PARAMETRIC PHASE EQUALIZERS FOR WARPED FILTER-BANKS

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ABSTRACT

In this paper, phase equalizers are investigated which aim to compensate the non-linear phase response of a frequency warped filter-bank and a new design is proposed. The frequency resolution of a warped filter-bank based on an allpass transformation can be adjusted by a single allpass coefficient. Thus, parametric phase equalizers are of special interest as their coefficients are given by a closed-form expression in dependence on this allpass coefficient. The approximation error for parametric FIR phase equalizers is analyzed. They can achieve a low phase error but introduce magnitude distortions. These distortions can be avoided by using allpass phase equalizers. A new parametric allpass phase equalizer is proposed. It has a lower complexity than a general allpass phase equalizer and leads to an equiripple approximation error for the desired phase response and group-delay.

1. INTRODUCTION

The technique of digital frequency warping by means of allpass transformation can be used for the design of filter-banks with non-uniform frequency resolution [1]. The frequency resolution of such filter-banks can be adjusted, for example, to approximate the critical Bark bands of the human auditory system [2]. This property is exploited, among others, by speech enhancement systems to achieve an improved (subjective) speech quality [3],[4].

A problem associated with warped filter-banks are phase distortions since the allpass transformation of a linear phase filter leads to a non-linear phase response. This effect can become noticeable for audio processing systems. A possible countermeasure is to process the output signal of a warped filter-bank by a phase equalizer to compensate the non-linear phase response, e.g., [3],[4].

A design procedure for phase equalizers to approximate an (almost) arbitrary phase response and group-delay has been proposed, e.g., in [5],[6]. The filter coefficients of these general phase equalizers are obtained by numerical solution of an optimization problem.

For allpass transformed filter-banks, parametric phase equalizers can be applied whose filter coefficients are given by a closed-form expression, which is dependent on the allpass coefficient used for the frequency warping, e.g., [3],[7],[8]. In contrast to general phase equalizers, such parametric phase equalizers can be directly adjusted if the frequency resolution (allpass coefficient) of the warped filter-bank is changed.

In this contribution, different parametric phase equalizers for warped filter-banks are investigated and a new phase equalizer design is proposed. The use of phase equalizers to obtain an allpass transformed filter-bank with near linear phase response is described in Sec. 2. The design and properties of two different parametric FIR phase equalizers are discussed in Sec. 3. Allpass phase equalizers are investigated in Sec. 4. The design of general allpass phase equalizers is outlined and a new parametric allpass phase equalizer is proposed. A design example for the discussed phase equalizers is given in Sec. 5. The results are summarized in Sec. 6.

2. ALLPASS TRANSFORMED FILTER-BANKS

Many algorithms for adaptive sub-band filtering are employing an *analysis-synthesis filter-bank* (AS FB) as shown in Fig. 1. The M spectral gains $W_i(k')$ are calculated at a re-

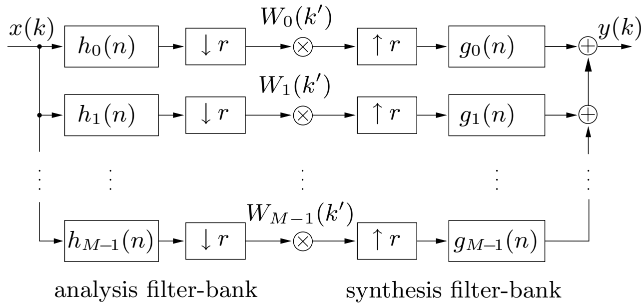

Figure 1: *M-channel analysis-synthesis filter-bank (AS FB) with down-sampling and spectral weighting.*

duced sampling rate where $k = r\,k'$. For the frequently used uniform DFT AS FB, the complex modulated analysis and synthesis FIR filters are given by

$$\begin{aligned} h_i(n) &= h_0(n)\, e^{j\,\frac{2\pi}{M}\,n\,i} \\ g_i(n) &= g_0(n)\, e^{-j\,\frac{2\pi}{M}\,n\,i} \\ n &= 0,1,\ldots,L \ \ ; \ \ i = 0,1,\ldots,M-1\,. \end{aligned} \tag{1}$$

These filters allow an efficient polyphase network implementation of the FB, e.g., [9]. The z-transform of the output sequence $y(k)$ (for time-invariant gains W_i) can be written as

$$Y(z) = X(z)\,\frac{1}{r}\sum_{i=0}^{M-1} H_i(z)\,G_i(z)\,W_i + \mathcal{E}_A(z)\,. \tag{2}$$

