DESIGN OF IIR QMF BANKS WITH NEAR-PERFECT RECONSTRUCTION AND LOW COMPLEXITY

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ABSTRACT

A novel design for a two-channel IIR quadrature-mirror filter (QMF) bank with near-perfect reconstruction (NPR) is presented. The analysis filter-bank is given by an efficient polyphase network (PPN) implementation based on allpass filters. The arising phase distortions are almost compensated by stable allpass filters, designed via analytical closed-form expressions. In a first design, the remaining aliasing, amplitude and phase distortions become arbitrarily small in dependence of the tolerable system delay and algorithmic complexity, respectively. In a second design, aliasing and amplitude distortions are completely canceled and phase distortions are minimized at the expense of an additional signal delay. The proposed QMF banks have a lower algorithmic complexity than comparable designs.

***Index Terms*—** QMF bank, allpass polyphase filters, IIR phase equalizer, closed-form design, low complexity

1. INTRODUCTION

Tree-structured filter-banks are used for a variety of applications such as speech and audio processing as well as subband coding. A common approach to construct such filter-banks is to employ a two-channel quadrature-mirror filter (QMF) bank with FIR analysis and synthesis filters, e.g., [1, 2, 3]. However, filters of high degree are required to achieve a steep transition band and a high stopband attenuation, which results in a high computational complexity and a high system delay, respectively. A more efficient design is that of an allpass-based polyphase network (PPN) implementation. The reconstructed signal of this IIR QMF bank is free of amplitude and aliasing distortions, but phase distortions are not compensated by the classical, allpass-based synthesis filter-bank [4].

These phase distortions can be equalized by means of anti-causal filtering, cf. [5]. A double buffering scheme can be employed for the processing of infinite-length sequences [6]. However, this rather complex procedure causes a high signal delay and requires the transfer of the filter states to the synthesis filter-bank.

These drawbacks are avoided by two-channel QMF banks based on IIR / FIR filters, e.g., [7, 8, 9]. FIR filters are used to (partly) compensate the phase distortions caused by the allpass filters of the PPN analysis filter-bank. The FIR filter coefficients are obtained by solving an optimization problem using either linear programming methods [7] or a least-squares error approach [8, 9].

A parametric design for a mixed IIR / FIR QMF bank is proposed in [10]. It has the advantage that the synthesis filters are designed by simple closed-form expressions so that no complex numerical optimization is required.

In this contribution, a closed-form design for a stable, allpass-based synthesis filter-bank is proposed. In contrast to the design of Galijašević [10], amplitude distortions can be completely avoided and a significantly lower computational complexity is achieved. The remaining phase distortions can be made arbitrarily small in dependence of the tolerable signal delay and computational complexity.

This paper is organized as follows: In Section 2, the design of IIR QMF banks is reviewed. The new synthesis filter-bank design is introduced in Section 3. A comparison with related QMF bank designs is provided by Section 4. The results are summarized in Section 5.

2. ALLPASS-BASED TWO-CHANNEL QMF BANKS

The general structure of a critically subsampled two-channel QMF bank is shown in Fig. 1 (e.g., [4]). The analysis subband filters have the transfer functions $H_0(z)$ and $H_1(z)$, and the synthesis filters are represented by $G_0(z)$ and $G_1(z)$. The input-output relation in the z-domain is given by

$$Y(z) = X(z)\, T_{\text{lin}}(z) + X(-z)\, E_{\text{alias}}(z) \tag{1}$$

with linear (distortion) transfer function

$$T_{\text{lin}}(z) = \frac{1}{2}\, [H_0(z)\, G_0(z) + H_1(z)\, G_1(z)] \tag{2}$$

and aliasing (distortion) transfer function

$$E_{\text{alias}}(z) = \frac{1}{2}\, [H_0(-z)\, G_0(z) + H_1(-z)\, G_1(z)]\, . \tag{3}$$

Aliasing distortions are completely eliminated by the choice

$$G_0(z) = H_1(-z) \quad \wedge \quad G_1(z) = -H_0(-z)\, . \tag{4}$$

If the analysis filters fulfill the requirement $H_0(z) = H_1(-z)$, they are referred to as quadrature-mirror filters.

