Deconvolution-based Physiological Signal Simplification for Periodical Parameter Estimation

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Abstract. The estimation of physiological parameters from raw signal recordings is absolutely crucial in modern clinical applications. A wide variety of these parameter incorporate a periodic nature, such as the heart or the respiration rate. Especially unobstructive, novel measurement techniques are characterized by complex waveforms, which are likely to change during the measurement. Simple peak detection algorithms are often not suited for these applications. One way to tackle these challenges is a preprocessing step for the simplification of the physiological signals. A novel deconvolution based approach for this preprocessing is introduced and evaluated in this paper. Two deconvolution methods are regarded, the Minimum Entropy Deconvolution (MED) and the Maximum Correlated Kurtosis Deconvolution (MCKD). Important parameters are outlined and examined. Finally the methods are validated using artificial as well as real clinical signals to demonstrate their potential.

Keywords

Deconvolution, preprocessing, pitch estimation, periodical parameter estimation, interbeat interval estimation, ballistocardiogram

1. Introduction

Nowadays, a wide variety of vital signs are measured in the clinical environment. Various of these are based on periodic physiological processes, such as the heart rate and the respiration rate. A reliable measurement of these periodical parameters is crucial for clinical decisions.

On the other hand, there is a trend towards monitoring of vital signs without supervision of clinical staff in the home environment of patients. Novel, easy to apply measurement systems are required for these conditions. One promissing technique is ballistocardiography, which measures the mechanical activity of the heart on the body surface, for example by pressure sensors. It has the capability to unobstructively measure the heart activity. This comes at the cost of a more complex waveform which impedes simple peak detection algorithms for the measurement of the heart rate.

Therefore, more sophisticated techniques are required. Various different approaches have been published in the previous years. Some operate in the frequency domain, e.q. [4], and some are correlation-based methods [1]. These methods also differ whether they solely measure an averaged interbeat interval or determined the individual heart beat locations.

In this paper, a novel approach for periodic parameter estimation using deconvolution techniques is proposed and investigated. This basic idea of their application is illustrated in Figure 1. The fundamentual assumption is that each heart



Fig. 1. Basic idea of the deconvolution-based methods for the example of the heart rate.

beat is represented by a dirac impulse. An FIR filter models the specific measurement setup and the physiological influences. It varies between different techniques (such as ECG, PPG or BCG), between different patients and also between different points in time. Measured is a convolution of the impulse sequence with this FIR filter, depicted at the last block to the right. The deconvolution technique tries to estimate the inverse of the FIR filter to compensate its influence and regain the impulse sequence. This impulse sequence is then further processed.

Two distinct deconvolution techniques are investigated, the Minimum Entropy Deconvolution (MED) and the Maximum Correlated Kurtosis Deconvolution (MCKD). The MED was introduced by Ralph A. Wiggins in the year 1978 for the analysis of seismic reflection recordings [8]. It is mainly used in the field of geophysics and seismic signal analysis. In 2012, Geoff McDonald published a paper on a novel variant of minimum entropy deconvolution for periodic signals, the Maximum Correlated Kurtosis Deconvolution [5]. It exploits the periodic nature of the input signals and was originally designed for gear tooth chip fault detection.

This paper is structured into a methods section, where the deconvolution techniques are presented, a results section with different experiments on the potential and limitations of the methods, their interpretation in the discussion section, and an overall conclusion in the last section.

2. Methods

The application of deconvolution techniques for interbeat interval estimation is based on the assumption that the heart beats are represented by an impulse sequence q, which is altered by an FIR filter w. The measurement signal x is the convolution of the two signals.

$$x = w * q \tag{1}$$

The deconvolution attempts to estimate an inverse filter f to reverse this process.

$$y = f * x = f * w * q \tag{2}$$

Therefore, the final goal is to regain the assumed impulse sequence q, where each impulse corresponds to one heart beat. In the ideal case, f is exactly the inverse of w. The challenge is to estimate an inverse filter f, which creates a good impulse sequence estimate y only based on the knowledge of x without any further information about q and w.

In the following, two methods are presented, which are based on the assumption of a simple, impulse-like target signal *y*.