The term $\mathcal{E}_A(z)$ represents non-linear aliasing distortions due to the down-sampling operation. It will be assumed that they are negligible ($\mathcal{E}_A(z) \approx 0$), which can be achieved by an appropriate prototype filter design and non-critically downsampling. For a paraunitary AS FB (cf. [9]), the frequency response can be expressed by

$$F_{\mathrm{AS\ FB}}(e^{j\,\Omega}) = \frac{Y(e^{j\,\Omega})}{X(e^{j\,\Omega})} = e^{-j\,L\,\Omega}\,\frac{1}{r}\sum_{i=0}^{M-1}|H_i(e^{j\,\Omega})|^2\,W_i \tag{3}$$

$$= e^{-j\,L\,\Omega}\,A_{\mathrm{AS\ FB}}(e^{j\,\Omega})\,. \tag{4}$$

For applications such as speech enhancement, only a modification of the magnitude spectrum is performed, e.g., [10]. In this case, the gains W_i are real and positive such that the AS FB has a linear phase response $\varphi_{\mathrm{AS\ FB}}(\Omega) = L\,\Omega$.

Aliasing distortions are avoided at all ($\mathcal{E}_A(z) = 0$), if no down-sampling is employed ($r = 1$) for the FB of Fig. 1. In this case, the output signal $y(k)$ can be obtained by summation of all weighted sub-band signals, that is, $G_i(z) = 1$. In [11],[8], an efficient implementation of this FB structure, termed as *filter-bank equalizer* (FBE), has been proposed. If the prototype lowpass $h_0(n)$ of Eq. (1) is of linear phase and the gains $W_i(k')$ constitute a linear phase DFT spectrum, the frequency response of the FBE is given by [8]

$$F_{\mathrm{FBE}}(e^{j\,\Omega}) = e^{-j\,\frac{L}{2}\,\Omega}\,A_{\mathrm{FBE}}(e^{j\,\Omega})\,. \tag{5}$$

The function $A_{\mathrm{FBE}}(e^{j\,\Omega})$ is real such that the frequency response has a generalized linear phase[1] (cf. [12]). A possible application of this filter(-bank) structure are speech enhancement systems with low signal delay [11],[13].

A technique to obtain a filter(-bank) with non-uniform frequency resolution is to employ digital frequency warping by means of an *allpass transformation* [1],[14]. Thereby, the delay elements of the discrete (sub-band) filters are substituted by allpass filters

$$z^{-1} \;\rightarrow\; H_A(z)\,. \tag{6}$$

For this allpass transformation, a complex causal allpass filter of first order with

$$H_A(z) = \frac{1-a^*z}{z-a} = \Big|_{z=e^{j\,\Omega}} \frac{1-a^*e^{j\,\Omega}}{e^{j\,\Omega}-a} = e^{-j\,\varphi_a(\Omega)} \tag{7}$$
$$|a| < 1 \;\;;\;\; a = \alpha\, e^{j\,\gamma} \in \mathbb{C}$$

will be regarded. (The asterisk * denotes the complex conjugate.) The phase response can be expressed by

$$\varphi_a(\Omega) = 2\,\arctan\left(\frac{\sin\Omega - \alpha\,\sin\gamma}{\cos\Omega - \alpha\,\cos\gamma}\right) - \Omega\,. \tag{8}$$

For the uniform FB with frequency response of Eq. (4), the allpass transformation according to Eq. (6) leads to

$$\widetilde{F}_{\mathrm{AS\ FB}}(e^{j\,\Omega}) = e^{-j\,L\,\varphi_a(\Omega)}\,A_{\mathrm{AS\ FB}}(e^{j\,\varphi_a(\Omega)}) \tag{9}$$

and correspondingly for the FBE of Eq. (5). Thus, the allpass transformation of a FB causes a frequency warping, which is solely determined by the phase response according to Eq. (8). The uniform FB is included as special case for an allpass coefficient $a\!=\!0$ since then $H_A(z)\!=\!z^{-1}$.

The frequency warping produces two undesirable side-effects. The first one is that (noticeable) aliasing distortions might occur ($\mathcal{E}_A \neq 0$). This effect can be reduced by a lower down-sampling factor r than for the uniform FB, e.g, [3],[4].

