IMPROVED DESIGN OF OVERSAMPLED ALLPASS TRANSFORMED DFT FILTER-BANKS WITH NEAR-PERFECT RECONSTRUCTION

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ABSTRACT

In this paper, the design of oversampled allpass transformed analysis-synthesis filter-banks is treated, which can be employed for non-uniform subband processing. Near-perfect reconstruction can be achieved by a synthesis filter-bank design which uses phase equalizers to compensate the phase modifications due to the allpass transformation of the analysis filter-bank. It is shown that an FIR phase equalizer, which is obtained by a least-squares approximation of the desired non-causal IIR phase equalizer, leads to almost perfect signal reconstruction and causes only a comparatively low signal delay. The devised synthesis filter-bank design can be described by a closed-form expression and allows to control the trade-off between reconstruction errors on the one hand and signal delay and computational complexity on the other hand.

1. INTRODUCTION

An approach to obtain an analysis-synthesis filter-bank (AS FB) with non-uniform frequency resolution is to employ digital frequency warping by means of an allpass transformation [1–4]. Such a frequency warped filter-bank can model the Bark frequency scale with great accuracy [5], which is beneficial for audio and speech processing applications, e.g., [6,7].

The straightforward synthesis filter-bank design for an allpass transformed filter-bank with perfect reconstruction (PR) leads to either unstable or anti-causal synthesis filters. For the allpass transformation of first order, a closed-form solution for an FIR synthesis filter-bank to achieve PR can be derived. This is shown in [8] for the critically subsampled warped DFT filter-bank and in [9,10] for arbitrary subsampling rates. However, the obtained synthesis subband filters exhibit an insufficient bandpass characteristic, which becomes problematic if subband processing takes place [10].

An FIR synthesis filter-bank design to obtain an oversampled allpass transformed DFT AS FB with near-perfect reconstruction has been proposed by Galijašević and Kliewer [11]. This approach has the advantage that the trade-off between tolerable reconstruction errors on the one hand and algorithmic complexity and signal delay on the other hand, can be controlled by the synthesis filter-bank design. Moreover, the synthesis subband filters possess a distinctive bandpass characteristic.

In this contribution, it is shown that this concept can be significantly improved by using a different phase equalizer design for the synthesis filter-bank without loosing the benefits of the original approach.

The paper is organized as follows: In Section 2, the construction of filter-banks with non-uniform frequency resolution by using an allpass transformation is treated. The synthesis filter-bank design is presented in Section 3 and suitable phase equalizers are investigated. Section 4 provides a design example and the paper concludes with Section 5.

2. ALLPASS TRANSFORMED DFT ANALYSIS FILTER-BANK

Digital frequency warping by means of allpass transformation is a well known technique to obtain a filter-bank with non-uniform frequency-bands [1–4]. The frequency transformation is achieved by replacing the delay elements of the subband filters with allpass filters

$$z^{-1} \;\rightarrow\; H_A(z)\,. \tag{1}$$

For this *allpass transformation*, a complex allpass filter of first order is commonly used. This filter has the z-transform

$$H_A(z) = \frac{1-a^* z}{z-a} \tag{2}$$

$$|z| > |a|; \quad |a| < 1 \;\; ; \;\; a = \alpha\, e^{j\gamma} \in \mathbb{C}$$

and frequency response

$$H_A(e^{j\Omega}) = \frac{1-a^* e^{j\Omega}}{e^{j\Omega}-a} = e^{-j\,\varphi_a(\Omega)}\,. \tag{3}$$

(The asterisk * denotes the complex conjugate.) The phase response can be expressed by

$$\varphi_a(\Omega) = 2\,\arctan\left(\frac{\sin\Omega - \alpha\,\sin\gamma}{\cos\Omega-\alpha\,\cos\gamma}\right) - \Omega\,. \tag{4}$$