An efficient realization of a QMF bank is obtained by an allpass-based PPN implementation according to Fig. 2. For the polyphase

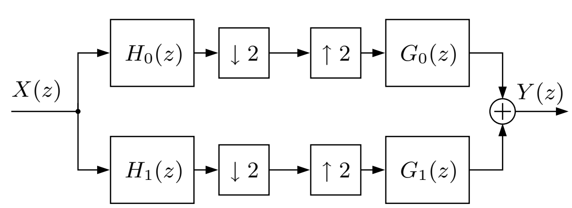

Fig. 1. General structure of a two-channel QMF bank.

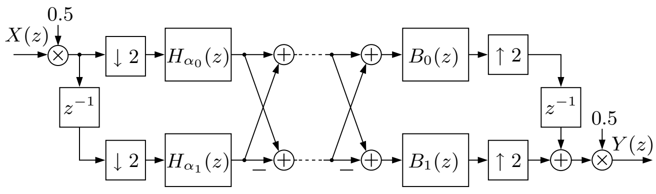

Fig. 2. PPN implementation of an allpass-based QMF bank.

analysis filters, allpass filters of first order with transfer functions

$$H_{\alpha_i}(z) = \frac{1-\alpha_i\, z}{z-\alpha_i}; \quad |\alpha_i| < |z| \tag{5}$$
$$-1 < \alpha_i < 1; \quad \alpha_i \in \mathbb{R}; \quad i \in \{0,1\}$$

are used, but the following designs can also be applied if allpass filters of higher order are employed. The subband filters read

$$H_0(z) = \frac{1}{2}\left[H_{\alpha_0}(z^2) + z^{-1}\, H_{\alpha_1}(z^2)\right] \tag{6}$$
$$H_1(z) = \frac{1}{2}\left[H_{\alpha_0}(z^2) - z^{-1}\, H_{\alpha_1}(z^2)\right] \tag{7}$$
$$G_0(z) = \frac{1}{2}\left[z^{-1}\, B_0(z^2) + B_1(z^2)\right] \tag{8}$$
$$G_1(z) = \frac{1}{2}\left[z^{-1}\, B_0(z^2) - B_1(z^2)\right]. \tag{9}$$

The real allpass coefficients α_i can be determined by minimizing the stopband energy $E_{\mathrm{S}} = \frac{1}{\pi}\int_{\Omega_{\mathrm{S}}}^{\Pi} |H_0(e^{j\,\Omega})|^2 \mathrm{d}\Omega$, e.g., [4]. The obtained power-complementary analysis filters for a stopband frequency of $\Omega_{\mathrm{S}} = 0.64\,\pi$ are shown in Fig. 3.

The linear transfer function and aliasing transfer function according to Eq. (2) and Eq. (3) are now given by

$$T_{\mathrm{lin}}(z) = \frac{z^{-1}}{4}\left[H_{\alpha_0}(z^2)\, B_0(z^2) + H_{\alpha_1}(z^2)\, B_1(z^2)\right] \tag{10}$$
$$E_{\mathrm{alias}}(z) = \frac{z^{-1}}{4}\left[H_{\alpha_0}(z^2)\, B_0(z^2) - H_{\alpha_1}(z^2)\, B_1(z^2)\right]. \tag{11}$$

It is obvious from these two equations that the requirement

$$H_{\alpha_i}(z)\, B_i(z) \stackrel{!}{=} z^{-\tau}; \quad \tau \geq 0; \quad i \in \{0,1\} \tag{12}$$

leads to perfect reconstruction (PR). Since $H_{\alpha_i}(z)$ represents allpass filters, Eq. (12) states a phase equalization problem. Different design approaches for the polyphase synthesis filters $B_i(z)$ are discussed in the following.

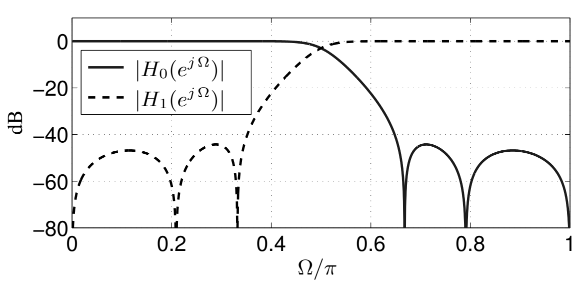

Fig. 3. Magnitude responses of the allpass-based analysis subband filters with $\alpha_0 = -0.1806$ and $\alpha_1 = -0.6485$ for $\Omega_{\mathrm{S}} = 0.64\,\pi$.