A second effect is that the overall phase response of the FB becomes non-linear. This can become perceivable for audio processing. For a long prototype filter degree L, the filtered audio signal might sound 'reverberant'. An approach to reduce this effect is to filter the FB output sequence $y(k)$ by means of a *phase equalizer* as sketched in Fig. 2. The task of the phase equalizer with frequency response $P(e^{j\,\Omega})$ is to compensate the non-linear phase response of the warped FB. The actual *transfer function*[2] $T(e^{j\,\Omega})$ should be equal to a desired transfer function $T_d(e^{j\,\Omega})$ with linear phase and constant magnitude response

$$T(e^{j\,\Omega}) = e^{-j\,L_c\varphi_a(\Omega)}\cdot P(e^{j\,\Omega}) \tag{10}$$
$$\stackrel{!}{=} C_T\cdot e^{-j\,\tau_d\,\Omega} = T_d(e^{j\,\Omega})\,. \tag{11}$$

[1] The AS FB has a generalized phase as well for real gains W_i, or if $H_i = H^*_{M-i}$ and $W_i = W^*_{M-i}$ for Eq. (4).

[2] This term is also used with different meaning in literature.

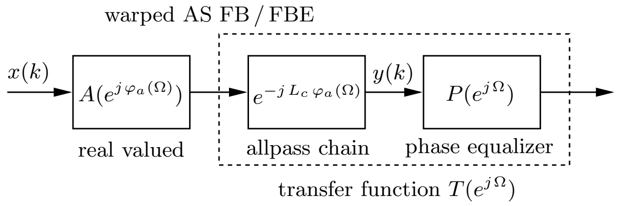

Figure 2: *Phase equalization of a warped filter-bank.*

The value for L_c depends on the underlying FB. For the two examples discussed before, the value is given by $L_c = L$ for the warped AS FB of Eq. (9), and $L_c = L/2$ for the warped FBE. The *nominal group-delay* τ_d (signal delay) depends on the used phase equalizer and its filter degree N_p, as well as the value for L_c. A positive value for τ_d is needed to ensure a causal phase equalizer. Magnitude and phase response of the transfer function are denoted by

$$T(e^{j\,\Omega}) = |T(e^{j\,\Omega})|\,e^{-j\,\varphi_T(\Omega)}\,. \tag{12}$$

The group-delay is calculated by $\tau_T(\Omega) = \frac{\partial\,\varphi_T(\Omega)}{\partial\,\Omega}$. The 'ideal' phase equalizer is obviously obtained for

$$P_{\mathrm{ideal}}(e^{j\,\Omega}) = e^{j\,L_c\,\varphi_a(\Omega)} \tag{13}$$

such that $T(e^{j\,\Omega}) = 1$. The impulse response of this phase equalizer is infinite and anti-causal (for $a \neq 0$), i.e., $p_{\mathrm{ideal}}(n) = 0$ for $n > 0$. Thus, Eq. (11) can only be approximately fulfilled in practice. In [15], a segment-wise processing scheme for the approximate realization of anti-causal filters is proposed. Here, phase equalizers for sample-wise processing will be regarded. Different phase equalizer designs and the resulting approximation errors are discussed in the following.

3. FIR PHASE EQUALIZERS

Two parametric phase equalizers with finite impulse response (FIR) will be investigated whose coefficients are given by a closed-form expression in dependence on the allpass pole a.

3.1 Equiripple FIR Phase Equalizer

The expression $\exp\{-j\,L_c\,\varphi_a(\Omega)\}$ in Eq. (10) represents an allpass chain, i.e., a cascade of L_c allpass filters given by Eq. (7). Therefore, the phase equalizer can be realized by a cascade of single phase equalizers $P_0(e^{j\,\Omega})$ designed for a single allpass filter $H_A(e^{j\,\Omega})$. This approach can be written as

$$\begin{aligned} T(e^{j\,\Omega}) &= e^{-j\,L_c\varphi_a(\Omega)}\cdot P(e^{j\,\Omega}) \\ &= \Big(H_A(e^{j\,\Omega})\cdot P_0(e^{j\,\Omega})\Big)^{L_c} = \Big(T_0(e^{j\,\Omega})\Big)^{L_c}\,. \end{aligned} \tag{14}$$

The filter degree of the single phase equalizer will be marked by N_0 to differentiate it from the degree $N_p = L_c\,N_0$ of the overall phase equalizer. An FIR phase equalizer for an allpass filter of first order and real pole $a = \alpha$ has been proposed in [7]. The phase equalization of a complex allpass filter is achieved accordingly by

$$\underbrace{\frac{1-a^*\,z}{z-a}}_{H_A(z)}\cdot\underbrace{(z-a)\,\sum_{i=0}^{N_0-1} z^{i-N_0}\,(a^*)^i}_{P_{0\,\mathrm{FIR}}(z)} = \underbrace{z^{-N_0} - (a^*)^{N_0}}_{T_{0\,\mathrm{FIR}}(z)}\,. \tag{15}$$

Such a phase equalizer can also be used for phase equalization of a *general allpass filter* of degree N_a

$$H_A^{[N_a]}(z) = \prod_{i=1}^{N_a} \frac{1 - a_i^* z}{z - a_i} = e^{-j\varphi_a^{[N_a]}(\Omega)} \tag{16}$$

$$|a_i| < |z| \;\; ; \;\; |a_i| < 1 \;\; ; \;\; a_i = \alpha_i \, e^{j\gamma_i} \in \mathbb{C}$$

by means of a cascade of N_a different single phase equalizers.