The allpass transformation of the subband filters of a uniform DFT analysis filter-bank leads to the transfer functions

$$\widetilde{H}_i(z) = \sum_{n=0}^{L} h(n)\, W_M^{n\,i}\, H_A(z)^n; \quad i = 0,1,\ldots,M-1 \tag{5}$$

$$\text{with} \quad W_M = \exp\left\{-j\,\frac{2\pi}{M}\right\}. \tag{6}$$

Without loss of generality, the length of the finite impulse response (FIR) of the analysis prototype filter $h(n)$ and synthesis prototype filter $g(n)$ is assumed to be an integer multiple of M, that is, $L+1 = l_M\, M$. The analysis filter-bank can be implemented efficiently by a (type 1) polyphase network (PPN) according to

$$\widetilde{H}_i(z) = \sum_{\lambda=0}^{M-1} H_\lambda^{(M)}\left(H_A(z)^M\right)\, W_M^{\lambda\,i}\, H_A(z)^\lambda \tag{7}$$

$$H_\lambda^{(M)}\left(H_A(z)^M\right) = \sum_{m=0}^{l_M-1} h(m\,M+\lambda)\, H_A(z)^{m\,M}. \tag{8}$$

A diagram of the described analysis filter-bank is provided by Fig. 1. The discrete Fourier transform (DFT) can be cal-

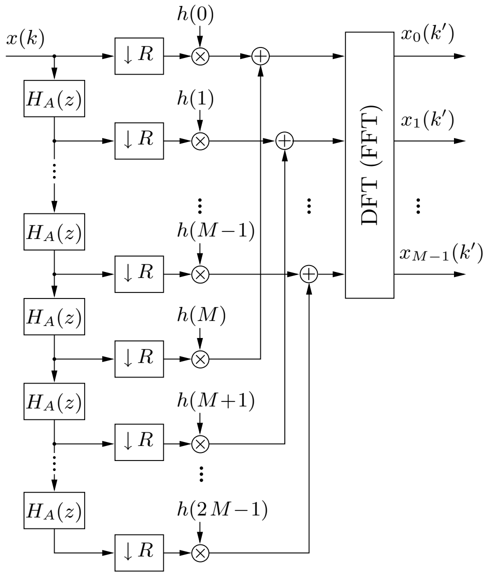

Figure 1: *Polyphase network implementation of the allpass transformed DFT analysis filter-bank with downsampling and* $L = 2M - 1$.

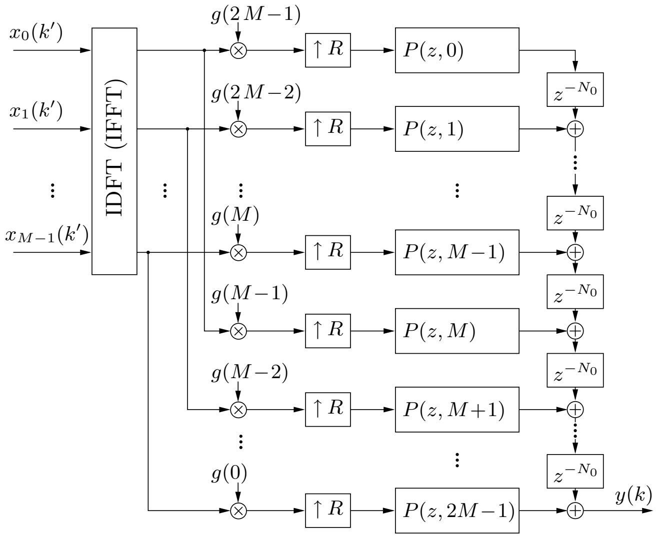

Figure 2: *Polyphase network implementation of the DFT synthesis filter-bank with upsampling and* $L = 2M - 1$.

culated efficiently by the fast Fourier transform (FFT), e.g., [12]. The downsampling by R can be performed directly after the allpass chain, which is not possible if individual downsampling rates R_i are considered for each subband signal $x_i(k')$. The uniform PPN DFT analysis filter-bank (cf. [13]) is included as special case for $a = 0$ where $H_A(z) = z^{-1}$ according to Eq. (2). Due to the allpass transformation, the frequency responses for the uniform subband filters $H_i(e^{j\Omega})$ are now frequency warped with

$$\widetilde{H}_i(e^{j\Omega}) = H_i(e^{j\varphi_a(\Omega)})\,. \tag{9}$$

This leads to non-uniform frequency bands without requiring a renewed prototype filter design.

