Generalized Filter-Bank Equalizer for Noise Reduction with Reduced Signal Delay

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Abstract

An efficient realization of a low delay filter-bank, termed as generalized filter-bank equalizer (FBE), will be proposed for noise reduction with low signal delay. The FBE is equivalent to a time-domain filter with coefficients adapted in the frequency-domain. This filter-bank structure ensures perfect signal reconstruction for a variety of spectral transforms with less restrictions than for an analysis-synthesis filter-bank (AS FB). A non-uniform frequency resolution can be achieved by an allpass transformation. In this case, the FBE has not only a lower signal delay than the AS FB, but also a lower algorithmic complexity for most parameter configurations. Another advantage of the FBE is the lower number of required delay elements (memory) compared to the AS FB. The noise reduction achieved by means of the AS FB and the FBE is approximately equal.

1. Introduction

Nowadays, noise reduction systems have found widespread application, for example, in mobile phones, hands-free telephony, speech recognition systems, and hearing aids. Most noise reduction systems are based on spectral weighting by using an *analysis-synthesis filter-bank (AS FB)*. Thereby, the DFT FB is often used, which can be efficiently implemented as polyphase network analysis-synthesis filter-bank (PPN AS FB) with the DFT calculated by means of the Fast Fourier Transform (FFT), cf. [1], [2]. This filter-bank structure is shown in Fig. 1. The spectral gain factors $W_i(k')$ are adapted here by the weighting rule of a noise reduction algorithm (mostly spectral subtraction), e.g., [3]. This adaptation, which has not been drawn in Fig. 1 for the sake of clarity, and the spectral weighting in the DFT-domain are performed at a lower rate than the sampling rate, indicated by the sample index k' with $k = r\,k'$.

A non-uniform frequency resolution can be obtained by an allpass transformation [4], [5]. The frequency resolution of an allpass transformed analysis-synthesis filter-bank (APT AS FB) can be adapted to the Bark scale, which has been applied for noise reduction in [6].

Noise reduction systems have to fulfill the (partly) conflicting requirements for low computational complexity, low memory consumption, high speech quality and high noise suppression, and, not at least, a low signal delay. The last requirement is important for most noise reduction systems which are part of a real-time speech transmission or speech processing system, respectively.

The (algorithmic) signal delay of the FB in Fig. 1 is determined by the analysis FB and the synthesis FB, assuming that

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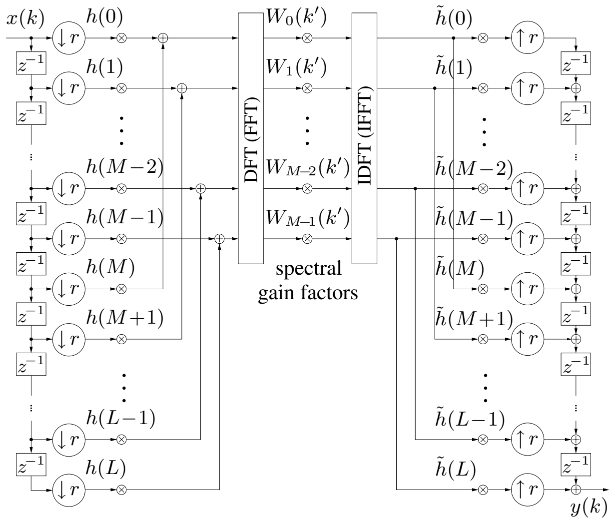

Figure 1: *DFT polyphase network analysis-synthesis filter-bank (PPN AS FB) with down-sampling for a prototype filter length of $L + 1 = 2\,M$. An allpass transformed filter-bank (APT FB) is obtained by replacing the delay elements with allpass filters.*

the computation of the gain factors causes no (significant) additional delay. An approach to reduce the signal delay is to employ a FB with short prototype filters which, however, reduces its frequency resolution. A noise reduction with reduced signal delay based on spectral weighting (spectral subtraction) has been proposed in [7].

The FB in Fig. 2 has a significantly lower signal delay than the AS FB of Fig. 1 as no synthesis FB is needed. Hence, this

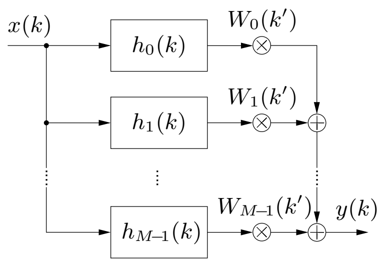

Figure 2: *Low delay filter-bank with time-varying gain factors.*

FB will be denoted as *low delay filter-bank.* It has a significantly higher algorithmic complexity, in terms of computational complexity and memory consumption, compared to the PPN AS FB of Fig. 1 as M separate sub-band filters are needed. An efficient implementation of the DFT/ DCT low delay filter-bank, termed as filter-bank equalizer, has been presented in [8], and a generalization and extension of this concept is given in [9].