The phase equalizer of Eq. (15) leads to the system response

$$T_{\mathrm{FIR}}(z) = (T_{0\,\mathrm{FIR}}(z))^{L_c} = \left(z^{-N_0} - (a^*)^{N_0}\right)^{L_c} \; . \tag{17}$$

The 'error term' $(a^*)^{N_0}$ can be made arbitrarily small by increasing the filter degree N_0 since $|a| < 1$ according to Eq. (7). The magnitude response for the transfer function according to Eq. (12) can be calculated as[3]

$$|T(e^{j\Omega})| = \left(1 - 2\,\alpha^{N_0}\,\cos(N_0\,(\Omega - \gamma)) + \alpha^{2\,N_0}\right)^{\frac{L_c}{2}} \; . \tag{18}$$

It can be shown that the approximation error for Eq. (11)

$$\Delta|T(e^{j\Omega})| = |T(e^{j\Omega})| - C_T \tag{19}$$

$$\text{with} \quad C_T = \frac{1}{2}\left((1 - \alpha^{N_0})^{L_c} + (1 + \alpha^{N_0})^{L_c}\right) \tag{20}$$

possesses alternating extrema of equal magnitude

$$\Delta|T(e^{j\Omega_i})| = -\Delta|T(e^{j\Omega_{i+1}})| \;\; ; \;\; i = 0, 1, \ldots, 2\,N_0 - 1 \tag{21}$$

$$\Omega_i = \frac{i\,\pi}{N_0} + \gamma \;\; ; \;\; i = 0, 1, \ldots, 2\,N_0 \tag{22}$$

within the frequency interval $\Omega \in [\gamma, 2\pi + \gamma]$. The $2\,N_0 + 1$ extrema at Ω_i constitute an *alternate* where

$$\max_{\Omega} \{|\,\Delta|T(\Omega)|\,|\} = ||\,|T(\Omega)|\,||_\infty = |\,\Delta|T(\Omega_i)|\,| \; . \tag{23}$$

Thus, the regarded FIR phase equalizer yields an equiripple error for the magnitude response of the transfer function $T(e^{j\Omega})$. The phase response of the transfer function

$$\varphi_T(\Omega) = L_c \arctan\left(\frac{\sin(N_0\,\Omega) - \alpha^{N_0}\,\sin(N_0\,\gamma)}{\cos(N_0\,\Omega) - \alpha^{N_0}\,\cos(N_0\,\gamma)}\right) \tag{24}$$

yields the approximation error function

$$\Delta\varphi_T(\Omega) = \varphi_T(\Omega) - L_c\,N_0\,\Omega \tag{25}$$

which possesses an alternate as well with the extrema

$$\Delta\varphi_T(\Omega_i) = -\Delta\varphi_T(\Omega_{i+1}) \;\; ; \;\; i = 0, 1 \ldots, 2\,N_0 - 1 \tag{26}$$

$$\Omega_i = \frac{1}{N_0}\left(2\,\pi\,\lfloor i/2 \rfloor - (-1)^i \arccos\left(\alpha^{N_0}\right)\right) + \gamma \tag{27}$$

$$i = 0, 1 \ldots, 2\,N_0$$

for $\Omega \in [\Omega_0, \Omega_{2N_0}]$. (The operation $\lfloor x \rfloor$ provides the greatest integer which is equal or smaller than x.) The nominal group-delay for Eq. (11) amounts to $\tau_d = N_p = L_c\,N_0$. It can be shown that the group-delay of the transfer function

$$\tau_T(\Omega) = L_c\,N_0\,\frac{1 - \alpha^{N_0}\,\cos(N_0\,(\Omega - \gamma))}{1 - 2\,\alpha^{N_0}\,\cos(N_0\,(\Omega - \gamma)) + \alpha^{2\,N_0}} \tag{28}$$

also exhibits an equiripple approximation error

$$\Delta\tau_T(\Omega) = \tau_T(\Omega) - \frac{L_c\,N_0}{1 - \alpha^{2\,N_0}} \tag{29}$$

with extrema at

$$\Omega_i = \frac{\pi\,i}{N_0} + \gamma \;\; ; \;\; i = 0, 1, \ldots, 2\,N_0 \; . \tag{30}$$

Thus, the approximation error functions for the magnitude response, phase response and group-delay between the desired and obtained transfer function show an equiripple behavior[4], which was not shown in [7]. Hence, this filter will be termed as *equiripple FIR phase equalizer*.