3. SYNTHESIS FILTER-BANK DESIGN

3.1 General Structure

The synthesis filter-bank design of [11] is shown in Fig. 2. The M subband filters of the synthesis filter-bank are given by

$$\bar{G}_i(z) = \sum_{\lambda=0}^{M-1} Q^{(M)}_{M-1-\lambda}(z)\, z^{-(M-1-\lambda)\,N_0}\, W_M^{-\lambda\, i} \qquad i = 0,1,\ldots,M-1 \tag{10}$$

with the 'modified' (type 1) polyphase components[1]

$$Q^{(M)}_\lambda(z) = \sum_{m=0}^{l_M-1} g(m\,M+\lambda)\, P\big(z, M\,(l_M-m)-1-\lambda\big) \times z^{-M\,N_0\,m};\quad \lambda = 0,1,\ldots,M-1\,. \tag{11}$$

(This modification of the synthesis filters in Eq. (10) is indicated by a bar.) The phase equalizers with transfer functions $P(z,n)$ for $n = 0,1,\ldots,L$ aim to compensate the phase modifications caused by the allpass transformation of the analysis filter-bank in order to achieve near-perfect signal reconstruction. The underlying principle can be demonstrated by looking at the non-subsampled AS FB ($R = 1$) whose prototype filters fulfill the requirement

$$\sum_{\lambda=0}^{M-1}\sum_{m=0}^{l_M-1}\sum_{k=0}^{l_M-1} h(m\,M+\lambda)\, g\left((k+1)\,M-1-\lambda\right) = \begin{cases}\frac{1}{M} & ;\ m+k+1 = l_M \in \mathbb{Z}\\ 0 & ;\ m+k+1 \in \mathbb{Z}\backslash\{l_M\}.\end{cases} \tag{12}$$

The transfer function of the filter-bank can now be written

$$\frac{Y(z)}{X(z)} = \sum_{n=0}^{L} h(n)\, g(L-n)\, H_A(z)^n\, P(z,n)\, z^{-(L-n)\,N_0} \tag{13}$$

if no modifications of the subband signals $x_i(k')$ are performed. The transfer function of allpass (filter) chain and phase equalizer

$$T_{AP}(z,n) = H_A(z)^n \cdot P(z,n);\quad n = 0,1,\ldots,L \tag{14}$$

should have a unit magnitude and linear phase response

$$T_{AP}(e^{j\Omega},n) = |T_{AP}(e^{j\Omega},n)|\, e^{-j\,\varphi_{AP}(\Omega,n)} \stackrel{!}{=} T_{\mathrm{des}}(e^{j\Omega},n) = e^{-j\,n\,N_0\,\Omega}\,. \tag{15}$$

The considered $L+1$ transfer functions $T_{AP}(z,n)$ cause different signal delays of $n\,N_0$ samples which is compensated by corresponding delays according to Eq. (13). These delay elements ceases to apply ($z^{-N_0} = 1$), if each transfer function $T_{AP}(z,n)$ has (approximately) the same linear phase response of $\varphi_{AP}(\Omega,n) = N_p\,\Omega$ for $n = 0,1,\ldots,L$.

If Eq. (15) can be fulfilled exactly, Eq. (13) simplifies to $Y(z)/X(z) = c \cdot z^{-d_0}$. Thereby, the overall signal delay of the filter-bank is given by $d_0 = L\,N_0$ or $d_0 = N_p$, respectively.

In general, the (finite energy) output signal $y(k)$ can be

[1]The original warped AS FB, cf. [4,6], uses an allpass transformed synthesis filter-bank which is obtained by the substitutions $z^{-N_0} \rightarrow H_A(z)$ and $P(z,n) \rightarrow 1$ for Eq. (10) and Eq. (11).