2.1. Minimum Entropy Deconvolution

Suppose the inputs are labeled by indizes $i = 1 \dots N_c$ with N_c the number of channels and $j = 1 \dots N_s$ with N_s the number of samples per channel. N_f corresponds to the number of filter samples. The deconvolution in Equation 2 may then be expressed in more detail as follows.

$$y_{ij} = \sum_{k=1}^{N_f} f_k \cdot x_{i,j-k} \tag{3}$$

The impulse-like structure and thereby the simplicity of the input can be measured by different norms. Wiggins used the varimax norm given in Equation 4, which is based on the varimax rotation from the field of factor analysis (see [7] for further information). It is the sum of normalized squares of the variances of the samples (see [8]).

$$V = \sum_{i} V_i = \sum_{i} \frac{\sum_{j} y_{ij}^4}{\left(\sum_{j} y_{ij}^2\right)^2} \tag{4}$$

In order to find the filter which maximizes this simplicity norm V, it is differentiated with respect to the filter coefficients f_k and set to zero.

$$\frac{\partial V}{\partial f_k} = 0 = \sum_i \frac{\partial V_i}{\partial f_k} \tag{5}$$

This results in the following Equation 6. For the detailed derivation the reader is referred to [8].

$$\sum_{l} f_{l} \sum_{i} V_{i} u_{i}^{-1} \sum_{j} x_{i,j-l} x_{i,j-k} = \sum_{i} u_{i}^{-2} \sum_{j} y_{ij}^{3} x_{i,j-k},$$
(6)

with $u_i = \sum_j y_{ij}^2$ or in a compact matrix form as

$$k_1 \boldsymbol{\phi}_{\boldsymbol{x}\boldsymbol{x}} \boldsymbol{f} = k_2 \boldsymbol{\phi}_{\boldsymbol{y}^3 \boldsymbol{x}}.$$
 (7)

 ϕ_{xx} is an autocorrelation matrix in Toeplitz form of the input signal x. ϕ_{y^3x} is a column vector of cross correlations of the input x and the cubed filter outputs y^3 . f is a column vector of the filter coefficients. The weighting factors k_1 and k_2 are scalar values depending on the channel i. They are defined as $k_1 = \frac{V(y_i)}{\sum_j y_{ij}^2}$ and $k_2 = (\sum_j y_{ij}^2)^{-2}$.

A closed form solution for f is not possible. However, equation 7 allows an iterative procedure based on the well known Levinson-Durbin-Algorithm to calculate the filter f. A convergence towards the optimal impulse sequence is not guaranteed and depends on different parameters, which are addressed later in this paper.

One of the major drawbacks of the MED for the application on periodical signals is the fact that the maximum of the norm is given for one single peak as illustrated in Figure 2. It may therefore happen that the techniques returns a filter which creates an output signal with one large peak rather than the desired periodic impulses. Therefore, we are also regarding the following method of MCKD.

2.2. Maximum Correlated Kurtosis Deconvolution

The MCKD method is based on the correlated kurtosis (CK) norm shown in Equation 8.

$$CK_{M}(T) = \frac{\sum_{j=1}^{N_{t}} \left(\prod_{m=0}^{M} y_{j-mT}\right)^{2}}{\left(\sum_{j=1}^{N_{s}} y_{j}^{2}\right)^{M+1}}$$
(8)

The norm consists of a multiplication of the signal with a shifted version of itself for M = 1. For higher orders, the signal is multiplied with M shifted versions. It combines the kurtosis calculation with the idea of correlation and thereby utilizes the periodic character of the input signal. The norm



Fig. 2. Example signals for demonstration of their effect on the different norms, which are given in Table 1. The image is inspired by [5].

Signal	V-Norm	$CK_1(T)$ -Norm
1	0.0028	0.0028
2	0.0015	0.0012
3	0.0040	0.0032
4	1.0000	0.0000
5	0.2500	0.1875
6	0.1667	0.0278

Tab. 1. Corresponding norm values for Figure 2

creates higher values in the presence of periodic impulses with a preselected period of interest T. For higher shift orders M, the sensitivity for the selected period of interest T is increased.

To get an impression of the norm values for different cases, Figure 2 is presented. It shows different input signals, which are examined. The corresponding Varimax (V) and Correlated Kurtosis (CK) values for the example signals are given in Table 1. The CK was calculated with a period of interest of T = 200 and a shift of M = 1. One can observe that the V-norm is maximal in case four, when only one large peak is present and the CK-norm reaches its maximum for case five, when the selected period fits the impulse sequences.

