Time-Domain Kalman Filter for Active Noise Cancellation Headphones

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***Abstract*—Noise pollution has a large negative influence on the health of humans, especially in case of long-term exposure. Various passive hearing protection approaches are available. However, they often lack good protection against low frequency noise. For these applications, the principle of Active Noise Cancellation (ANC) offers a promising supplement. It relies on anti-phase compensation of the noise signal. Within the area of ANC, only few publications deal with the Kalman filter approach. The state-of-the-art in literature is briefly reviewed. The algorithm presented in this contribution is inspired by the time-domain Kalman filter. The Kalman filter has the favorable property of fast convergence as well as good tracking properties. Especially the tracking of time-varying noise conditions is often a drawback of least-mean-square (LMS) and recursive-least-square (RLS) approaches. The proposed algorithm uses the Kalman equations which are extended by online model parameter estimation based on observable signals. This results in faster convergence and higher robustness against dynamically changing noise conditions. The performance of the algorithm is evaluated by means of convergence, tracking and stability with measured acoustic paths from a real-time system.**

I. Introduction

Noise pollution is an increasing problem in modern society, due to densely populated areas and raising traffic volume. Convenient methods for noise avoidance or cancellation are required. Passive approaches, e.g. hearing protection, can be supplemented by active methods, widely known as Active Noise Cancellation (ANC). ANC uses anti-phase signals to compensate existing noise sources. It has the advantage of being able to compensate low frequency noises without the need for heavy absorption materials.

Current research on ANC systems mostly concentrates on adaptive digital system, as they offer greater flexibility than analog fixed systems. On the other hand, digital systems require analog-digital (AD) conversion, which introduces additional delay that may limit the achievable performance. Thus, the real-time system design is a demanding task.

The general structure of the adaptive digital system is shown in Fig. 1. We regard the noise cancelling headphone as an example. The headphone comprises of two microphones, one capturing the external noise reference signal $x(n)$ and one measuring the internal residual error signal $e(n)$. The direct transmission from the outer reference signal to the inner error signal is described by the so called primary path $P(z)$. The internal loudspeaker is used to play the antiphase cancellation signal $y(n)$. The transmission between this loudspeaker and the inner microphone is given by the secondary path $S(z)$.[1] The feedback from the loudspeaker to the outer microphone

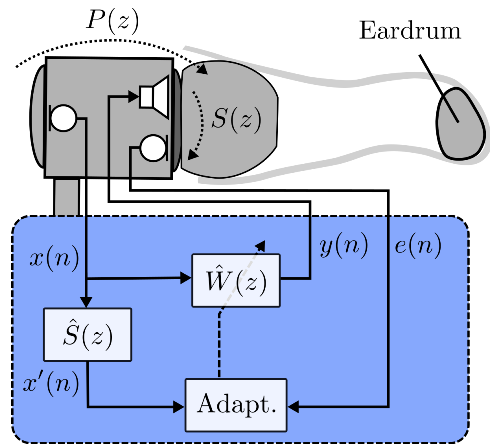

Fig. 1. Functional structure of an in-ear headphone for an adaptive feedforward ANC system with Filtered-X (FX) structure.

is neglected here, due to lower sound levels inside and the passive attenuation of the headphone. The cancellation signal $y(n)$ is created by a filter $\hat{W}(z)$ using the reference signal $x(n)$. $\hat{W}(z)$ is adjusted by an adaptive algorithm. The most common adaptive algorithm in ANC is the least-mean-squares algorithm (LMS). It has the advantage of being simple and computationally inexpensive. However, it suffers from slow convergence and poor tracking due to the dependency on the Eigenvalue spread of the input signal [1]. In this paper, we describe active cancellation as a state estimation problem using the well-known Kalman filter. It has better convergence, tracking and stability properties for the price of a higher computational complexity. The Kalman filter can also be interpreted as an LMS algorithm with optimal variable step size [2].

For convergence, the reference signal $x(n)$ of the adaptive algorithm needs to be prefiltered with an estimate of the secondary path $\hat{S}(z)$, also depicted in Fig. 1. This leads to the well-known Filtered-X structure (FX). Introducing this prefiltering results in an exchange of the sequential order of the secondary path filter function $S(z)$ and the cancellation filter $\hat{W}(z)$ within the adaptive algorithm. This is usually argued as allowable due to slowly changing filter coefficients. For fast adaptive algorithms, such as the Kalman filter, an additional error occurs as this assumption is violated. This additional error leads to instabilities and thus needs to be corrected by the extension to the so-called Modified Filtered-X structure (MFX) [3].