expressed in the z-domain according to (cf. [13])

$$Y(z) = X(z)\, \underbrace{\frac{1}{R} \sum_{i=0}^{M-1} \widetilde{H}_i(z)\, \bar{G}_i(z)}_{T_{\mathrm{lin}}(z)} + \sum_{\rho=1}^{R-1} X(z\, W_R^{\rho})\, \underbrace{\frac{1}{R} \sum_{i=0}^{M-1} \widetilde{H}_i(z\, W_R^{\rho})\, \bar{G}_i(z)}_{U_{\rho}(z)} \qquad (16)$$

if no spectral modifications are performed. Here, $T_{\mathrm{lin}}(z)$ marks the linear transfer function of the AS FB. The amount of aliasing distortions due to the subsampling can be expressed by the peak aliasing distortions [13]

$$\mathcal{E}_{A_{\mathrm{peak}}}(\Omega) = \sqrt{\sum_{\rho=1}^{R-1} |U_{\rho}(e^{j\,\Omega})|^2}\,. \qquad (17)$$

The aliasing distortions are limited by a sufficient stopband attenuation of the prototype filters, which becomes especially important if spectral modifications of the subband signals are performed.

For the treated warped AS FB, a suitable pair of prototype lowpass filters (with linear phase responses) is given by

$$h(n) = g(n) = \frac{\sqrt{R}}{2\,M} \left(1 - \sqrt{2}\cos\left(\frac{\pi}{M}(n + 0.5)\right)\right) \qquad (18)$$
$$n = 0, 1, \ldots, L; \quad L = 2\,M - 1$$

which yields perfect signal reconstruction, if no subsampling is performed ($R{=}1$) and if Eq. (15) is fulfilled, cf. [11].

3.2 Phase Equalizer Designs

The phase equalizer design is crucial for the above synthesis filter-bank concept. The 'ideal' phase equalizer to fulfill Eq. (15) is obviously given by the inverse transfer function of the allpass chain

$$P_{\mathrm{ideal}}(z, n) = H_A(z)^{-n} \qquad (19)$$
$$|z| < \frac{1}{|a|}; \quad n = 0, 1, \ldots, L\,.$$

The impulse response of this phase equalizer is infinite and anti-causal (for $a \neq 0$), that is, $p_{\mathrm{ideal}}(k, n) = 0$ for $k > 0$. An approach to implement anti-causal filters is to buffer the input samples in order to process them in time-reversed order [14, 15]. This buffering technique, however, leads to a very high signal delay and internal filter states must be transmitted to the synthesis filter-bank.

Design I

A buffering scheme is avoided by the FIR phase equalizer presented in [11], which considers the phase equalization of a single real allpass filter. The extension to a complex allpass filter is straightforward and can be expressed by

$$\underbrace{\frac{1 - a^*\, z}{z - a}}_{H_A(z)} \cdot \underbrace{(z - a) \sum_{l=0}^{N_0 - 1} z^{l - N_0} (a^*)^l}_{P_{\mathrm{equi}}(z,1)} = \underbrace{z^{-N_0} - (a^*)^{N_0}}_{T_{\mathrm{AP}}(z,1)}\,. \qquad (20)$$

The phase equalizer for an allpass chain of length n is obtained by the cascade

$$P_{\mathrm{equi}}(z, n) = P_{\mathrm{equi}}(z, 1)^n\,. \qquad (21)$$

The maximal filter degree for this FIR phase equalizer is given here by $N_p = L\, N_0$ according to Eq. (14). The approximation error for the desired transfer function $T_{\mathrm{des}}(e^{j\,\Omega}, n)$ of Eq. (15) can be made arbitrarily small by increasing the filter degree N_0, since $|a| < 1$. In [16] it is shown that this phase equalizer achieves an equiripple approximation error for the desired transfer function of Eq. (15), thus termed as *equiripple FIR phase equalizer.*

Design II

We propose an alternative approach which approximates the desired anti-causal phase equalizer of Eq. (19) by means of a causal FIR filter. Its coefficients can be obtained by a time-shift and truncation of the impulse response $p_{\mathrm{ideal}}(k, n)$ according to

$$p_{\mathrm{LS}}(k, n) = \begin{cases} p_{\mathrm{ideal}}(k - N_p, n) & ; \;\; k = 0, 1, \ldots, N_p \\ 0 & ; \;\; \text{else}\,. \end{cases} \qquad (22)$$

The transfer function of an inverse allpass filter chain $H_A(z)^{-n}$ is identical to the para-conjugate transfer function, i.e., the z-variable is replaced by z^{-1} and the complex conjugate filter coefficients are taken. Thus, the impulse response of the ideal phase equalizer $p_{\mathrm{ideal}}(k, n)$ can be obtained by the time-reversed impulse response of an allpass chain of length n with complex conjugate allpass coefficient a^*.