3.2 Least-Squares FIR Phase Equalizer

The 'ideal' phase equalizer of Eq. (13) can be approximated by truncating and shifting of its impulse response $p_{\mathrm{ideal}}(n)$. This filter design by windowing (cf. [12]) results in a causal FIR phase equalizer of degree N_p with impulse response

$$p_{\mathrm{FIR}}(n) = \begin{cases} p_{\mathrm{ideal}}(n - N_p) & ; \;\; n = 0, 1, \ldots, N_p \\ 0 & ; \;\; \text{else} \, . \end{cases} \tag{31}$$

The least-squares norm (2-norm) of the approximation error between the frequency response of the ideal phase equalizer and its approximation

$$||\Delta P_{\mathrm{FIR}}(e^{j\Omega})||_2 = ||P_{\mathrm{FIR}}(e^{j\Omega}) - P_{\mathrm{ideal}}(e^{j\Omega})\,e^{-j\Omega\,N_p}||_2 \tag{32}$$

becomes minimal in this case, cf. [16]. The nominal group-delay amounts to $\tau_d = N_p$. The use of this least-squares (LS) FIR phase equalizer for an allpass transformed DFT AS FB can be found in [3],[4]. The application for the allpass transformed filter-bank equalizer is proposed in [8].

4. ALLPASS PHASE EQUALIZERS

FIR phase equalizers always introduce amplitude distortions. This can be avoided by the use of allpass phase equalizers.

4.1 General Allpass Phase Equalizers

The design of allpass phase equalizers to approximate a prescribed phase response (and group-delay) has been proposed, e.g., in [5],[6]. One application of such phase equalizers is the construction of recursive filters with linear phase response.

The regarded phase equalizer is given by a real allpass filter of degree $N_a = N_p$ according to Eq. (16). The N_p real allpass coefficients α_i can be determined by the requirement

$$\varphi_d(\Omega_i) \stackrel{!}{=} \varphi_\alpha^{[N_p]}(\Omega_i) \;\; ; \;\; i = 1, 2, \ldots, K_f \;\; ; \;\; K_f \geq N_a \, . \tag{33}$$

The desired phase response is given here by $\varphi_d(\Omega_i) = \tau_d\,\Omega_i - L_c\,\varphi_a(\Omega_i)$ with τ_d to be chosen appropriately. The solution of Eq. (33) yields a set of K_f linear equations, which is overdetermined for $K_f > N_p$. In [5], an iterative algorithm is proposed to obtain an unbiased least-squares error solution. This design will be referred to as general *LS allpass phase equalizer*.

An equiripple design has been published in [6]. The coefficients of the allpass filter are obtained by a modified Remez iteration. The extrema of the phase error function are equal to the Chebyshev norm ($\infty-$norm) of the phase error within a given range of frequencies. A stable allpass filter cannot

[3] The indication of the used phase equalizer by a suffix will be omitted if obvious from the context.

[4] This property has been found here by analysis of a given filter but not by solving a Chebyshev approximation for a set of filters based on the alternate theorem, cf. [16].

be guaranteed in general for these two design methods, but has been obtained for many design examples [5],[6].

For the phase equalization of an allpass chain, the design of a single allpass phase equalizer can lead to numerical problems due to the large number of coefficients. To avoid this, the phase equalization of an allpass chain can be done by a cascade of single phase equalizers according to Eq. (14).

It should be noted that the design of an optimal phase equalizer for a given approximation criterion does not imply that this also provides an optimal group-delay equalizer for the same criterion, cf. [5],[6].

4.2 New Equiripple Allpass Phase Equalizer

A closed-form solution for a stable allpass phase equalizer should be developed. The phase response of the allpass filter of Eq. (7) can be 'linearized' by a second allpass filter with the same pole but of opposite sign according to

$$\frac{1-a^*z}{z-a}\,\frac{1+a^*z}{z+a} = \frac{1-(a^*z)^2}{z^2-a^2}\,. \qquad (34)$$

This principle can be applied to the obtained allpass of second order once again by multiplication with an allpass filter given by $(1+(a^*z)^2)/(z^2+a^2)$. Employing this kind of phase equalization d times yields

$$P_{0\mathrm{AP}}(z) = \prod_{i=0}^{d-1} \frac{1+(a^*z)^{2^i}}{z^{2^i}+a^{2^i}} \quad ; \quad d \in \{1,2,3,\ldots\} \qquad (35)$$

which is an *allpass phase equalizer* of degree $N_0 = 2^d - 1$. A general allpass phase equalizer of degree $N_0 = 2^d - 1$ according to Eq. (16) needs $2\,N_0$ multipliers, $2\,N_0$ adders and N_0 delay elements. The new parametric allpass phase equalizer only needs $2\,d$ adders, $2\,d$ multipliers and $2^d - 1$ delay elements. Moreover, no complex numerical computation of its coefficients is required.