To derive a solution for f, the norm given in Equation 8 is again differentiated with respect to the filter coefficients f_k and set to zero. This leads to the following matrix Equation 9. For the whole derivation the reader is referred to [5].

$$X_0 X_0^T f = \frac{\|y\|^2}{2\|\beta\|^2} \sum_{m=0}^M X_{mT} \alpha_m$$
(9)

with

$$\mathbf{X}_{r} = \begin{pmatrix} x_{1-r} & x_{2-r} & x_{3-r} & \dots & x_{N-r} \\ 0 & x_{1-r} & x_{2-r} & \dots & x_{N-1-r} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & x_{N-L-r+1} \end{pmatrix}, \quad (10)$$



Fig. 3. Various steps of the deconvolution process from the input signal, an intermediate signal to the final output. The last row contains the ideal impulse sequence as a reference. The corresponding norm values are given in Table 2.

Signal	V-Norm	$CK_1(T)$ -Norm
Х	0.0040	0.0032
y_2	0.0198	0.0155
y _{final}	0.0791	0.0547
ideal	0.1000	0.0700

Tab. 2. Corresponding norm values for Figure 3

$$\boldsymbol{\alpha}_{\boldsymbol{M}} = \begin{pmatrix} y_{1-MT}^{-1} (y_1^2 y_{1-T}^2 \dots y_{1-MT}^2) \\ \vdots \\ y_{N_t-MT}^{-1} (y_{N_t}^2 y_{N_t-T}^2 \dots y_{N_t-MT}^2) \end{pmatrix}$$
(11)

and

$$\boldsymbol{\beta}_{\boldsymbol{M}} = \begin{pmatrix} y_1 y_{1-T} \cdots y_{1-MT} \\ \vdots \\ y_{N_t} y_{N_t-T} \cdots y_{N_t-MT} \end{pmatrix}.$$
 (12)

The result shown in Equation 9 again requires an iterative calculation of the filter f.

2.3. Iterative Procedure

The convergence of the iterative algorithms is indicated by the Figure 3. After the first processing step shown by the signal y_2 the neighboring peaks have already been attenuated compared to the input signal x. The main peaks, on the other hand, were emphasized, which is also reflected by the increasing norm values in Table 2.

The final result of the MED method, given by y_{final} , is quite similar the ideal impulse sequence. Looking at the norm values the reader can observe that the method tends to maximize these in order to create impulse sequences. This example illustrates the convergence of the algorithm towards a good estimate of the impulse sequence.

3. Results

The following section addresses the parameter influences on the deconvolution result. Furthermore, two short evaluations are carried out with artificial and real clinical signals to investigate the peak detection and interbeat interval estimation capabilities.

3.1. Influences on the Deconvolution

There are some parameters that need to be chosen and several influences to be kept in mind, when using the MED and the MCKD. These are, namely, the local validity of the filter estimate, the choice of the initial filter f, as well as the filter length. Additionally, the MCKD is influenced by a preassigned period and M-shift parameter. In this paper, only the most significant influences should be outlined.

One important parameter both methods have in common is the filter length. A shorter filter length limits the capabibility of precise modeling of longer wavelets, but decreases the computational effort. With a longer filter length, the procedure tends to misestimate the filter and only reaches relatively poor local maxima. Additionally, it increases the computational effort. The influence is illustrated in Figure 4 for three different filter lengths. The input signal was a clean sequence with varying pitch period and absence of any noise. The left column contains the inverse filter estimates f^{-1} and



Fig. 4. Influence of the filter length on the estimation of the inverse filter estimate f^{-1} and the MED output.

the right the corresponding output of the MED method. In the first row, the ideal filter estimate of length 100 samples and the ideal impulse sequence are given as a reference. It is easy to observe that an underestimation of the filter length, as shown in the second row, leads to an unsatisfying filter and impulse train estimate. The filter f is too short to adequately adapt to the assumed FIR filter w. An overestimation, as shown in the bottom row, also leads to insufficient estimates. The algorithm overfits the filter to the given signal and is no longer able to consider individual intervals. Solely an adequately chosen filter length leads to satisfying results as shown in the third row from the top.

Another aspect for the estimation of a longer impulse signal is the local validity of the filter. The waveform is likely to vary during a measurement. Reasons for this are changes is the measurement modalities as well as background noise influences. Therefore it is necessary to perform local blockwise processing. In order to avoid sharp edges at the block borders the use of a window function, which attenuates the borders of each segment, is crucial. The employed window function is illustrated in Figure 5 with the attenuating area at the borders and the passband in the middle part (according to [6] p.55). The attenuated parts are



Fig. 5. Window function with 80% plateau of unchanged signal. The grey colored edges contain the attenuated signal parts.

cut off after the processing and need to be considered within an overlapping of successive blocks. Furthermore additional overlapping can lead to better estimates and to an improved robustness.