3. SYNTHESIS FILTER BANK DESIGN

3.1. Concept of IIR and Mixed IIR / FIR Designs

The classical, allpass-based PPN QMF bank uses polyphase synthesis filters given by $B_0(z) = H_{\alpha_1}(z)$ and $B_1(z) = H_{\alpha_0}(z)$ so that $E_{\mathrm{alias}}(z) = 0$ and $T_{\mathrm{lin}}(z) = 0.5\, z^{-1} H_{\alpha_0}(z^2)\, H_{\alpha_1}(z^2)$ [4]. In this case, amplitude distortions are avoided but (significant) phase distortions remain.

The choice $B_i(z) = H_{\alpha_i}(z)^{-1}$ with region of convergence $1/|\alpha_i| < |z|$ for $i \in \{0,1\}$ achieves PR according to Eq. (10) and Eq. (11), but the corresponding impulse responses are infinite and anti-causal. The (approximate) realization of anti-causal filters by time-reversed processing [5, 6] requires large buffers and leads to a very high signal delay. An alternative is to use an FIR design to approximate the desired anti-causal filters, with coefficients obtained by solving an optimization problem, e.g., [7, 8, 9].

In [10], the phase equalization problem is solved by FIR filters which can be expressed by the closed-form expression

$$\underbrace{\frac{1-\alpha_i\, z}{z-\alpha_i}}_{=H_{\alpha_i}(z)} \cdot \underbrace{(z-\alpha_i)\sum_{n=0}^{d_i-1} z^{n-d_i}\,(\alpha_i)^n}_{=P_{\mathrm{FIR}}^{(i)}(z)} = \underbrace{z^{-d_i} - (\alpha_i)^{d_i}}_{=T_{\mathrm{FIR}}^{(i)}(z)}. \tag{13}$$

As shown in [11], this FIR phase equalizer leads to an equiripple approximation error for the desired transfer function z^{-d_i}.

3.2. Concept of the New IIR Design

Here, a pure IIR QMF bank design is devised. The idea is to use allpass filters for the polyphase synthesis filters to solve the phase equalization problem stated in Eq. (12). One method to construct these filters is to employ a general allpass filter design, e.g., [12]. However, such numerical approaches have a comparatively high design complexity and do not always provide stable filters. To circumvent these problems, we propose the following closed-form design for a stable allpass phase equalizer

$$P_{\mathrm{AP}}^{(i)}(z) = \prod_{n=0}^{N_i-1} \frac{1+(\alpha_i z)^{2^n}}{z^{2^n}+\alpha_i^{2^n}}; \quad N_i \in \{1,2,3,\ldots\}, \tag{14}$$

which has been developed originally for the design of frequency warped DFT filter-banks [11]. The transfer function of allpass filter and phase equalizer is given by

$$T_{\mathrm{AP}}^{(i)}(z) = H_{\alpha_i}(z) \cdot P_{\mathrm{AP}}^{(i)}(z) = \frac{1-(\alpha_i\, z)^{d_i}}{z^{d_i}-\alpha_i^{d_i}}; \quad d_i = 2^{N_i}. \tag{15}$$

This allpass filter is always stable (since $|\alpha_i| < 1$) and tends to a pure delay z^{-d_i} for an increasing value of d_i. This phase equalizer concept can be easily adapted, if a cascade of first order allpass filters is used for the allpass filter of Eq. (5) in order to improve the frequency selectivity of the analysis subband filters.

The proposed allpass phase equalizer of Eq. (14) of degree $d_i - 1$ requires only $2\,\log_2 d_i$ real multipliers, $2\,\log_2 d_i$ real adders and $d_i - 1$ delay elements.[1] In contrast, a general allpass phase equalizer (e.g., [12]) needs $2\,d_i$ multipliers, $2\,d_i$ adders and d_i delay elements. The FIR phase equalizer of Eq. (13) requires d_i multipliers, $d_i - 1$ adders and $d_i - 1$ delay elements.