With Eq. (35), the single system response for Eq. (14) can be written as

$$T_{0\mathrm{AP}}(z) = H_A(z)\cdot P_{0\mathrm{AP}}(z) = \frac{1-(a^*z)^D}{z^D-a^D} \quad ; \quad D = 2^d \qquad (36)$$

which is an allpass filter of degree $D = 2^d$. Since $|a| < 1$, this filter is always stable and tends to a pure delay z^{-D} for an increasing filter degree D.

An interesting link between the transfer functions obtained by the equiripple FIR phase equalizer of Sec. 3.1 and those obtained by the new allpass phase equalizer can be observed. The system response $T_{\mathrm{FIR}}(z)$ of Eq. (17) for $N_0 = D$ can be decomposed into the allpass system response $T_{\mathrm{AP}}(z) = (T_{0\mathrm{AP}}(z))^{L_c}$ derived from Eq. (36) and a minimum phase filter $H_{\min}(z)$ according to

$$\underbrace{(z^{-D}-(a^*)^D)^{L_c}}_{T_{\mathrm{FIR}}(z)} = \underbrace{\left(\frac{z^{-D}-(a^*)^D}{1-a^D\,z^{-D}}\right)^{L_c}}_{T_{\mathrm{AP}}(z)} \cdot \underbrace{(1-a^D\,z^{-D})^{L_c}}_{H_{\min}(z)}\,. \qquad (37)$$

The phase response and group-delay of the transfer function $T_{\mathrm{AP}}(e^{j\,\Omega})$ can be expressed by

$$\varphi_T(\Omega) = 2\,L_c\,\mathrm{arctan}\left(\frac{\sin(D\,\Omega)-\alpha^D\,\sin(D\gamma)}{\cos(D\,\Omega)-\alpha^D\,\cos(D\gamma)}\right) - L_c\,D\,\Omega \qquad (38)$$

$$\tau_T(\Omega) = L_c\,D\,\frac{1-\alpha^D}{1-2\,\alpha^D\,\cos(D\,(\Omega-\gamma))+\alpha^{2\,D}}\,. \qquad (39)$$

The nominal group-delay is given by $\tau_d = L_c\,D$. It can be shown that the approximation error between the achieved phase and the desired phase

$$\Delta\varphi_T(\Omega) = \varphi_T(\Omega) - L_c\,D\,\Omega \qquad (40)$$

possesses an alternate with extrema

$$\Delta\varphi_T(\Omega_i) = -\Delta\varphi_T(\Omega_{i+1}) \quad ; \quad i = 0,1,\ldots,2D-1 \qquad (41)$$

$$\Omega_i = \frac{1}{D}\left(2\,\pi\,\lfloor i/2\rfloor - (-1)^i \mathrm{arccos}\left(\alpha^D\right)\right) + \gamma \qquad (42)$$

$$i = 0,1,\ldots,2D$$

for $\Omega \in [\Omega_0, \Omega_{2D}]$. Accordingly, the group-delay error

$$\Delta\tau_T(\Omega) = \tau_T(\Omega) - L_c\,D\,\frac{1+\alpha^{2\,D}}{1-\alpha^{2\,D}} \qquad (43)$$

possesses an alternate at the extremal points

$$\Omega_i = \frac{\pi\,i}{D} + \gamma \quad ; \quad i = 0,1,\ldots,2D \qquad (44)$$

for $\Omega \in [\gamma, 2\pi+\gamma]$. Therefore, the approximation errors of the phase response $\Delta\varphi_T(\Omega)$ and the group-delay $\Delta\tau_T(\Omega)$ for the desired transfer function $T_d(e^{j\,\Omega}) = \exp\{-j\,L_c\,D\,\Omega\}$ exhibit an *equiripple behavior.* The extremal frequencies Ω_i are the same as for the equiripple FIR phase equalizer. Comparison of the calculated error functions $\Delta\varphi_T(\Omega)$ and $\Delta\tau_T(\Omega)$ also reveals that the allpass phase equalizer has a higher phase and group-delay error than the equiripple FIR phase equalizer, but avoids magnitude distortions.

The proposed phase equalizer can also be used in cascade for the phase equalization of a general allpass filter according to Eq. (16).