The L_2-norm of the error function between the frequency response $\hat{P}(e^{j\,\Omega})$ of an arbitrary causal FIR filter of degree N_p and the frequency response of the desired non-causal IIR phase equalizer with impulse response $p_{\mathrm{ideal}}(k - N_p)$ is given by

$$\left|\left|\Delta \hat{P}(e^{j\,\Omega})\right|\right|_2 = \left|\left|\hat{P}(e^{j\,\Omega}) - P_{\mathrm{ideal}}(e^{j\,\Omega})\, e^{-j\,\Omega\, N_p}\right|\right|_2. \qquad (23)$$

This error becomes minimal for the approximation according Eq. (22), cf. [17]. Therefore, this filter provides a least-squares (LS) error approximation and is hence termed as *LS FIR phase equalizer.*

The use of the LS phase equalizer of Eq. (22) causes a signal delay of N_p samples independent of n. Thus, Eq. (15) applies with the substitution $n\, N_0 \to N_p$, and $z^{-N_0} \to 1$ for Eq. (10) and Eq. (11) since no different delays occur now for each branch.

The impulse response of a short allpass chain decreases more rapidly towards zero than those for a long allpass chain. Hence, the LS phase equalizers for these shorter allpass chains ($n = 0, 1, 2, \ldots$) contain many coefficients which are (almost) equal to zero such that their effective filter degree is lower than N_p. The incorporation of LS phase equalizers with different degrees to reduce the computational complexity is straightforward and has been omitted to ease the treatment.

If a complex allpass filter is used for the allpass transformation[2], the output signal $y(k)$ of the filter-bank becomes complex in the case of an imperfect phase equalization. Thereby, the imaginary part of the output signal (which is usually discarded) becomes negligible, if the the desired transfer function of Eq. (15) is approximated with a low error.

The discussed phase equalizers are applied to the phase equalization of an allpass chain of length $L = 63$ with allpass coefficient $a = 0.4$. The approximation error between the desired transfer function $T_{\mathrm{des}}(e^{j\,\Omega}, L)$ and the actual transfer function $T_{AP}(e^{j\,\Omega}, L)$ is plotted in Fig. 3 for different phase equalizers. The LS phase equalizer achieves a significantly

[2] The complex allpass transformation provides an enhanced flexibility for the adjustment of the frequency resolution, e.g., [9].

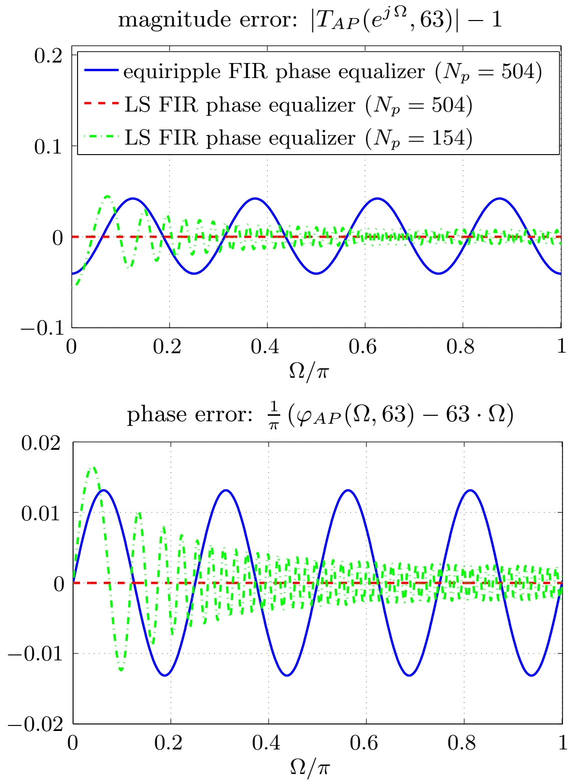

Figure 3: *Approximation error for the desired transfer function of Eq. (15) obtained by phase equalization of an allpass chain of length $L = 63$ ($a = 0.4$) with phase equalizers of filter degree N_p.*