[1] The implementation of an allpass filter of first order by two multiplication, two adders and one delay element is taken as basis.

Moreover, the proposed design is especially suitable for a fixed-point implementation: The allpass property is maintained even for quantized coefficients, and the poles of the allpass phase equalizers of Eq. (14) are closer to the origin than for the allpass filters $H_{\alpha_i}(z)$.

3.3. Design I: Minimizing of Phase, Amplitude and Aliasing Distortions

Near-perfect reconstruction (NPR) is achieved by the following design for the polyphase synthesis filters

$$B_0(z) = 2\, P_{\mathrm{AP}}^{(0)}(z)\, z^{-(d_1-d_0)} \tag{16}$$
$$B_1(z) = 2\, P_{\mathrm{AP}}^{(1)}(z)\,. \tag{17}$$

The delay $d_1 - d_0$ compensates the time difference which occurs for different values for d_0 and d_1. Without loss of generality, it is assumed that $|\alpha_1| \geq |\alpha_0|$ and $d_1 \geq d_0$, cf. Eq. (15). The linear transfer function is obtained by inserting Eq. (16) and Eq. (17) into Eq. (10) and applying Eq. (15) which leads to

$$T_{\mathrm{lin}}(z) = \frac{z^{-2\,(d_1-d_0)-1}}{2}\,\frac{1-\alpha_0^{d_0}\,z^{2\,d_0}}{z^{2\,d_0}-\alpha_0^{d_0}} + \frac{z^{-1}}{2}\,\frac{1-\alpha_1^{d_1}\,z^{2\,d_1}}{z^{2\,d_1}-\alpha_1^{d_1}}. \tag{18}$$

This transfer function tends to a pure delay of $2\max\{d_0, d_1\} + 1$ samples, if the values for d_0 and d_1 are increased.

Accordingly, the aliasing transfer function is obtained by inserting Eq. (16) and Eq. (17) into Eq. (11):

$$E_{\mathrm{alias}}(z) = \frac{z^{-2\,(d_1-d_0)-1}}{2}\,\frac{1-\alpha_0^{d_0}\,z^{2\,d_0}}{z^{2\,d_0}-\alpha_0^{d_0}} - \frac{z^{-1}}{2}\,\frac{1-\alpha_1^{d_1}\,z^{2\,d_1}}{z^{2\,d_1}-\alpha_1^{d_1}}. \tag{19}$$

Obviously, the aliasing distortions can be made arbitrarily small by using higher values for d_0 and d_1. A design example is given in Fig. 4. For this and the following examples, the analysis filters according to Fig. 3 are employed and the parameters $d_0 = 8$ and $d_1 = 16$ are taken, if not mentioned otherwise. Fig. 4 reveals that the new design causes only negligible amplitude distortions.

3.4. Design II: Cancellation of Aliasing and Amplitude Distortions, and Minimizing of Phase Distortions

Complete aliasing cancellation is achieved by the choice

$$B_0(z) = 2\, P_{\mathrm{AP}}^{(0)}(z)\, T_{\mathrm{AP}}^{(1)}(z) \tag{20}$$
$$B_1(z) = 2\, P_{\mathrm{AP}}^{(1)}(z)\, T_{\mathrm{AP}}^{(0)}(z)\,. \tag{21}$$

The linear transfer function is now given by

$$T_{\mathrm{lin}}(z) = z^{-1}\,\frac{1-\alpha_0^{d_0}\,z^{2\,d_0}}{z^{2\,d_0}-\alpha_0^{d_0}}\,\frac{1-\alpha_1^{d_1}\,z^{2\,d_1}}{z^{2\,d_1}-\alpha_1^{d_1}}\,. \tag{22}$$

This is an allpass filter of order $2\,(d_0 + d_1) + 1$. Thus, no amplitude distortions occur. The overall phase response becomes increasingly linear for higher values of d_0 and d_1. In contrast to the previous design, amplitude and aliasing distortions are completely eliminated at the expense of an increased group delay and a higher algorithmic complexity. An example for the group delay is given by Fig. 5.