5. DESIGN EXAMPLE

The phase equalization of an allpass transformed DFT AS FB with $L = M = 64$ for Eq. (1) and allpass coefficient $a = 0.5$ is regarded (cf. Fig. 2). This FB configuration requires the phase equalization of an allpass chain with $L_c = L = 64$ according to Eq. (9) and Eq. (10). This warped AS FB has a higher frequency resolution for low frequencies and vice versa, and approximates the Bark scale for a sampling frequency of 11.8 kHz, cf. [1],[2]. A typical use for such a FB are noise reduction systems, which are employed, e.g., in mobile phones or hearing aids. For such applications, a low computational complexity and low signal delay is required. An almost linear phase response is not mandatory since the human ear is relatively insensitive towards phase distortions [10]. Therefore, a comparatively low degree N_p has been chosen for the phase equalizers such that all systems achieve a nominal group-delay (signal delay) of $\tau_d = 128$. (The underlying uniform AS FB has a signal delay of $\tau_d = L = 64$.) Apart from the LS FIR phase equalizer, all phase equalizers are realized by a cascade of $L_c = 64$ single phase equalizers according to Eq. (14). The coefficients of the LS allpass phase equalizer [5] have been calculated by 4 iterations for $K_f = 50$ frequency points. The general equiripple (or Chebyshev) allpass phase equalizer [6] provides no full-band design and, hence, has not been considered here. The results of the phase equalization are shown in Fig. 3.

The plots show the differences between the approximation errors for the equiripple phase equalizers and the LS phase equalizers. The FIR phase equalizers yield a lower phase error than the allpass phase equalizers. The LS FIR phase equalizer causes significantly lower magnitude distortions than the equiripple FIR phase equalizer. Magnitude distortions are avoided at all, if an allpass phase equalizer is

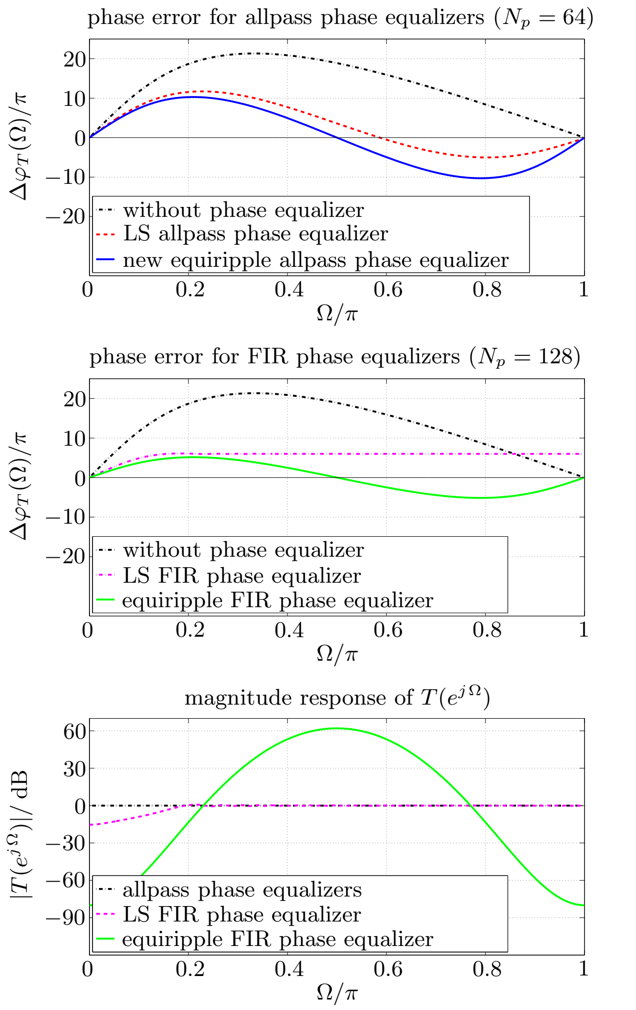

Figure 3: *Approximation errors of the phase and magnitude response of* $T(e^{j\Omega})$ *obtained for phase equalization of an allpass chain of length* $L_c = 64$ *and* $a = 0.5$. *The equiripple FIR phase equalizer (Sec. 3.1), the LS FIR phase equalizer (Sec. 3.2), the (general) LS allpass phase equalizer (Sec. 4.1), and the new equiripple allpass phase equalizer (Sec. 4.2) are used with* $\tau_d = 128$.

used. Thereby, it should be noted that the proposed allpass phase equalizer provides a closed-form solution in contrast to the design procedure for the general allpass phase equalizer.

Clearly, if a higher signal delay and computational complexity can be allowed, the approximation error for the phase equalization can be decreased by increasing the degree of the phase equalizer N_p. In this case, the new allpass phase equalizer has a significantly lower computational complexity than a general allpass phase equalizer (cf. Sec. 4.2).