Lopes [1] compared the Kalman filter to the LMS, as well as

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the recursive-least-squares (RLS) algorithm and demonstrated its advantageous convergence properties. The applicability of the Kalman filter for ANC is limited due to computational complexity and the demanding requirements for real-time. With the goal of reduced complexity, Fraanje proposed a fast-array form of the Kalman filter based on the shift property of the reference signal $x(n)$ [4]. Ophem and Berkhoff extended this approach to MIMO systems [5], [6], and pursued investigations on tracking and convergence, however they encountered numerical stability problems [7], [8]. They also included the secondary path estimation in their model. Recently, Lopes proposed a random walk Kalman filter [9]. It relies on white Gaussian noise signals to model state changes. In all of these publications the model incorporates static parameters, which need to be empirically chosen and tuned. They have a large influence on performance and stability and finding an optimal time-invariant solution is often not possible due to variability of real-world problems. To improve this downside, we propose an online estimation method of the process noise, the measurement noise, and the transition matrix. The algorithm exploits temporal signal statistics. As the algorithm it is purely data-driven and deviates from the Kalman filter principle of combining data with prior knowledge, it should be denoted as Kalman-like.

II. Methods

A Kalman filter requires a dynamic model of an underlying system as well as knowledge of its process variables. It is represented by a set of equations aimed to estimate the current state of a dynamic system based on disturbed measurements [10].

The dynamics of the control filter $\boldsymbol{w}(n) \in \mathbb{R}^{L\times 1}$ with the filter length L are described using a Markov model of first order

$$\boldsymbol{w}(n) = \boldsymbol{A}(n)\boldsymbol{w}(n-1) + \boldsymbol{v}_\mathrm{p}(n), \tag{1}$$

with transition matrix $\boldsymbol{A}(n) \in \mathbb{R}^{L\times L}$ and zero-mean white Gaussian process noise $\boldsymbol{v}_\mathrm{p}(n) \in \mathbb{R}^{L\times 1}$. The single channel Kalman equations for estimating the state $\boldsymbol{w}(n)$, divided in prediction and correction step, are given in the following. A-priori estimations are denoted with $^-$ and a-posteriori estimations are marked with $^+$.

Prediction step:

$$\hat{\boldsymbol{w}}^-(n) = \boldsymbol{A}(n)\hat{\boldsymbol{w}}^+(n-1) \tag{2a}$$

$$\boldsymbol{P}^-(n) = \boldsymbol{A}(n)\boldsymbol{P}^+(n-1)\boldsymbol{A}(n)^T + \boldsymbol{Q}(n) \tag{2b}$$

Correction step:

$$\boldsymbol{K}(n) = \frac{\boldsymbol{P}^-(n)\boldsymbol{H}(n)}{\boldsymbol{H}^T(n)\boldsymbol{P}^-(n)\boldsymbol{H}(n) + R(n)} \tag{2c}$$

$$\boldsymbol{P}^+(n) = \left[\boldsymbol{I} - \boldsymbol{K}(n)\boldsymbol{H}^T(n)\right]\boldsymbol{P}^-(n) \tag{2d}$$

$$\hat{\boldsymbol{w}}^+(n) = \hat{\boldsymbol{w}}^-(n) + \boldsymbol{K}(n)\cdot e(n) \tag{2e}$$

In the prediction step the modeled system dynamics are applied to the previous a-posteriori state $\hat{\boldsymbol{w}}^+(n-1)$ and covariance matrix $\boldsymbol{P}^+(n-1)$ to obtain an estimate of the current state. The model is typically based on prior system knowledge, such as prior measurements, statistical analysis or physical insight. This estimate is corrected using a-posteriori knowledge, here the measurement of the error $e(n)$, in the correction step. This combination of a system model and measurements is one of the central properties of a Kalman filter. The error, which represents the innovation, is weighted by the Kalman gain $\boldsymbol{K}(n)$. The Kalman gain weights the correction of the state estimates $\hat{\boldsymbol{w}}^-(n)$ as well as the covariance estimate $\boldsymbol{P}^-(n)$, which describes the uncertainty of the state estimates. The observation matrix $\boldsymbol{H}(n)$ contains the past L values of the filtered reference signal $x'(n)$. The parameter $\boldsymbol{Q}(n)$ denotes the process noise covariance, describing the uncertainty of the system model, and $R(n)$ represents the measurement noise variance, containing the uncertainty of the measurements. $\boldsymbol{I}$ is the identity matrix. The control filter $\hat{\boldsymbol{w}}(n)$ for creating the cancellation signal $y(n)$ will be set to $\hat{\boldsymbol{w}}^+(n)$. Neither $\boldsymbol{A}(n)$, $\boldsymbol{Q}(n)$ nor $R(n)$ are directly observable and are often chosen as constants for filtering applications. In the following, we will derive methods to estimate these process variables using observable data. The concept for the measurement noise estimation was presented in [11] for the application of echo cancellation. In [12] the individual online process noise estimation for each filter coefficient is proposed. These insights from the field of echo cancellation are transfered to the field of active noise cancellation.