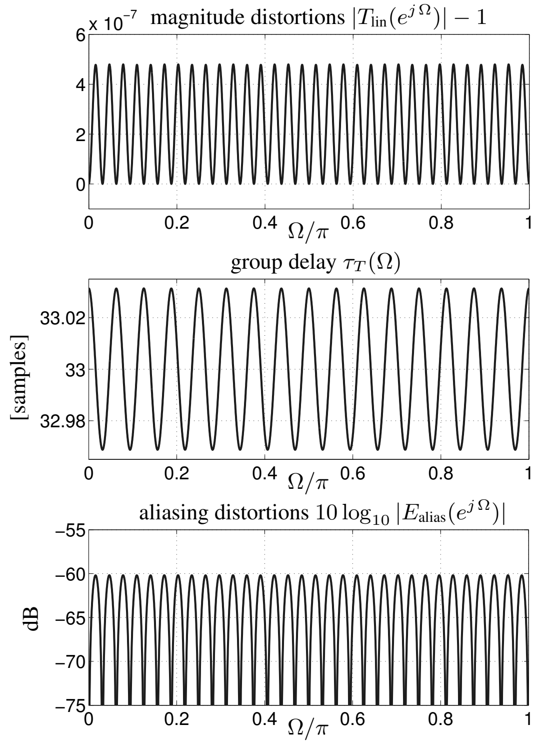

Fig. 4. Signal reconstructions errors for the IIR QMF bank design I with $d_0 = 8$ and $d_1 = 16$.

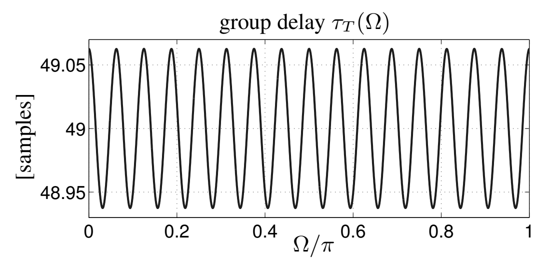

Fig. 5. Overall group delay for the IIR QMF bank design II with $d_0 = 8$ and $d_1 = 16$.

3.5. Design III: Subband Filters with Linearized Phase

A more linear phase characteristics of the subband filter can be achieved by means of the following PPN implementations

$$H_0'(z) = \frac{1}{2}\,[T_{\mathrm{AP}}^{(0)}(z^2) + z^{-1}\,H_{\alpha_1}(z^2)\,P_{\mathrm{AP}}^{(0)}(z^2)] \tag{23}$$
$$H_1'(z) = \frac{1}{2}\,[T_{\mathrm{AP}}^{(0)}(z^2) - z^{-1}\,H_{\alpha_1}(z^2)\,P_{\mathrm{AP}}^{(0)}(z^2)] \tag{24}$$
$$G_0'(z) = z^{-1}\,T_{\mathrm{AP}}^{(1)}(z^2) + H_{\alpha_0}(z^2)\,P_{\mathrm{AP}}^{(1)}(z^2) \tag{25}$$
$$G_1'(z) = z^{-1}\,T_{\mathrm{AP}}^{(1)}(z^2) - H_{\alpha_0}(z^2)\,P_{\mathrm{AP}}^{(1)}(z^2)\,. \tag{26}$$

The relation to the subband filters of the classical, allpass-based PPN QMF bank (described before) is given by

$$H_i'(z) = H_i(z)\,P_{\mathrm{AP}}^{(0)}(z^2) \tag{27}$$
$$G_i'(z) = 2\,G_i(z)\,P_{\mathrm{AP}}^{(1)}(z^2);\quad i \in \{0,1\}\,. \tag{28}$$

Hence, the magnitude responses of the subband filters are the same as for the classical design since we use allpass phase equalizers. The

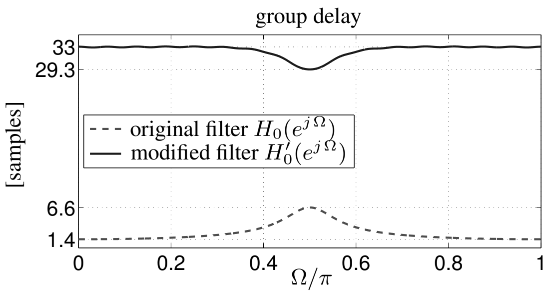

Fig. 6. Group delays of the original and new analysis subband filter for $d_0 = 16$. (The curves for the highpass filters and lowpass filters are identical.)

phase responses of the new subband filters become more linear because the multiplication with $P_{\text{AP}}^{(i)}(z^2)$ leads to a partial phase compensation. This improvement is exemplified by Fig. 6.