6. CONCLUSIONS

Parametric phase equalizers have been investigated, which can be used to obtain a warped FB with linear phase transfer function. The analysis of the FIR phase equalizer of [7] has shown that the approximation error for the magnitude response, phase response and group-delay between the desired and the obtained transfer function show an equiripple behavior. For phase equalization of an allpass chain (warped FB), the approximation error obtained by this equiripple FIR phase equalizer is higher compared to the discussed LS FIR phase equalizer for the same filter degree.

An FIR phase equalizer can achieve a low phase and group-delay error, but introduces magnitude distortions. These distortions can be avoided by using an allpass phase equalizer. A new closed-form solution for a stable allpass phase equalizer has been proposed. The approximation error of the phase response and group-delay for the desired transfer function show an equiripple behavior. The new allpass phase equalizer has a lower computational complexity than a general allpass phase equalizer.

An application of the discussed parametric phase equalizers are warped FBs used for adaptive filtering. Allpass phase equalizers are an interesting choice for speech enhancement systems where phase distortions are often less problematic than magnitude distortions.

REFERENCES

[1] A. V. Oppenheim, D. H. Johnson, and K. Steiglitz, “Computation of Spectra with Unequal Resolution Using the Fast Fourier Transform,” *Proc. of the IEEE*, vol. 59, no. 2, pp. 299–301, Feb. 1971.

[2] J. O. Smith and J. S. Abel, “Bark and ERB Bilinear Transforms,” *IEEE Transactions on Speech and Audio Processing*, vol. 7, no. 6, pp. 697–708, Nov. 1999.

[3] A. Engelsberg, *Transformationsbasierte Systeme zur einkanaligen Störunterdrückung bei Sprachsignalen*, Ph.D. thesis, Christian-Albrechts-Universität, Kiel, 1998, (in German).

[4] T. Gülzow, A. Engelsberg, and U. Heute, “Comparison of a discrete wavelet transformation and a nonuniform polyphase filterbank applied to spectral-subtraction speech enhancement,” *Signal Processing, Elsevier*, vol. 64, pp. 5–19, Jan. 1998.

[5] M. Lang and T. I. Laakso, “Simple and Robust Method for the Design of Allpass Filters Using Least-Squares Phase Error Criterion,” *IEEE Transactions on Circuits and Systems II*, vol. 41, no. 1, pp. 40–48, Jan. 1994.

[6] M. Lang, “Optimal weighted phase equalization according to the L_∞-norm,” *Signal Processing, Elsevier*, vol. 27, pp. 87–98, 1992.

[7] E. Galijašević and J. Kliewer, “Design of Allpass-Based Non-Uniform Oversampled DFT Filter Banks,” in *Proc. of Intl. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, Orlando, USA, May 2002, pp. 1181–1184.

[8] H. W. Löllmann and P. Vary, “Efficient Non-Uniform Filter-Bank Equalizer,” in *Proc. of European Signal Processing Conf. (EUSIPCO)*, Antalya, Turkey, Sept. 2005.

[9] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice-Hall, Upper Saddle River, New Jersey, 1993.

[10] P. Vary, “Noise Suppression by Spectral Magnitude Estimation - Mechanism and Theoretical Limits -,” *Signal Processing, Elsevier*, vol. 8, no. 4, pp. 387–400, July 1985.

[11] P. Vary, “An adaptive filter-bank equalizer for speech enhancement,” *Signal Processing, Elsevier, Special Issue on Applied Signal and Audio Processing*, vol. 86, pp. 1206–1214, 2006.

[12] A. V. Oppenheim, R. W. Schafer, and J. R. Buck, *Discrete-Time Signal Processing*, Prentice-Hall, Upper Saddle River, New Jersey, 2nd edition, 1999.

[13] H. W. Löllmann and P. Vary, “Generalized Filter-Bank Equalizer for Noise Reduction with Reduced Signal Delay,” in *Proc. of European Conf. on Speech Communication and Technology (Interspeech)*, Lisbon, Portugal, Sept. 2005.

[14] P. Vary, “Digital Filter Banks with Unequal Resolution,” in *Short Communiation Digest of European Signal Processing Conf. (EUSIPCO)*, Lausanne, Switzerland, Sept. 1980, pp. 41–45.

[15] R. Czarnach, “Recursive Processing by Noncausal Digital Filters,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 30, no. 3, pp. 363–370, June 1982.

[16] T. W. Parks and C. S. Burrus, *Digital Filter Design*, John Wiley & Sons, Inc., New York, 1987.