A. Process noise estimation

The covariance matrix $\boldsymbol{Q}(n)$ of the system process noise $\boldsymbol{v}_\mathrm{p}(n) = [v_{\mathrm{p},0}, ..., v_{\mathrm{p},L-1}]^T$ is defined as

$$\boldsymbol{Q}(n) = \mathrm{E}\left\{\boldsymbol{v}_\mathrm{p}^T(n)\cdot\boldsymbol{v}_\mathrm{p}(n)\right\}. \tag{3}$$

Since the process noise is assumed to be white Gaussian noise, $v_{\mathrm{p},i}(n)$ and $v_{\mathrm{p},j}(n)$ are uncorrelated for $i \neq j$, and $\boldsymbol{Q}(n)$ can be simplified to

$$\boldsymbol{Q}(n) = \mathrm{diag}\left\{\sigma^2_{v_{\mathrm{p},0}}, \sigma^2_{v_{\mathrm{p},1}}, \dots, \sigma^2_{v_{\mathrm{p},L-1}}\right\}. \tag{4}$$

Eq. 1 results in the following context for $\sigma^2_{v_{\mathrm{p},l}}$ with $l = 0, \dots, L-1$

$$\sigma^2_{v_{\mathrm{p},l}}(n) = \mathrm{E}\left\{\left[w_l(n) - A_l(n)\cdot w_l(n-1)\right]^2\right\}, \tag{5}$$

with $A_l(n)$ being the l-th entry of the main diagonal of $\boldsymbol{A}(n)$. Since $\boldsymbol{w}(n)$ is unknown, a good approximation is to substitute it by the estimated state $\hat{\boldsymbol{w}}(n)$. Using Eq. 2a this results in

$$\hat{\sigma}^2_{v_{\mathrm{p},l}}(n) = \mathrm{E}\left\{\left[\hat{w}_l^+(n) - A_l(n)\cdot\hat{w}_l^+(n-1)\right]^2\right\} \tag{6a}$$

$$= \mathrm{E}\left\{\left[\hat{w}_l^+(n) - \hat{w}_l^-(n)\right]^2\right\}. \tag{6b}$$

The expectation can then be approached using a first order IIR low-pass filter with control parameter $0 \ll \alpha \leq 1$

$$\hat{\boldsymbol{Q}}(n) = \alpha\cdot\hat{\boldsymbol{Q}}(n-1) + (1-\alpha)\cdot\mathrm{diag}\left\{\left[\hat{\boldsymbol{w}}^+(n) - \hat{\boldsymbol{w}}^-(n)\right]^2\right\} \tag{7a}$$

$$= \alpha\cdot\hat{\boldsymbol{Q}}(n-1) + (1-\alpha)\cdot\mathrm{diag}\left\{\left[\boldsymbol{K}(n)\cdot e(n)\right]^2\right\}. \tag{7b}$$

B. Measurement noise estimation

The estimation of the variance $R(n)$ of the measurement noise $v_\mathrm{m}(n)$ plays an important role in characterizing the

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stability properties of the Kalman filter. This is due to the fact that it directly influences the denominator of the Kalman gain vector $\boldsymbol{K}(n)$, as seen in Eq. 2c. The variance $R(n)$ is defined as

$$R(n) = E\left\{v_\mathrm{m}^2(n)\right\}. \tag{8}$$

To obtain a robust estimation, the measurement noise $v_\mathrm{m}(n)$ is interpreted as comprising all errors, including those remaining during and after convergence. This assumption results in the following estimation for the variance of the measurement noise

$$\hat{R}(n) = \alpha \cdot \hat{R}(n-1) + (1-\alpha) \cdot e^2(n), \tag{9}$$

using the same first order IIR filter structure as in Eq. 7b, with $0 \ll \alpha \leq 1$.