The MCKD contains additional parameters, of which solely the influence of the period estimate is regarded in this paper. The impact of different period parameters is depicted in Figure 6. In order to adequately demonstrate the influence an input signal with a fixed period is used and additive white Gaussian noise (AWGN) with an SNR of 10dB was added. The Correlated Kurtosis depending on the period T and the MCKD output are shown. The selected period T is marked in the CK plot by a red cross. The cases of T = 0 and T = 50 do not give any usable results. This is equal to the MED. If the period is adequately chosen near the real interval of 100 samples, the output contains emphasized peaks as expected. This small experiment shows that an suitable selection of T is necessary to gain good results.

In order to adapt to T this additional parameter, for each frame a separate period estimation is performed with the Correlated Kurtosis norm. Through this, the period T is automatically adapted.



Fig. 6. Influence of the period parameter estimation on the Correlated Kurtosis (CK) and the MCKD output. Marked in red is the current period position in the CK. The Shift parameter M is set to 1.

3.2. Quantitative Analysis with Artificial Signals

After introducing the different influences on the deconvolution, a quantitative analysis is done on how good the methods reconstruct an impulse-like signal and how well the period can be estimated.

To create a controlled environment for examinations, an artificial BCG signal with 10000 samples and 112 simulated heart beats is generated. It is composed by a combination of sine waves and a sawtooth signal based on [3]. Thereafter, different levels of white Gaussian noise are added to the signal to evaluate the robustness of the methods.

In order to determine the capability to detect the peaks in the processed signal, a new metric is introduced. It takes into account the height of the peaks compared to the level of the background noise. This is done by dividing the mean peak height by the standard deviation of the remaining background values. The peak-to-background-ratio (PBR) is therefore defined as

$$PBR = \frac{\frac{1}{K} \sum_{i=1}^{K} p_i}{\sqrt{\frac{1}{L-1} \sum_{j=1}^{L} (b_j - \bar{b})^2}}$$
(13)

with p_i being the K detected peaks, b_j the L remaining samples and \overline{b} the mean of the remaining background samples.

This metric theoretically approaches the value one for peaks that are the same height as the surrounding background samples and infinity if solely clean peaks are present and all background samples are zero. The following Figure 7 depicts the PBR values of the two processed signals for different signal-to-noise ratios (SNR). The curves indicate that the peak dominance is in-



Fig. 7. Peak to Background Ratio (PBR) for the output of the two methods MED and MCKD with artificial input signal.

creasing towards good SNRs. Interestingly, the MED seems to produce clearer peaks than the MCKD method for moderate to high SNRs. Due to missing information on which PBR level is necessary for a reliable estimation, other measures have to be considered. However, the PBR gives an indication on how distinctive the resulting peaks are.

In order to determine the necessary SNR level for a good estimation, the peak detection rate (PDR) and the peak misdetection rate (PMR) are valuable measures. The PDR indicates how many of the reference peaks have been found by the algorithm and the PMR represents how many of the detected peaks do not correspond to a reference peak and are therefore falsely detected.

Figure 8 shows how the methods perform regarding these metrics. From an SNR of 4 dB on both methods are able to create an output signal, where 100 % of the reference peaks are found. However, both incorporate a slight fluctuation concerning the PMR. This means that there are some falsely detected peaks. One must keep in mind that the PDR and the PMR strongly depend on the peak detection algorithm and therefore contain an additional influence. Still, it belongs to the process chain and therefore should also be regarded. Further tuning of the parameters of the peak detection algorithms will probably result in more reliable results. For this investigation the *findpeaks* algorithm of MATLAB 2012a was chosen.

Eventually, the interbeat interval is the parameter of interest. In order to judge when an estimation is satisfying, the mean interbeat interval error (\overline{E}) is regarded. It incorporates all reasonable interbeat intervals and measures the error with respect to the reference. In this case the reference is given by the perfect knowledge of the artificial signal.



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Fig. 8. Peak Detection Rate (PDR) and Peak Misdetection Rate (PDR) depending on the SNR.

Figure 9 supports the conclusions already drawn from Figure 8. The minimum SNR to get appropriate estimations is 4 dB. From this value on \overline{E} lies consistently below 4 %.



Fig. 9. Mean beat-to-beat interval error (\overline{E}) depending on the SNR.

3.3. Real Clinical Signals

In the following short evaluation, the capability of the presented deconvolution methods with respect to real signals is regarded. For this purpose, physiological signals are taken from a sleep lab database [2]. The database contains simultaneous recordings of ECG, PPG and BCG sensors. As commonly accepted, the ECG sensor defines the goldstandard for interbeat interval measurement and is used as the reference. An artifact free segment of t = 1000 s with 853 detected heart beats was chosen for the evaluation.