In this paper, the application of the filter-bank equalizer (FBE) for noise reduction with reduced signal delay will be investigated. In Sec. 2, the uniform and non-uniform generalized FBE will be presented as far as needed for this contribution. In Sec. 3, implementation and design aspects are discussed, and the properties of the FBE are investigated. Noise reduction by using the FBE and the common AS FB will be contrasted in Sec. 4. A summary is given in Sec. 5.

2. Generalized Filter-Bank Equalizer

In this section, the concept of the generalized filter-bank equalizer (FBE) will be presented based on [9].

2.1. Uniform Filter-Bank Equalizer

The low delay filter-bank of Fig. 2 with M sub-bands will be considered. The impulse response $h_i(n)$ of the i-th sub-band filter shall be given by a modulation of the prototype lowpass filter with impulse response $h(n)$ of length $L+1 \geq M$

$$h_i(n) = \begin{cases} h(n)\,\Phi(i,n) & ;\ i=0,\dots,M-1;\ n=0,\dots,L \\ 0 & ;\ \text{else}\,. \end{cases} \tag{1}$$

The sequence $\Phi(i,n)$ is a general modulation sequence which represents the transformation kernel of the FB. The modulation sequence and the prototype filter determine the spectral selectivity and time resolution of the FB. The periodicity of the modulation sequence in general can be written

$$\Phi(i,n)\,\tilde{p}(m) = \Phi(i, n+m\,M)\ ;\ m \in \mathbb{Z}\,. \tag{2}$$

$\mathbb{Z}$ marks the set of all integer numbers, $\mathbb{R}$ the set of all real numbers, and $\mathbb{C}$ the set of all complex numbers.

An important transformation kernel is given by the generalized discrete Fourier transform (GDFT)

$$\Phi_{\text{GDFT}}(i,n) = \exp\left\{-j\,\frac{2\pi}{M}\,(i-i_0)\,(n-n_0)\right\};\ i_0, n_0 \in \mathbb{R}. \tag{3}$$

The DFT is given for $i_0 = n_0 = 0$. The GDFT possesses the periodicity of Eq. (2) with $\tilde{p}(m) = \exp\{2\pi\,m\,i_0\}$. The GDFT with $i_0 = 0$ and $n_0 = L/2$ will be used for the adaptive FBE presented in Sec. 3.

The output sequence of the low delay filter-bank of Fig. 2 can be expressed in the z-domain for time-independent gain factors $W_i(k') = W_i$ as follows

$$Y(z) = \sum_{i=0}^{M-1} W_i \left(\sum_{n=0}^{L} X(z)\,z^{-n}\,h_i(n)\right)\,. \tag{4}$$

With Eq. (1), the transfer function $F_0(z) = Y(z)/X(z)$ reads

$$F_0(z) = \sum_{n=0}^{L} h(n) \underbrace{\sum_{i=0}^{M-1} W_i\,\Phi(i,n)}_{w_n \doteq T\{\,W_i\,\}} z^{-n} \tag{5}$$

$$= \sum_{n=0}^{L} h(n)\,w_n\,z^{-n} \doteq \sum_{n=0}^{L} h_s(n)\,z^{-n}\,. \tag{6}$$

The (time-domain) weighting factors w_n are calculated by spectral transformation $T\{\,W_i\,\}$ of the gain factors. The derived filter(-bank) structure is named as *generalized filter-bank equalizer (FBE)* [9] since this concept applies for a broad class of spectral transforms including, e.g., the GDFT, Hadamard and Walsh transform. The FBE consists of a single filter whose impulse response $h_s(n)$ is the product of the impulse response of the prototype filter $h(n)$ and the weighting factors w_n (window sequence) adapted in the spectral-domain.

