 2 is well approximated. It is not identical since the frequency response is approximated by a time domain filter in this case. The reduction of the magnitude response in the “LP critical area” from the white noise reference (dotted line) is due to the difference between  $F_{\text{new}}(z)$  and  $A(z)$ . Moreover, due to the zero-mean property of  $(1 - F(z))/(1 - A(z))$ , a slightly increased noise level (again compared to the white noise reference) occurs at the other (lower) frequencies.

For reference, also the noise envelope that results from the use of the conventional error weighting filter  $F_{\text{conv}}(z) = A(z/\gamma)$  is shown in Figure 3 (square markers). In order to have a comparable impact as with  $F_{\text{new}}(z)$ ,  $\gamma$  was chosen such that the spectral envelope of the processed quantization noise is below the spectral envelope of the input signal at all frequencies and in particular in the *LP critical areas* from Figure 2 ( $\gamma = 0.55$ ).

As a conclusion of this diagram, the new approach approximates the spectral envelope which is optimal according to rate-distortion theory much better than the conventional weighting filter. The quantization noise level is significantly lower in low frequency areas, and, for the given example, the overall SNR is improved by approximately 6 dB.

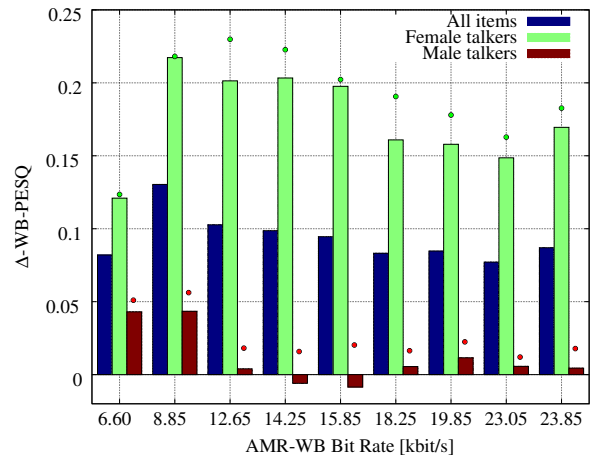


Figure 5: WB-PESQ-evaluation of the mod. AMR-WB codec

## 4 Application to CELP & AMR-WB

So far, the quantizer has been modeled by a *scalar* additive noise source. Yet, the results can to a large extent be generalized to the case of LPC with gain-shape vector quantization (VQ) (i.e. CELP [2]) because also here, due to the gain-shape decomposition, a constant  $\text{SNR}_0$  ensues, see [11]. CELP coding with a scalar quantizer is in fact equivalent to closed-loop LPC if the CELP error weighting filter is configured as  $W(z) = (1 - A(z))/(1 - F(z))$ , cf. [5]. Also, it is clear that for the same effective bit rate per sample, VQ has a higher performance than scalar quantization.

### 4.1 Implicit Error Weighting Effect

Investigations were carried out with a basic CELP encoder (using a fixed codebook only) for  $W(z) = 1$  (and hence  $F(z) = A(z)$  in the scalar model). Measurements show that the CELP encoder in fact behaves as predicted by the new model: For lower bit rates, the quantization error signal is fed back over the error weighting filter and the (equivalent) signal  $d'(k)$  deviates from  $d(k)$ . Therefore, the overall system can become unstable. However, CELP seems to be less sensitive to feedback problems than scalar LPC.

The reason for this well-tempered behavior is that, in CELP coding, the joint optimization of sequential samples (codevector search) implies a certain interdependence of the quantized samples which can not be achieved by scalar quantization. This interdependence, effectively, acts as an *implicit error weighting* which must be considered in addition to the *explicit weighting* by the noise shaping filter. The implicit error weighting filter actually helps to better match the optimal reverse waterfilling and reduces the variance of the signal  $e_f(k)$  and hence also the feedback gain  $G_{\Delta, e_f}$ . This effect can be observed in the CELP decoder output: The quantization noise in the decoder output, usually expected to be spectrally white ( $\text{SE}_{X-\tilde{X}}(e^{j\Omega}) = \text{const}$ ) for  $W(z) = 1$ , in fact exhibits a spectral shape that is very similar to the reverse waterfilling curve. This behavior can be explained with the hypothesized implicit error weighting filter. Its impact is, however, restricted by the vector dimension and the, in practice, non-exhaustive codebook search. It is therefore not a good substitute for the near-optimal weighting filter of Section 2, in particular for small vector dimensions.