C. Prediction estimation (PE) of the transition matrix

Many realizations of the Kalman filter implement the transition matrix using an empirical forgetting factor $0 \ll \gamma \leq 1$, resulting in $\boldsymbol{A}(n) = \gamma \cdot \boldsymbol{I}$. A closer look at the evolution of estimated filter coefficients $\hat{w}_l(n)$, as shown in Fig. 2, yields that the initial convergence is slowed down using this approach. The knowledge of the underlying adaption behaviour, the convergence can be accelerated. For a stationary reference signal, the filter coefficients converge in a predictable manner, as illustrated in Fig. 2 In the following, this knowledge will be used to deduce the proposed method for estimating the transition matrix $\boldsymbol{A}(n)$.

The proposed method is based on predicting the current filter state using an extrapolation technique. The discrete analogue to the Taylor polynomial, the Newton series, uses finite differences to extrapolate smooth signals around a given point of evaluation, as described in [13]. Using this approach the current estimate $\tilde{w}(n|n-1)$, with $n-1$ being the point of evaluation, results in

$$\tilde{\boldsymbol{w}}(n|n-1) = \sum_{k=0}^{\mathcal{O}} \Delta_\mathrm{R}^k \hat{\boldsymbol{w}}(n-1), \tag{10}$$

with the order of newton polynomial $\mathcal{O}$. Here finite differences are used in a backwards oriented fashion, since only past values of $\hat{\boldsymbol{w}}(n)$ are known. The k-th finite backwards difference $\Delta_\mathrm{R}^k \hat{\boldsymbol{w}}(n)$ is recursively defined as

$$\Delta_\mathrm{R}^k \hat{\boldsymbol{w}}(n) = \Delta_\mathrm{R}^{k-1} \hat{\boldsymbol{w}}(n) - \Delta_\mathrm{R}^{k-1} \hat{\boldsymbol{w}}(n-1) \tag{11}$$

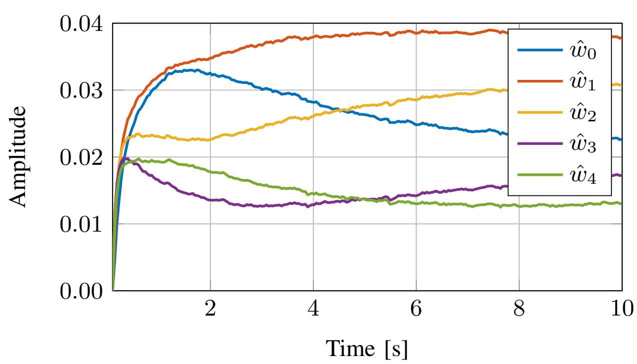

Fig. 2. Evolution of the first filter coefficients of $\hat{\boldsymbol{w}}(n) = (\hat{w}_0, ..., \hat{w}_{L-1})^T$ over time for white Gaussian noise as reference signal $x(n)$.

with

$$\Delta_\mathrm{R}^1 \hat{\boldsymbol{w}}(n) = \hat{\boldsymbol{w}}(n) - \hat{\boldsymbol{w}}(n-1) \tag{12}$$

Eq. 10 can also be interpreted as applying a FIR filter $\Lambda(n)$ of length $\mathcal{O}+1$ to each past coefficient of $\hat{\boldsymbol{w}}(n)$, such that

$$\tilde{\boldsymbol{w}}(n|n-1) = \sum_{k=0}^{\mathcal{O}} \Lambda(k) \cdot \hat{\boldsymbol{w}}(n-k-1) \tag{13}$$

holds. With Eq. 10 and Eq. 11 this filters impulse response can be concluded to

$$\Lambda(n) = (-1)^n \binom{\mathcal{O}+1}{n+1}, \qquad \text{for } n = 0, \ldots, \mathcal{O} \tag{14}$$