The curves for the synthesis filters are similar and show the same deviations. Increasing the values for d_i improves further the linear phase characteristic. The signal delay and signal reconstruction errors are the same as for the previous design II, but the algorithmic complexity is higher.

4. EVALUATION

In this section, the proposed IIR QMF bank designs I & II are compared to the related mixed IIR / FIR designs of [10, 9] which all achieve NPR. In addition, the paraunitary QMF Lattice design of [3] is considered to include an FIR QMF bank with PR, cf. [4]. The different filter-banks are designed in such a manner that they have all the same (nominal) signal delay τ_0 and similar aliasing cancellation properties. Table 1 contrasts the reconstruction errors and algorithmic complexity of the different filter-bank (FB) designs.

The considered NPR QMF banks achieve a similar aliasing cancellation and have a significantly lower complexity than the PR FIR QMF bank at the expense of small reconstruction errors. The new IIR QMF bank causes no or negligible magnitude distortions and has the lowest algorithmic complexity of all considered designs at the expense of only slightly higher group delay distortions.

Table 1. Comparison of different two-channel QMF banks w.r.t. the maximal aliasing distortions (MALD), the maximal amplitude distortions (MAMD) and the maximal group delay deviations (MGDD). The last column contains the overall number of real multipliers ($\mathcal{M}$), real adders ($\mathcal{A}$) and delay elements ($\mathcal{D}$).

QMF bank design	MALD [dB]	MAMD for $\lvert T_{\text{lin}}(\Omega)\rvert - 1$	MGDD for $\tau_T(\Omega) - \tau_0$	$\mathcal{M}/\mathcal{A}/\mathcal{D}$
signal delay $\tau_0 = 33$				
new design I	-60.2	$+4.8 \cdot 10^{-7}$	$\pm 3.1 \cdot 10^{-2}$	9/12/24
NPR FB [10]	-66.2	$\pm 5 \cdot 10^{-4}$	$\pm 1.6 \cdot 10^{-2}$	15/17/36
NPR FB [9]	-61.9	$\pm 8 \cdot 10^{-4}$	$\pm 2.3 \cdot 10^{-2}$	15/17/36
PR FB [3]	none	≈ 0	± 0	36/35/37
signal delay $\tau_0 = 49$				
new design II	none	± 0	$\pm 6.3 \cdot 10^{-2}$	11/14/40
NPR FB [10]	none	$\pm 9.8 \cdot 10^{-4}$	$\pm 3.1 \cdot 10^{-2}$	16/18/52
NPR FB [9]	none	$\pm 16 \cdot 10^{-4}$	$\pm 4.6 \cdot 10^{-2}$	27/29/52
PR FB [3]	none	≈ 0	± 0	52/51/52

5. CONCLUSIONS

A new design for a two-channel IIR QMF bank with NPR is presented. The synthesis filters are obtained by allpass filters, designed by simple closed-form expression. Compared to mixed IIR / FIR QMF bank designs with NPR, the new filter-bank has a significantly lower algorithmic complexity and causes no or negligible amplitude distortions. The trade-off between signal reconstruction errors on the one hand, and signal delay and algorithmic complexity on the other hand, can be controlled in a simple and flexible manner. Aliasing can be completely canceled and subband filters with a more linear phase can be achieved at the expense of an increased signal delay. The slightly higher group delay distortions compared to IIR/FIR QMF banks can usually be tolerated for speech and audio processing systems, which are one possible application for the proposed QMF bank designs. The analysis filter-bank has a lower complexity and signal delay than the synthesis filter-bank so that their exchange can be beneficial for coding and transmission applications.

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