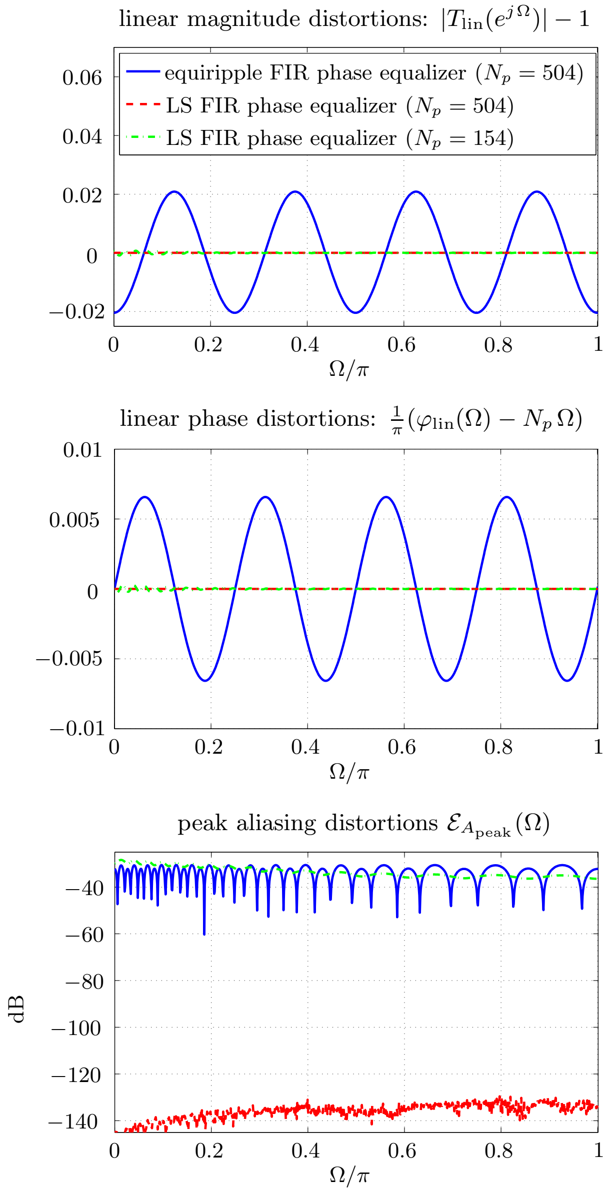

Figure 4: *Reconstruction errors for a 32-channel allpass transformed AS FB ($a = 0.4$, $R = 4$) with synthesis filter-banks based on different FIR phase equalizers having a maximal filter degree of N_p.*

lower approximation error than the equiripple phase equalizer for the same filter degree and signal delay, respectively. Alternatively, the LS phase equalizer can obtain a similar approximation error than the equiripple phase equalizer with a considerably lower filter degree.

It is not contradictory that the LS FIR phase equalizer achieves a lower approximation error than the equiripple FIR phase equalizer. The equiripple phase equalizer leads to an overall transfer function $T_{\mathrm{AP}}(z, N_p)$ which shows an equiripple approximation error for the desired transfer function $T_{\mathrm{des}}(z, N_p)$ [16]. In contrast, the LS FIR phase equalizer achieves a least-squares approximation error for the desired phase equalizer according to Eq. (23). Thus, different approximation errors are considered for these phase equalizers.

4. DESIGN EXAMPLE

The design of a warped DFT analysis-synthesis filter-bank with $M = 32$ frequency bands is investigated. An allpass coefficient of $a = 0.4$ is taken for the allpass transformation of the analysis filter-bank, which yields a good approximation of the Bark frequency bands for 8 kHz sampling frequency [5]. The synthesis filter-bank is constructed by means of the phase equalizers of Section 3.2 as exemplified in Fig. 3. The prototype filters of Eq. (18) are used and a downsampling rate of $R = 4$ is chosen.[3]

[3] In [11], the lifting scheme is applied to use longer prototype filters while constraining the overall signal delay. This causes a high algorithmic complexity as the filter-bank is operated at the non-decimated sampling rate in contrast to the considered filter-bank according to Fig. 1 and Fig. 2. Besides, the lifting scheme can also be applied to the proposed filter-bank design.