To break the closed loop induced by this method, the estimation $\tilde{\boldsymbol{w}}(n|n-1)$ will be based on the additional auxiliary signal

$$\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \boldsymbol{K}(n) \cdot e(n), \tag{15}$$

which accumulates only the update of $\hat{\boldsymbol{w}}(n)$ from the correction step (Eq. 2e) of the Kalman filter. It thus reduces the influence of $\boldsymbol{A}(n)$ on the estimation of itself. To further reduce jittering effects, as seen in Fig. 2, $\boldsymbol{v}(n)$ will be smoothed using a first order IIR low-pass filter with $0 \ll \lambda \leq 1$ resulting in

$$\bar{\boldsymbol{v}}(n) = \lambda \cdot \bar{\boldsymbol{v}}(n-1) + (1-\lambda) \cdot \boldsymbol{v}(n). \tag{16}$$

In matrix notation the estimate, now denoted as $\tilde{\boldsymbol{v}}(n|n-1)$, can then be written as

$$\tilde{\boldsymbol{v}}(n|n-1) = \bar{\boldsymbol{V}}(n-1)\boldsymbol{\Lambda}, \tag{17}$$

with the matrix $\bar{\boldsymbol{V}}(n)$ and the vectorized FIR filter $\boldsymbol{\Lambda}$ defined as

$$\bar{\boldsymbol{V}}(n) = [\bar{\boldsymbol{v}}(n), \bar{\boldsymbol{v}}(n-1), \ldots, \bar{\boldsymbol{v}}(n-\mathcal{O})] \tag{18}$$

$$\boldsymbol{\Lambda} = [\Lambda(0), \Lambda(1), \ldots, \Lambda(\mathcal{O})]. \tag{19}$$

Lastly we are using the update

$$\Delta\tilde{\boldsymbol{v}}(n) = \tilde{\boldsymbol{v}}(n|n-1) - \bar{\boldsymbol{v}}(n-1) \tag{20}$$

as an additive contribution to derive the main diagonal of the transition matrix $\boldsymbol{A}(n)$, according to Eq. 2a, for $l = 0, \ldots, L-1$

$$\bar{v}_l(n-1) + \Delta\tilde{v}_l(n) = A_l(n) \cdot \bar{v}_l(n-1) \tag{21a}$$

$$\Leftrightarrow \qquad A_l(n) = 1 + \frac{\Delta\tilde{v}_l(n)}{\bar{v}_l(n-1)}. \tag{21b}$$

To avoid divisions by zero, this term should be rewritten as

$$A_l(n) = 1 + \operatorname{sgn}\{\bar{v}_l(n-1)\} \cdot \frac{\Delta\tilde{v}_l(n)}{|\bar{v}_l(n-1)| + \varepsilon} \tag{22}$$

with ε being a small positive value.

III. Evaluation

In the following, we evaluate the influence of the proposed extensions with a focus on convergence and tracking.

The transmission paths for the simulations have been measured with a real-time system based on dSPACE hardware (DS1005, dSPACE GmbH, Paderborn, Germany), with the DS2004 AD-extension and the DS2102 DA-extension board. The round-trip delay, including AD/DA-conversion, is 1 sample latency at 48 kHz. The electro-acoustic front-end is realized

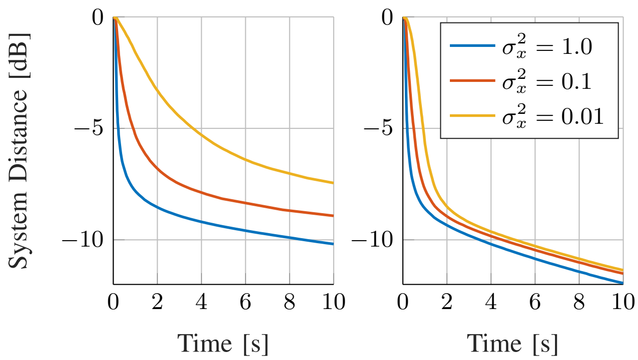

Fig. 3. Effect of online process parameter estimation. Left: Static parameters. Right: Online parameter estimation.

