Approximating EXIT Characteristics of Soft Demodulators in OFDM BICM-ID Systems for Time Variant Frequency Selective Fading Channels

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) in combination with Bit-Interleaved Coded Modulation with Iterative Decoding (BICM-ID) is known to provide very good BER performance: While OFDM decomposes a frequency selective broadband transmission channel causing intersymbol interference (ISI) into narrow band flat fading subchannels, the iterative BICM-ID scheme enables demodulation and decoding of the transmitted data at nearly optimum Bit Error Rates (BERs). To provide a stopping criterion for the iterative process, the Extrinsic Information Transfer (EXIT) chart is an efficient tool. For the use in time-varying ISI environments however it is impossible to provide EXIT characteristics of the Soft Demodulator (SDM) as these change with the channel. In this paper we describe a simple but effective and accurate method for constructing SDM characteristics for ISI environments based on a limited set of SDM characteristics prerecorded under AWGN conditions.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has proven to be a modulation technique that provides high spectral efficiency while keeping the complexity at a low level. The concept of Turbo coding [1] which allows data transmission near the Shannon limit was extended in many ways, e.g., towards *bit-interleaved coded modulation with iterative decoding* (BICM-ID) [2]–[4]. The high spectral efficiency of OFDM and the good Bit Error Rate (BER) performance of Turbo codes are the reason why these concepts are nowadays widely applied in new and evolving communication standards such as IEEE 802.16-2004 [5].

As the energy consumption of iterative schemes increases linearly with the number of iterations, iterative processing of soft information is always more complex and thereby more energy consuming than non-iterative hard decided demodulation, channel decoding and source decoding. Analysis of the Extrinsic Information Transfer (EXIT) characteristics is an easy way of estimating the number of necessary decoding/demodulation iterations using knowledge of components of the transmission system and of the channel. The concept was introduced in [6], [7] for the convergence analysis and optimization of modulation schemes in BICM-ID systems and later in [8] extended to the analysis of parallel channel codes.

In [7] it has been shown that the EXIT characteristic of the soft demodulator (SDM) is dependent on the channel characteristics. In case of a flat fading AWGN channel, the characteristics depend on the channel *Signal-to-Noise Ratio* (SNR). Unfortunately, the assumption of an flat fading AWGN channel is not valid when analyzing broadband mobile communication systems. Multi-path propagation causing *intersymbol interference* (ISI) has to be taken into account as well as time-variant local reflections, diffractions and Doppler shifts leading to fast fading in every single propagation path. These variations of the channel result in steady variations of the SDM characteristics which can hardly be predicted.

In this paper we suggest a simple approach to approximate EXIT characteristics of an SDM used in an OFDM BICM-ID system under ISI conditions by channel dependent weighted summation of SDM characteristics drawn from a limited set of SDM characteristics recorded from AWGN simulations. If such a limited set of AWGN SDM characteristics is present at the receiver together with *Channel State Information* (CSI) such as the actual SNR and *Channel Impulse Response* (CIR), the convergence behavior of the BICM-ID process can be predicted and the number of necessary iterations can be estimated. Furthermore, if the CSI is slowly varying and fed back to the transmitter, EXIT-optimized resource allocation becomes possible by assigning modulation types (contingently with *non-regular signal constellation sets* [9], [10]), symbol mappings, and transmit powers individually per subcarrier.

The remainder of this paper is organized as follows: Section 2 describes the model of the OFDM BICM-ID system under consideration and explains the channel model followed by the introduction of different approximations of SDM characteristics for OFDM BICM-ID systems under ISI conditions in Sec. 3. Simulation results are presented in Sec. 4 depicting the BER performance of the OFDM BICM-ID system as well as confirming the accuracy and applicability of the suggested approximation method.

2 System Model

In the following we define and describe the transmission system under consideration (see Fig. 1). A block of binary information bits $\mathbf{u}$ is encoded by a convolutional encoder to become the encoded data stream $\mathbf{x}$ which then is interleaved by a pseudo-random bit-interleaver π resulting in the interleaved encoded data stream $\tilde{\mathbf{x}}$. The bit-interleaved data bits $\tilde{\mathbf{x}}$ are grouped into sets $\underline{\tilde{x}}$ of I bits and mapped to modulation symbols $Y \in \mathcal{Y} \subset \mathbb{C}$ of a constellation $\mathcal{Y}$ using a certain mapping rule μ. That creates the symbol stream $\mathbf{Y}$ consisting of symbols $Y \in \mathbb{C}$. The complex modulation symbols are grouped to form blocks of size M with M indicating the FFT/IFFT size and therefore the number of subchannels