As shown in [9], the uniform generalized FBE ensures *perfect reconstruction* $(W_i = 1)$ with a delay of d_0 samples, if the general transformation kernel of Eq. (1) has the property

$$\sum_{i=0}^{M-1} \Phi(i,n) = \begin{cases} c \neq 0\ ; & n = n_0 \\ 0 & ;\ n \neq n_0 \end{cases}\ ;\ n, n_0 \in \{0,\dots,M-1\} \tag{7}$$

(where mostly $c = M$), and if the prototype lowpass filter is a generalized M-th band filter with

$$h(n) = \begin{cases} (\,c\,\tilde{p}(m_c)\,)^{-1}; & n = n_0 + m_c\,M \doteq d_0;\ \tilde{p}(m_c) \neq 0 \\ 0 & ;\ n = n_0 + m\,M;\ m \in \mathbb{Z}\backslash\{m_c\} \\ \text{arbitrary} & ;\ \text{else}\,. \end{cases} \tag{8}$$

Hence, perfect reconstruction is obtained for a variety of spectral transforms with less restrictions compared to the AS FB.

2.2. Non-Uniform Filter-Bank Equalizer

A non-uniform frequency resolution of a filter(-bank) can be achieved by digital frequency warping based on an allpass transformation [4], [5]. Thereby, the delay elements of the discrete (sub-band) filters are replaced by allpass filters

$$z^{-1} \;\rightarrow\; H_A(z)\,. \tag{9}$$

An allpass filter of first order with frequency response (cf. [2])

$$H_A(z = e^{j\,\Omega}) = \frac{1 - a^*\,e^{j\,\Omega}}{e^{j\,\Omega} - a} = e^{-j\,\varphi_a(\Omega)} \qquad |a| < 1\ ;\ a \in \mathbb{C} \tag{10}$$

is used for the allpass transformation here. For a real pole a, this allpass filter can be realized by two real multiplications, two real summations, and one delay element.

The frequency response of the *allpass transformed filter-bank equalizer (APT FBE)* is obtained by applying Eq. (9) and Eq. (10) to Eq. (6) which results

$$F_a(z = e^{j\,\Omega}) = \sum_{n=0}^{L} h(n)\,w_n\,e^{-j\,n\,\varphi_a(\Omega)}\,. \tag{11}$$

The allpass transformation causes a frequency mapping (warping) $\Omega \rightarrow \varphi_a(\Omega)$ which is solely determined by the allpass pole a. The uniform filter-bank equalizer is included as special case because $H_A(z) = 1/z$ for $a = 0$.

2.3. Polyphase Network Implementation

The polyphase components of the prototype lowpass filter are given in the z-domain by (cf. [1])

$$H_\lambda^{(M)}(z) = \sum_{m=0}^{\lceil \frac{L+1}{M} \rceil - 1} h(\lambda + m\,M)\,z^{-m}\ ;\ 0 \leq \lambda \leq M-1. \tag{12}$$

(The operation $\lceil . \rceil$ provides the smallest integer which is greater or equal than the argument.)

The overall transfer function of the uniform FBE, given by Eq. (5) and Eq. (6), can then be expressed by using Eq. (2) and Eq. (12) which leads to

$$F_0(z) = \sum_{\lambda=0}^{M-1} w_\lambda \sum_{m=0}^{\lceil \frac{L+1}{M} \rceil - 1} \tilde{p}(m)\, h(\lambda + m\, M)\, z^{-(\lambda+mM)} \,. \tag{13}$$

For the DFT or the GDFT with $i_0 = 0$, $\tilde{p}(m) = 1$ which yields the following *polyphase network filter-bank equalizer (PPN FBE)*

$$F_0(z) = \sum_{\lambda=0}^{M-1} w_\lambda\, H_\lambda^{(M)}(z^M)\, z^{-\lambda} \,. \tag{14}$$

The PPN FBE for other modulation sequences, or the allpass transformed FBE of Eq. (11), are obtained alike [9].

3. Application and Properties

The low delay filter-bank of Fig. 2 and the efficient implementation by means of the filter-bank equalizer according to Eq. (6) are (exactly) equivalent for time-invariant gain factors $W_i(k') = W_i$. The *adaptive* filter-bank equalizer is obtained for time-varying gain factors. Its application for noise reduction is exemplified by Fig. 3.

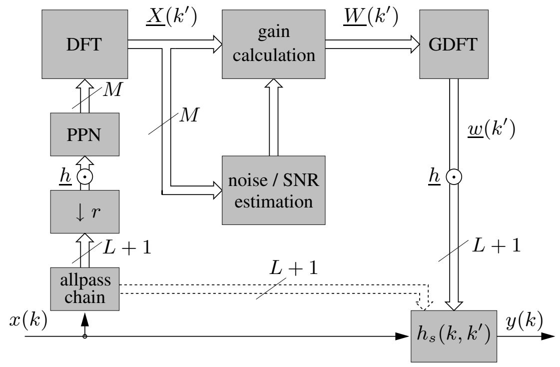

Figure 3: *Adaptive FBE applied for noise reduction. (The 'circle-dot' represents an element-wise vector multiplication.)*

An allpass transformation with $a = 0.5$ has been used for the APT FBs. By this, a higher frequency resolution is achieved for lower frequency bands to account for the non-uniform frequency resolution of the human ear, cf. [6]. Due to the allpass transformation, the phase response of the FBE and AS FB are altered, which can become audible for long prototype filters.