The following Table 3 contains the results after processing the input signals of two different sensors with both methods. The parameterization of the methods was done as

Input	Method	\overline{E} [%]	$\overline{E_{95}}$ [%]	PDR [%]	PMR [%]
PPG	MED	1.25	3.06	94.25	0.99
	MCKD	1.69	4.53	93.78	0.99
BCG	MED	1.27	2.53	99.30	2.20
	MCKD	4.12	32.14	98.94	8.37

Tab. 3. Interval-based and peak-based measures for an artifact free segment of t = 1000 s with 853 detected heart beats.

follows. For a sampling frequency of 200 Hz, a reasonable heart rate between 40 and 160 bpm would result in a period length between 75 and 300 samples. Therefore, the MED and the MCKD filter length was chosen to be 100 samples. The overlapping of neighboring frames of the size of 1500 samples is 75 % for the MED, which corresponds to a feed rate of 375 samples, and 50 % for the MCKD, which corresponds to a feedrate of 750 samples. The framesize is chosen to guarantee the coverage of five full periods with the maximum period length of 300 samples. Consequently, the M-shift of the MCKD is set to 4. To avoid sharp edges, the earlier presented window function with an 80 % plateau of unattenuated signal part is used.

The evaluation parameters are grouped into intervalbased and peak-based measures. The mean beat-to-beat interval error (\overline{E}), which describes the mean deviation from the interbeat interval reference, and the 95th error percentile ($\overline{E_{95}}$), which represents the spread of the error, embody the interval-based measures. Only reasonable interbeat intervals are considered for this metric, described by a period length between 75 and 300 samples for this example. On the other hand, the Peak Detection Rate (PDR) and the Peak Misdetection Rate (PMR) characterize how succesful the peaks are detected.

The PDR is over 93 % for all cases and in the same time the PMR is less than 2.5 % for all but one case. This indicates that most of the heart beats and the corresponding peaks have been found and only little misdetections happen. In the same time the interval-based parameters are all below 2 %, except for the BCG sensor in combination with the MCKD method. Overall, most of the results are well within very good boundaries and indicate the applicability of these methods for physiological signals.

4. Discussion

The results of the evaluation with real clinical data depicts good estimation performance with regard to the reference. Consequently, they demonstrate the appropriateness of deconvolution methods for physiological signal simplification in order to obtain interbeat interval estimates.

However, adequate selection of the presented method's parameters is crucial in order to obtain usable impulse se-

quences. As the deconvolution methods are not guaranteed to converge to the absolute maximum, the parameters must define a suitable initial state. If, for example, the filter length is estimated poorly, the output is likely deteriorating the signal rather than improving it. The applicability is therefore limited for cases in which a careful selection of the parameters is difficult. This is the case if large variations in reasonable interval values complicate an adequate parameter selection.

The overall goal of the MED and the MCKD method is to create an output signal, which maximizes the corresponding norm. Both norms are taking high values for impulse sequences with sharp spikes and therefore lead to the desired output signal. However, the estimated filter f solely has the purpose to create these output signals. This is revealed especially in the case of noise interfering with the input signals. The estimated filter f will then have an arbitrary and not meaningful shape to achive the goal of large spikes. Therefore an interpretation of this output is not useful.

In general, the presented methods provide precise heart beat positions connected with the interbeat intervals rather than averaged estimations. Therefore they establish the possibility to identify individual irregular periods or can be used for long term heart rate variability measurements.

Another advantage is the independence of the method from the given signal morphology.

5. Conclusion

We presented a novel deconvolution-based approach for the preprocessing of physiological signals in order to estimate periodic vital signs.

Two iterative deconvolution methods, which are based on the maximization of a simplicity norm, are introduced; the Minimum Entropy Deconvolution (MED) based on the varimax norm and the Maximum Correlated Kurtosis Deconvolution (MCKD) based on the correlated kurtosis norm.

Both methods have been evaluated with regard to the influence of their parameters and the necessity for a careful choice of these was motivated. A short investigation on artificial BCG signals with additive white Gaussian noise indicated a good peak detection rate for SNRs higher than 4 dB. Furthermore, an evaluation with real clinical signals from one PPG and two BCG sensors, depicted a high peak detection performance and a precise interbeat interval estimation with less than 2 % mean error for almost all regarded cases. In general, the deconvolution methods have shown promising results for signal simplification to estimate periodic parameters.

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