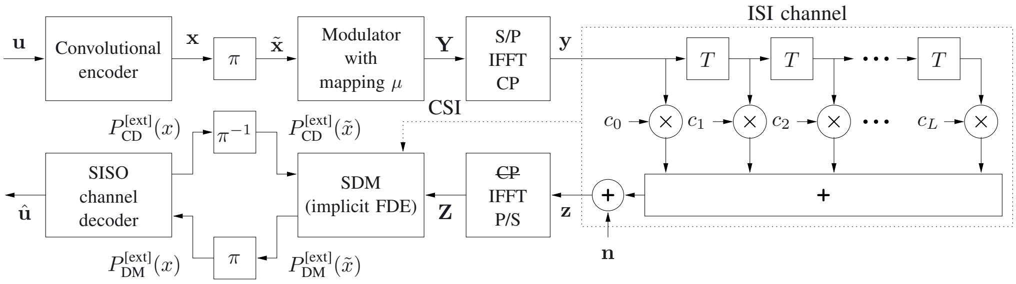

Fig. 1. Considered OFDM BICM-ID transmission system with ISI channel.

of the OFDM system. If the size of block $\mathbf{Y}$ is not an integer multiple of M, the block is resized to the next integer multiple of M and filled with dummy symbols Y. Each block is processed with an IFFT and a Cyclic Prefix (CP) of length L is added, resulting in OFDM symbols that form a complex baseband signal which is transmitted over the channel. Considering a sample rate $r = 1/T$ for the baseband system one OFDM symbol $\mathbf{y}$ comprising $M+L$ complex samples y has a duration of $(M + L) \cdot T$.

Throughout this work we are considering a frequency selective multi-path propagation channel causing inter-symbol interference with AWGN. This ISI channel is modeled by a tapped delay line. One complex multiplication in each tap models the individual fading comprising the effects of the static attenuation, shadowing, and fast fading. The channel impulse response $c(t)$ of the channel is given by:

$$c(t) = \sum_{k=0}^{L} c_k(t)\delta(kT - t)\,, \quad c_k \in \mathbb{C}\,. \tag{1}$$

Despite the time-variant nature of the fading coefficients of each tap, firstly, we assume the channel to be constant for the duration of the transmission one data block $\mathbf{u}$. The channel is then given by its CIR consisting of its complex fading coefficients

$$\mathbf{c} = [c_0, c_1, \ldots, c_L]\,. \tag{2}$$

We will loosen the restriction of a constant CIR to the duration of one OFDM symbol $\mathbf{y}$ later in the end of Sec. 3.

Figure 1 depicts the ISI channel as a finite impulse response (FIR) filter with impulse response $\mathbf{c}$, order L (same as CP length), and constraint length $L + 1$ plus AWGN. Here we assume the smallest possible delay T to be the duration of one complex sample y of the transmitted symbol stream $\mathbf{y}$ which is usually fulfilled for existing systems and channel specifications. By choosing $c_k \in \mathbb{C}$ additional small delays in the range of one carrier wave length can be modeled. Longer delays are consequently modeled by using a greater filter order.

Given the impulse response $\mathbf{c}$ of this FIR filter the corresponding frequency transfer function $C_m = \mathrm{FFT}_M\{\mathbf{c}\}$ is given by the Fourier transform of length M, thus illustrating the frequency selectivity of the channel. Assuming that $L + 1 < M$, the channel impulse response $\mathbf{c}$ is zero padded before its application to the FFT resulting in the known correlation of the fading coefficients of neighboring subcarriers.

Due to the use of a cyclic prefix, one noisy time domain OFDM symbol $\mathbf{z}$ observed at the receiver can be written as

$$\mathbf{z} = \mathbf{y} * \mathbf{c} + \mathbf{n}\,, \tag{3}$$

where '$*$' denotes the cyclic convolution of the OFDM symbol $\mathbf{y}$ with the channel impulse response $\mathbf{c}$ while $\mathbf{n}$ denotes a block of additive white Gaussian noise samples of power σ_n^2 and size $M + L$. After removal of the CP and application of the FFT at the receiver, (3) can be reformulated in the transform domain:

$$Z_m = Y_m \cdot C_m + N_m\,, \quad m \in 1, \ldots, M\,. \tag{4}$$

The Fourier transformed noise samples $N_m = \mathrm{FFT}_M\{\mathbf{n}\}$ remain white and Gaussian distributed with power σ_n^2. Frequency domain equalization (FDE) is implicitly realized in the soft demodulator in our system (see below) but can equivalently be modeled by division of the single observations Z_m on subchannel m with the corresponding values of the channel transfer function C_m yielding

$$\frac{Z_m}{C_m} = Y_m + \frac{N_m}{C_m},\ m \in \{1, \ldots, M\} \tag{5}$$

for every received OFDM symbol $\mathbf{z}$. This of course assumes, that the CIR is known (perfect channel state information, CSI) or estimated at the receiver $\mathbf{c}$.