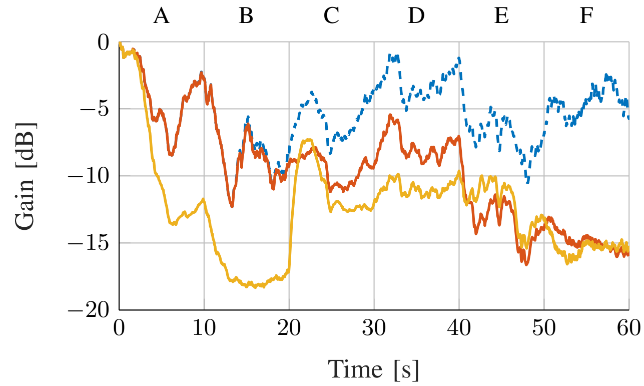

Fig. 4. Gain for combination of real-world scenarios for different low-pass parameters $\alpha = 1.0$ (- - -) (Reference system, comparable to [16]), $\alpha = 0.99999$ (——), $\alpha = 0.9999$ (——). The utilized scenarios from the ETSI background database [15] are **A** - Cafeteria, **B** - Inside Train, **C** - Outside Traffic Road, **D** - Pub, **E** - Kindergarten, **F** - Midsize Car1 100kmh. $\boldsymbol{P}_0 = 1\text{e}{-5}$, $\boldsymbol{Q}_0 = 1\text{e}{-4}$, $R_0 = 1\text{e}{-3}$, $f_s = 44,1$ kHz. For better visibility, the plot has been smoothed with a first-order IIR low-pass.

by *Bose QC 20* headphone hardware (without the Bose ANC electronics) [14]. Two sets of primary and secondary paths have been measured with human test persons. We focus on the cancellation filter adaptation and for an isolated consideration of the proposed methods assume a perfect secondary path estimation ($\hat{S}(z) = S(z)$) for the MFX-filter structure.

The quality of the system identification process of $P(z)$ at time instance n may be measured by the system distance $\text{SD}(n)$ according to

$$\text{SD}(n) = 10 \cdot \log_{10}\left(\frac{\|\boldsymbol{p} - \hat{\boldsymbol{p}}(n)\|^2}{\|\boldsymbol{p}\|^2}\right), \tag{23}$$

with $\hat{p}(n) = \hat{w}(n) * s(n)$. The overall goal of ANC is minimizing the power of the error signal $e(n)$. The active component of the attenuation, here named the gain, corresponds to the linear version of the SD for white Gaussian reference signals.

If not specified further, the Kalman filter with the MFX-structure has been used with the following parameters for all simulations with a sampling rate of $f_s = 48$ kHz: $\boldsymbol{P}_0 = 1\text{e}{-4}$, $\boldsymbol{Q}_0 = 1\text{e}{-8}$, $R_0 = 1.0$, $L = 64$, $\boldsymbol{A} = \boldsymbol{I}$. The parameters have been empirically chosen for fastest conversion without getting instabilities in all the experiments. The process noise for the static parameters is chosen constant for all coefficients, in contrast to the individual adjustment with the online parameter estimation.

A. Process and measurement noise estimation

For the parameterization of the Kalman filter in ANC applications, a suitable modeling of the process noise $\boldsymbol{Q}(n)$ and the measurement noise $R(n)$ is crucial. As indicated earlier, they are not directly observable and thus need to be estimated. In Fig. 3 we compare the convergence for empirically chosen static parameters on the left and the proposed online estimation on the right. The reference signal $x(n)$ is a zero-mean white Gaussian noise signal with different variances. The plot shows the system distance. We may observe that for static parameters the convergence speed varies for different reference signal powers. The online parameter estimation achieves roughly the same convergence speed for all three cases. For specific cases, the static parameters might be better tuned than in this example, however, it lacks the illustrated flexibility. Inappropriately chosen static parameters may also lead to instabilities in certain circumstances. However, stability of the algorithm is one of the most important properties. Thus, the online estimation reduces the problem of mistuning and the chance for instabilities to occur.

For realistic evaluation signals, we used the ETSI background noise database [15]. It includes binaural recordings from various noise scenarios. We have selected six representative scenarios and combined 10s of each to one 60s long signal. The transitions between the six scenarios were realized by linear fading of 100 ms between adjacent cases. The following cases were combined: **A** - Cafeteria, **B** - Inside Train, **C** - Outside Traffic Road, **D** - Pub, **E** - Kindergarten, **F** - Midsize Car1 100kmh. They were chosen to be varying in noise power and frequency characteristic from one signal to the other. Fig. 4 shows the gain, which represents the achieved active noise attenuation, for three different cases. $\alpha = 1.0$ (- - -) is the performance of the Kalman filter without the proposed extensions. The other two curves use different low-pass filtering for the online parameter estimation. The parameter α controls the speed of adjustment to the current state. The lower it is, the less smoothing is performed. For $\alpha = 0.99999$ (——), we can see an improved performance from scenario **C** on. However, we observe a significant improvement of convergence speed, tracking and the achieved gain for all scenarios **A** to **F** for $\alpha = 0.9999$ (——).