### 4.2 Implementation in the AMR-WB Codec

To assess the practical value of the proposed noise weighting filter  $F_{\text{new}}(z)$ , we have replaced the standard weighting filter  $F_{\text{conv}}(z) = A(z/\gamma)$  in the 3GPP AMR-WB encoder [7, 8] with our new proposal, whereby the additionally standardized tilt compensation  $1/(1 - \beta z^{-1})$  was left in place. A second run of the

Levinson-Durbin algorithm (based on the modified autocorrelation coefficients according to (11)) delivers the coefficients of  $F_{\text{new}}(z)$  for the fourth AMR-WB subframe, whereby the standardized interpolation procedure is used to obtain the coefficients for the first three subframes. These modifications only affect the encoder. The standard AMR-WB decoder can still be used.

The modified codec was tested with the 96 American language utterances of the NTT corpus [12] whereby the white noise correction factor  $C_{\Delta}$  has been optimized individually for each codec mode. The SNR was measured for the core CELP codec of AMR-WB running at 12.8 kHz sampling rate. The results are depicted in Figure 4 and contrasted with the standard codec. For example, the modified AMR-WB encoder at 12.65 kbit/s now achieves an SNR performance that is better than the SNR of the 15.85 kbit/s mode of the standard codec. Furthermore, the decreasing SNR gain with higher bit rates is consistent with theory.

Figure 5 shows the obtained improvements of the modified codec over the standard in terms of a “ $\Delta$ -WB-PESQ score”, i.e. the difference between the respective wideband PESQ scores [13]. Clear improvements can actually be observed in particular for female talkers over the entire range of AMR-WB bit rates. In contrast, the quality of male talkers can only benefit for the very low bit rate modes. Interestingly, for WB-PESQ evaluations of the standard codec, the quality for female talkers is always rated significantly lower than for male talkers. The new weighting filter  $F_{\text{new}}(z)$  helps to close this gap.

## 5 Discussion & Conclusions

Retrospectively, a lot of common techniques are employed in state-of-the-art speech codecs that actually help to prevent the evolution of artifacts caused by unstable feedback loops. These techniques, however, were originally introduced for other reasons:

- *White-noise-correction* has been introduced to avoid ill-conditioned matrices [9]. It is now clear that this technique is a very important approach to combat feedback problems and to maximize the SNR.
- *Spectral bandwidth widening* in the LP spectrum has been introduced to better match formant frequencies [9]. This technique also reduces the feedback gain in LPC and hence feedback problems.
- The conventional weighting filter  $F(z) = A(z/\gamma)$  was introduced to achieve a better perceptual speech quality [14]. However, we can now conclude that values of  $\gamma < 1.0$  also limit the feedback gain and therefore lead to performance benefits with respect to the overall coding SNR. The approach is, however, a suboptimal alternative to the proposed noise weighting filter.
- Typical CELP codecs use comparatively large vector dimensions for the quantization of the LP residual (e.g. 40 in the AMR speech codec [15, 16]). In fact, VQ with large vector dimensions reduces the feedback gain due to the implicit error weighting filter.

Now, as a new alternative, the proposed optimization criterion for closed-loop LPC with noise feedback leads to a different design for the noise feedback filter  $F_{\text{new}}(z)$ .

It was shown that this filter design facilitates the approximation of the reverse waterfilling procedure known from rate-distortion theory in the time-domain. It therefore leads to a higher coding SNR than the conventional weighting with  $F_{\text{conv}}(z) = A(z/\gamma)$ , cf. [6]. A practically relevant quality impact of the filter  $F_{\text{new}}(z)$  could be observed within a modified version of the

3GPP AMR-WB codec. It is finally worth to note that audio (or music) signals often exhibit more extreme characteristics than speech signals (e.g. segments with very high prediction and feedback gains). The new noise shaping filter is expected to be even more beneficial in such cases.

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