The SISO channel decoder (CD) and the soft demodulator (SDM) exchange extrinsic information in a Turbo process. The SDM computes extrinsic probabilities $P_{\mathrm{DM}}^{[\mathrm{ext}]}(\tilde{x})$ for each bit $\tilde{x}^{(i)}$ [11]

$$\begin{aligned} &P_{\mathrm{DM}}^{[\mathrm{ext}]}(\tilde{x}^{(i)} = b) \\ &\sim \sum_{\hat{Y} \in \mathcal{Y}_b^i} P(Z_m|\hat{Y}) \prod_{j=1, j\neq i}^{I} P_{\mathrm{CD}}^{[\mathrm{ext}]}\Big(\tilde{x}^{(j)} = \bar{\mu}^{-1}(\hat{Y})^{(j)}\Big) \end{aligned} \tag{6}$$

with $b \in \{0,1\}$. Each $P_{\mathrm{DM}}^{[\mathrm{ext}]}(\tilde{x})$ consists of the sum over all possible channel symbols $\hat{Y}$ for which the i^{th} bit of the corresponding bit pattern $\underline{\tilde{x}} = \bar{\mu}^{-1}(\hat{Y})$ is b. These channel symbols form the subset $\mathcal{Y}_b^i$ with $\mathcal{Y}_b^i = \{\mu([\tilde{x}^{(1)}, \ldots, \tilde{x}^{(I)}]) | \tilde{x}^{(i)} = b\}$. In the first iteration the feedback probabilities $P_{\mathrm{CD}}^{[\mathrm{ext}]}(\tilde{x})$ are initialized as equiprobable, i.e., $P_{\mathrm{CD}}^{[\mathrm{ext}]}(\tilde{x}) = 0.5$. The conditional probability density $P(Z_m|\hat{Y}) = (1/\pi\sigma_{m,n}^2)\exp(-d_{Z_m\hat{Y}}^2/\sigma_{m,n}^2)$ with $d_{Z_m\hat{Y}}^2 = \|Z_m - C_m\hat{Y}\|^2$ describes the complex subchannel $m \in \{1, \ldots, M\}$. $\sigma_{m,n}^2$ denotes the white noise power resulting from σ_n^2 on subcarrier m, see also (5).

After appropriately deinterleaving the $P_{\mathrm{DM}}^{[\mathrm{ext}]}(\tilde{x})$ to $P_{\mathrm{DM}}^{[\mathrm{ext}]}(x)$, the $P_{\mathrm{DM}}^{[\mathrm{ext}]}(x)$ are fed into a Soft-Input Soft-Output (SISO) channel decoder (CD), which computes extrinsic probabilities $P_{\mathrm{CD}}^{[\mathrm{ext}]}(x_t^{(i)})$ for the encoded bits $x_t^{(i)} = \{0, 1\}$ in addition to the preliminary estimated decoded data bits $\hat{\mathbf{u}}$.

3 SDM EXIT Characteristics

EXIT charts [8] are a powerful tool to analyze and optimize the convergence behavior of iterative systems utilizing the Turbo principle, i.e., systems exchanging and refining extrinsic information. The goal of optimizing an iterative system with EXIT charts is to achieve a "tunnel" between the EXIT characteristics of the two components exchanging extrinsic information. The number of necessary demodulation and decoding iterations can be determined from the width and length of that decoding tunnel.

Here we focus on the characteristics of the SDM, creating extrinsic information $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ using noisy channel observations $\mathbf{Z}$ and extrinsic a priori information generated by the SISO module $P_{\text{CD}}^{[\text{ext}]}(\tilde{x})$. The generation of the characteristics of extrinsic information transfer by simulation is described in [7]. The author also shows that the function $\mathcal{I}_{\text{DM}}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]})$ depends on the channel. In case of AWGN it depends on the noise power σ_n^2, i.e.,

$$\mathcal{I}_{\text{DM}}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \sigma_n^2) := \mathcal{T}_{\text{DM},n} \,. \qquad (7)$$

For an AWGN channel the SDM characteristic in (7) can be obtained from simulations for different values of σ_n^2. In the same way it is also possible to obtain the transfer characteristic $\mathcal{I}_{\text{DM}}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]})$ of SDMs for different kinds of channels like Rayleigh fading channels. The only prerequisite is that the distribution of the a priori information $P_{\text{DM}}^{[\text{ext}]}$ has to be Gaussian distributed. If that is not the case, the simulated values for $P_{\text{DM}}^{[\text{ext}]}$ have to be adapted according to the given distribution (see, e.g., [12]).