The output sequence of the APT FB $y(k)$ can be filtered by a *phase equalizer (PEQ)* to combat this effect. As shown in [9], the filter degree of the PEQ L_{pe} can be lower for the APT FBE than for the APT AS FB for about the same phase distortions, because no allpass transformed synthesis FB is needed for the FBE. An FIR PEQ is obtained by shifting and truncating the time-reversed, infinite impulse response of the APT FB for $W_i = 1$, [6], [9]. An FIR PEQ with degree $L_{\text{pe}} = 2\,L$ for the APT AS FB, and a PEQ of degree L for the APT FBE yields a good phase compensation with about the same reconstruction error for both APT FBs (near perfect reconstruction).

For the FBE of Fig. 3, the vector $\underline{X}(k')$ with M DFT coefficients is calculated in the same way as for the PPN AS FB of Fig. 1. The DFT instead of the GDFT with $i_0 = 0$ is used, which makes no difference since the computation of $W_i(k')$ (cf. Sec. 4) only requires the magnitude of $\underline{X}$ but not the phase. The DFT can be calculated by the in-place (radix-2) FFT to save memory (and complexity), e.g., [2]. Thereby, the FFT of a real sequence of M samples can be computed by a complex FFT of size $M/2$ with approximately half the algorithmic complexity.

The weighting rule of most noise reduction algorithms yields gain factors with $0 \leq W_i(k') \leq 1$, i.e., the gain factors are of zero phase. The GDFT has to be used (and not the IDFT) to obtain weighting factors $w_n(k')$ with non zero phase response, i.e., a causal window sequence. The GDFT with $i_0 = 0$ of the M real gain factors $\underline{W}(k')$ can by calculated by the FFT as well followed by a cyclic shift of the obtained time-domain sequence by n_0 samples.

For the simulations in Sec. 4, a Hanning window (e.g. [2]) with $L = M = 256$ has been taken for the prototype lowpass filters of both FBs. Hence, the filter $h(n)$ has a linear phase and the GDFT of Eq. (3) with $n_0 = M/2$ ($i_0 = 0$) has to be used to obtain weighting factors $w_n(k')$ with a linear phase as well. However, the use of prototype filters with non-linear phase, e.g., to further reduce the signal delay is also possible, if the conditions for perfect reconstruction of Sec. 2 are fulfilled.

The *algorithmic complexity* of the regarded four FBs is listed in Table 1. The complexity for the computation of the

	uniform FB with M channels	additional OPs for APT + PEQ
polyphase network analysis-synthesis filter-bank		
mult.	$\frac{1}{r}(2M \log_2 M + 2L + 2 + M)$	$4L + (L_{\text{pe}} + 1)$
add.	$\frac{1}{r}(3M \log_2 M + L + 1 - M) + L$	$4L + L_{\text{pe}}$
mem.	$2(M+L)$	L_{pe}
polyphase network filter-bank equalizer		
mult.	$\frac{1}{r}(2M \log_2 M) + L + 1 + M$	$2L + (L_{\text{pe}}/2 + 1)$
add.	$\frac{1}{r}(3M \log_2 M) + L$	$2L + L_{\text{pe}}/2$
mem.	$2\,M + L$	$L_{\text{pe}}/2$

Table 1: *Algorithmic complexity in terms of required average number of real multiplications and real additions per sample, and number of delay elements (memory) for different realizations of a DFT FB with real prototype lowpass.*

spectral gain factors $W_i(k')$ is independent of the FB and has not been considered in Table 1.

The implementation of the actual time-domain filter in Fig. 3 is done by the efficient polyphase network implementation according to Sec. 2.3. The implementation of the (uniform or APT) FBE can be based on different *filter structures*. This becomes important, e.g., for time-varying filter coefficients and / or filter implementations in fixed-point arithmetic. As the filter coefficients are time-varying, the direct (filter) forms with coefficients $h_s(n)$ are of more interest than the parallel form or the cascade form, which require a conversion of the filter coefficients [2]. For the direct forms, considered in Table 1, the allpass chain can be used for the time-domain filter as well, indicted by the dashed line in Fig. 3. For the transposed direct forms, a separate allpass chain is needed. This filter realization has been shown to be beneficial to avoid filter-ringing effects, which occur for strongly time-varying filter coefficients.