The error between the frequency response of the linear transfer function of the filter-bank

$$T_{\mathrm{lin}}(e^{j\Omega}) = |T_{\mathrm{lin}}(e^{j\Omega})|\,\exp\{-j\,\varphi_{\mathrm{lin}}(\Omega)\} \tag{24}$$

according to Eq. (16), and the desired transfer function is displayed in Fig. 4 for different designs. In addition, the peak aliasing distortions according to Eq. (17) are plotted for these filter-bank designs.

The new synthesis filter-bank based on the LS phase equalizer achieves a significantly lower reconstruction error than by means of the equiripple phase equalizer for the same overall signal delay $d_0 = N_p = 504$. For a maximal filter degree of $N_p = 154$, the LS phase equalizer achieves similar peak aliasing distortions than the original filter-bank design using the equiripple phase equalizer with $N_p = 504$, but the proposed design still causes almost no linear distortions.

original filter-bank design

new filter-bank design

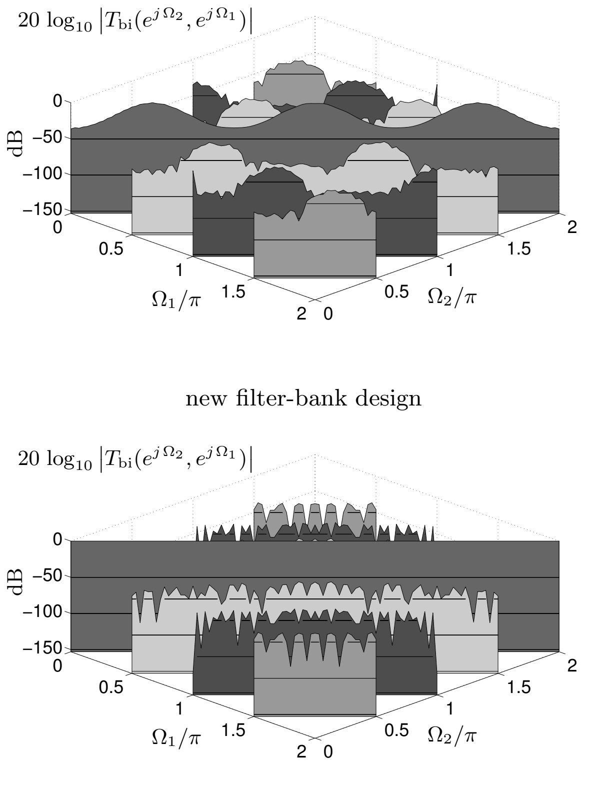

Figure 5: *Magnitude of the bifrequency system function for the original equiripple filter-bank design and the new least-squares design ($M = 32$, $R = 4$, $a = 0.4$, $d_0 = N_p = 189$).*

Another form to visualize the linear distortions and the aliasing distortions of an AS FB is to calculate its bifrequency system function [18]. The magnitude of the bifrequency system function of the two considered filter-bank designs for the same overall system delay of $d_0 = 189$ samples are plotted in Fig. 5. Again, the new LS filter-bank design achieves significantly lower reconstruction errors especially for the linear distortions.

5. CONCLUSIONS

An improved FIR synthesis filter-bank design for an oversampled allpass transformed DFT analysis filter-bank has been presented. The synthesis filter-bank employs phase equalizers which are an optimal least-squares FIR filter approximation of the desired non-causal IIR phase equalizers. This approach achieves a significantly lower signal reconstruction error with a lower overall signal delay and lower algorithmic complexity than the original design of Galijašević and Kliewer [11]. The coefficients for the proposed synthesis filter-bank are given by analytical closed-form expressions such that no complex numerical optimization is required. This allows to control efficiently the trade-off between reconstruction errors on the one hand and signal delay and algorithmic complexity on the other hand. The linear distortions of the filter-bank can be made arbitrarily small. These properties are of interest for algorithms using non-uniform subband processing such as (perceptual based) speech enhancement algorithms or subband coding systems.

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