B. Prediction estimation of the transition matrix

Hereafter, we regard the isolated influence of the proposed prediction estimation (PE) of the transition matrix according to Eq. 22. Fig. 5 illustrates the convergence speed with and without the PE-method for a stationary jackhammer noise signal [15]. For studying the convergence properties, we chose a realistic stationary signal. We observe improved convergence speed when using the PE-method. This method can be applied when a fast convergence for stationary signals is desired. Its application for rapidly changing noise conditions is limited due to the inherent low-pass filtering shown in Eq. 16. Therefore, it may also have a degrading influence on the tracking properties depending on the noise signal.

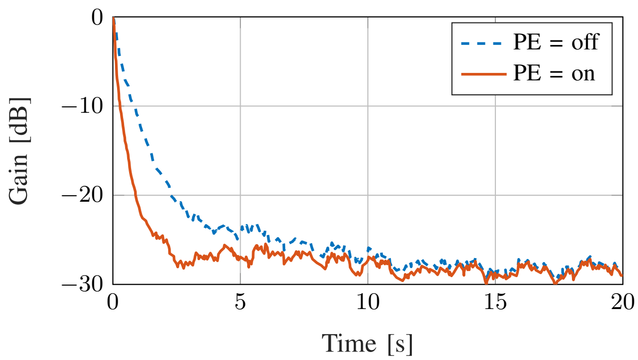

Fig. 5. Convergence for stationary jackhammer noise with (λ estimated online) and without ($\lambda = 1$) proposed PE method ($f_s = 44,1$ kHz).

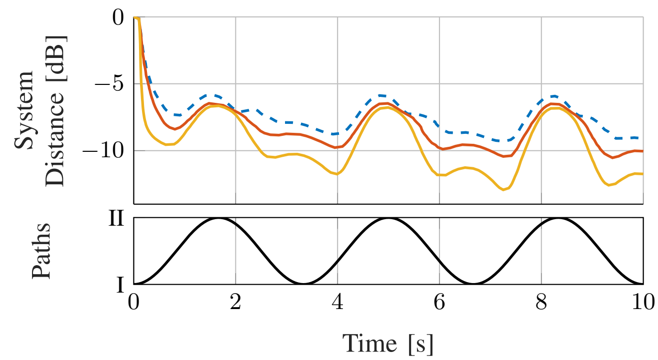

Fig. 6. Tracking of continuous path changes using static parameters - - - ($\alpha = 1.0, \lambda = 1.0$); estimates of $\boldsymbol{Q}$ and R —— ($\alpha = 0.9999, \lambda = 1.0$); estimates of $\boldsymbol{Q}$, R and $\boldsymbol{A}$ —— ($\alpha = 0.9999, \lambda = 0.999$). The primary and the secondary paths are continuously crossfaded between the measurements of the two test persons as indicated in the lower plot.

The influence of primary and secondary path changes on the adaptive algorithm is illustrated in Fig. 6. Two sets of measured paths of human test persons are continuously faded as indicated in the lower plot of Fig. 6. This cancellation filter still constantly needs to adapt, as the ideal filter $W^0(z) = P(z)/S(z)$ is changing [17]. We compare three different cases: static parameters (- - -), online estimation of $\boldsymbol{Q}$ and R (——), and online estimation of $\boldsymbol{Q}$, R and $\boldsymbol{A}$ (——) for white Gaussian noise as reference $x(n)$. The system distance shows a consistent improvement with all extensions.

IV. Conclusion

In this contribution we regarded the extension of a Kalman filter algorithm for ANC applications. For improving the three main objective properties convergence, tracking and stability, with respect to algorithm tuning in view of the great variability of real applications, we proposed the online estimation of the process noise $\boldsymbol{Q}(n)$ and the measurement noise $R(n)$. Usually these parameters are empirically chosen. Our approach uses the difference of the a-priori and the a-posteriori filter coefficients $\hat{\boldsymbol{w}}^-(n)$ and $\hat{\boldsymbol{w}}^+(n)$ for estimating the process noise. The measurement noise is estimated using the variance of the error signal. Both estimations are smoothed with a first order recursive low-pass filter. The online parameter estimation has shown clear improvements in convergence and tracking. Instabilities have not been observed, however, a detailed analysis remains for further studies.