Strongly time-varying coefficients can usually be avoided by the (implicit or explicit) smoothing of the spectral gain factors over time by the noise reduction algorithm (e.g. by the recursive decision-directed estimation of the a priori SNR [3]).

The FBE has a lower *signal delay* for sample-wise processing than the corresponding AS FB with the same values for L, M and r. For a prototype lowpass filter with linear phase, the signal delay is equal to $L/2$ samples for the uniform FBE, and equal to L samples for the corresponding AS FB [9]. For the APT AS FB with FIR PEQ of degree L_{pe}, the signal delay is approximately given by L_{pe} samples. For the APT FBE, the delay is equal to $L_{\mathrm{pe}}/2$ samples as the PEQ for the APT FBE can have half the filter degree than the PEQ for the APT AS FB for about the same reconstruction error. This different degrees for the PEQ have also been considered for the comparison in Table 1, whose last column contains the additional operations (OPs) needed for the allpass transformation (APT) of the FB and the phase equalizer (PEQ).

4. Comparison with AS FB

The FBE and the AS FB, as described in Sec. 3, have been used for a noise reduction with $M = 256$ frequency channels and sampling frequency of 8 kHz. The gain factors $W_i(k')$ are adapted after $r = M/2$ sample instants by the MMSE STSA estimator [3] with an SNR estimation based on minimum statistics [10]. Volvo car noise from the noisex-92 database and white noise has been added to a female speech signal. The noise amplification has been varied to obtain different (global) SNRs for the noisy input speech.

In the simulations, the noise and speech signal have been processed separately based on gain factors $W_i(k')$ adapted for the noisy input speech in order to calculate the *noise attenuation (NA)* and the *cepstral distance (CD)* [11]. The NA is the ratio between the power of the input noise and the power of the filtered noise. It is a time-domain measure for the amount of achieved noise suppression. The CD is a frequency-domain distortion measure to indicate the speech quality. The results are plotted in Fig. 4. The instrumental measures vary significantly with the kind of noise and exhibit the trade-off between speech quality and noise suppression. However, for both noise types the differences between the two FB types are negligible for the CD and NA, respectively, with respect to the sensitivity of the human ear. Informal listening test have shown no differences for the subjective speech quality. The employed uniform AS FB has a signal delay of 256 sample according to Sec. 3. In contrast, the uniform FBE causes a delay of only 128 samples.

The instrumental measures for the APT FBs are similar to those of Fig. 4. However, the subjective quality of the enhanced speech signals was found more favorable than for the uniform FBs. The denoised speech sounds more natural which complies with the findings in [6]. The regarded APT AS FB has a signal delay of 512 samples and the APT FBE a delay of 256 samples.

5. Conclusions

In this contribution, the concept of the generalized filter-bank equalizer has been applied for noise reduction. The main advantage of this uniform and non-uniform filter(-bank) is its reduced signal delay compared to the corresponding AS FB. The achieved noise reduction is approximately equal for both FB structures. The FBE needs less delay elements (memory) than the AS FB dependent on its filter structure. The incorporation of other spectral transforms than the GDFT is easily possible.

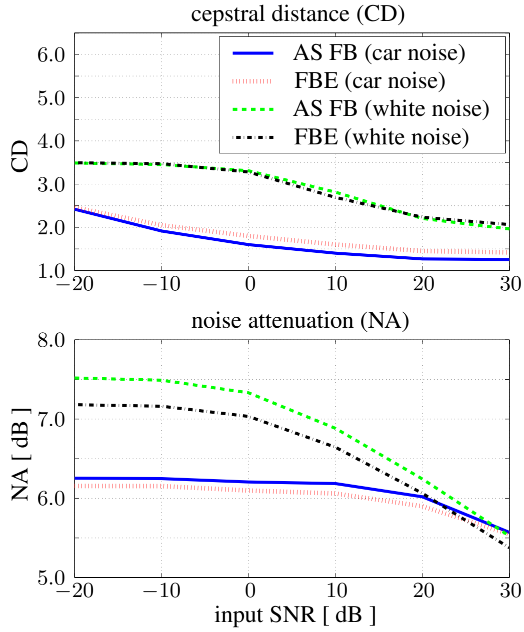

Figure 4: *Instrumental comparison of the achieved noise attenuation and speech quality for the uniform FBE and the uniform AS FB. (The first 40 cepstral coefficients are regarded for the CD measure.)*

6. References

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