Furthermore, we propose a prediction method for the transition matrix $\boldsymbol{A}$, rather than designing a random walk Kalman filter ($\boldsymbol{A} = \boldsymbol{I}$) or including a forgetting factor ($\boldsymbol{A} = \gamma\boldsymbol{I}$ with $0 \ll \gamma \leq 1$). This method is based on observations of the evolution of the filter coefficients.

The goal of improving convergence speed and maintaining good tracking properties while simplifying the tuning could be confirmed within simulations for stationary noise signals.

We focused on the cancellation filter adaptation and assumed a perfect secondary path estimation ($\hat{S}(z) = S(z)$) to isolate the improvements of these extensions.

References

[1] P. A. Lopes and M. Piedade, "The Kalman filter in active noise control," *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, no. 5, pp. 1111–1124, 1999.

[2] D. P. Mandic, S. Kanna, and A. G. Constantinides, "On the Intrinsic Relationship Between the Least Mean Square and Kalman Filters [Lecture Notes]," *IEEE Signal Processing Magazine*, vol. 32, no. 6, pp. 117–122, 2015.

[3] E. Bjarnason, "Active noise cancellation using a modified form of the filtered-x lms algorithm," in *Proceedings of Eusipco 92, 6th European Signal Processing Conference*, 1992, pp. 1053–1056.

[4] P. R. Fraanje, *Robust and fast schemes in broadband active noise and vibration control*. University of Twente, 2004.

[5] S. van Ophem and A. P. Berkhoff, "Multi-channel Kalman filters for active noise control," *Journal of the Acoustical Society of America*, vol. 133, no. 4, pp. 2105–2115, 2013.

[6] ——, "Performance of a multi-channel adaptive Kalman algorithm for active noise control of non-stationary sources," in *Inter-Noise 2012*. USA: The Institute of Noise Control Engineering of the USA, 2012, pp. 4809–4819.

[7] A. P. Berkhoff and S. van Ophem, "Tracking and convergence of multi-channel Kalman filters for active noise control," in *Proceedings Internoise 2013*. INCE, 2013, pp. 1–10.

[8] S. van Ophem and A. P. Berkhoff, "Active control of time-varying broadband noise and vibrations using a sliding-window Kalman filter," in *Proceedings of ISMA 2014 including USD 2014*, P. Sas, D. Moens, and H. Denayer, Eds. Katholieke Universiteit Leuven, 2014, pp. 107–118.

[9] P. A. Lopes and J. A. Gerald, "Auxiliary Noise Power Scheduling Algorithm for Active Noise Control with Online Secondary Path Modeling and Sudden Changes," *Signal Processing Letters, IEEE*, vol. 22, no. 10, pp. 1590–1594, 2015.

[10] P. A. Lopes, J. A. Gerald, and M. S. Piedade, "The Random Walk Model Kalman Filter in Multichannel Active Noise Control," *IEEE Signal Processing Letters*, vol. 22, no. 12, pp. 2244–2248, 2015.

[11] C. Paleologu, J. Benesty, and S. Ciochin, "Study of the General Kalman Filter for Echo Cancellation," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 21, no. 8, pp. 1539–1549, 2013.

[12] C. Paleologu, J. Benesty, S. Ciochin, and S. L. Grant, "A Kalman filter with individual control factors for echo cancellation," in *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2014, pp. 5974–5978.

[13] K. Jordán, *Calculus of finite differences*. American Mathematical Soc, 1965.

[14] K. P. Annunziato, J. Harlow, M. Monahan, A. Parthasarathi, R. C. Silvestri, and E. M. Wallace, "In-ear active noise reduction earphone," Patent US8 682 001 B2, 2014. [Online]. Available: https://www.google.com/patents/US8682001

[15] European Telecommunications Standards Institut, "Speech Processing, Transmission and Quality Aspects (STQ); and background noise database; Speech quality performance in the presence of background noise; Part 1: Background noise simulation technique (ETSI EG 202 396-1)," vol. 1.2.3 (2009-03), 2008.

[16] P. A. Lopes and M. Piedade, "A Kalman filter approach to active noise control," *EUSIPCO 2000 Proceedings*, 2000.

[17] Colin H. Hansen and Scott Snyder, Xiaojun Qiu, Laura Brooks, Danielle Monreau, *Active control of sound and vibration*, 2nd ed. London: CRC Press, 2012.