As seen in Sec. 2 the complex channel impulse response $\mathbf{c}$ is time variant due to Doppler induced fast fading effects. Recording EXIT characteristic of the soft demodulator would consequently result in a different characteristic for each realization and channel noise power which results in high computational effort as the number of possible channel realizations may be extremely large, depending on the number of taps and distributions of the complex fading coefficients c_k. Moreover, regarding a mobile receiver that tries to estimate the number of necessary demodulation and decoding iterations, the characteristics need to be known in advance and thus have to be stored in the memory of the receiver which is hardly realizable nor reasonable. In the following we describe three methods for approximating the SDM characteristics $\mathcal{T}_{\text{DM},n}$ for the complex ISI channel with given CIR using a limited set of SDM characteristics obtained by AWGN simulations.

A first approximation is picking the characteristic $\mathcal{T}_{\text{DM},n}$ for each transmitted block with the according noise power σ_n^2 from a set of stored EXIT characteristics recorded beforehand from AWGN simulations. This, of course, ignores the impact of the frequency selective channel which, as expected, results in an overestimation of the capability of the SDM (c.f. simulation results in Sec. 4.2).

Due to the ability of OFDM to split up the bandwidth of a broadband frequency selective transmission channel into narrow band flat fading channels, the received symbols Z_m with $m \in 1 \dots M$ after FDE can be regarded as outputs of M independent AWGN channels with different (though correlated) noise powers $\sigma_{n,m}^2$. It is easy to show via the theoretical capacity of each subcarrier and the resulting overall capacity R that an equivalent noise power $\overline{\sigma}_n^2$ can be calculated with the help of the individual noise powers of each subcarrier $\sigma_{n,m}^2$, assuming unity signal powers $\sigma_{y,m}^2 = \sigma_y^2 = 1$ on all subcarriers:

$$R = \frac{1}{2}\log_2\left(1 + \frac{\sigma_y^2}{\overline{\sigma}_n^2}\right) = \frac{1}{2\,M}\sum_{m=1}^{M}\log_2\left(1 + \frac{\sigma_{y,m}^2}{\sigma_{n,m}^2}\right)$$

$$\Rightarrow\ \overline{\sigma}_n^2 = \left[\prod_{m=1}^{M}\left(1 + \frac{1}{\sigma_{n,m}^2}\right)^{1/M} - 1\right]^{-1} . \qquad (8)$$

Using (5) the individual noise powers $\sigma_{n,m}^2$ are obtained by

$$\mathrm{E}\left\{\left(\frac{N_m}{C_m}\right)^2\right\} = \sigma_{n,m}^2 = \frac{\sigma_n^2}{|C_m|^2} \,, \qquad (9)$$

where $\mathrm{E}\{\cdot\}$ denotes expectation.

Using (8) and (9) SDM characteristics $\mathcal{T}_{\text{DM},\overline{n}}$ obtained from AWGN simulations can approximate the ISI SDM characteristic of an OFDM BICM-ID system. However, the SDM characteristic $\mathcal{T}_{\text{DM},\overline{n}}$ still falls short of the SDM characteristic directly recorded for a certain realization $\mathbf{c}$ of the ISI channel (c.f. simulation results in Sec. 4.2).

Therefore we propose a method similar to the generation of EXIT characteristics of decoders for irregular codes by weighted superposition as used, e.g., in [13], [14]. Using the assumption of flat fading AWGN channels for each subcarrier m, for one channel realization $\mathbf{c}$ it is possible to calculate the noise powers $\sigma_{n,m}^2$ as seen before. For each subcarrier m there exists an SDM characteristic $\mathcal{T}_{\text{DM},n,m} := \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \sigma_{n,m}^2)$ which is easily recorded in AWGN simulations. A very good approximation of the SDM characteristic of the multi-carrier system is now obtained by weighted summation of the AWGN SDM characteristics $\mathcal{T}_{\text{DM},n,m}$ according to their occurrence for the given channel $\mathbf{c}$:

$$\mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \mathbf{c}) = \frac{1}{M}\sum_{m=1}^{M}\mathcal{T}_{\text{DM},n,m} \,,$$

which using (9) can be written as

$$\mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \mathbf{c}) = \frac{1}{M}\sum_{m=1}^{M}\mathcal{T}\left(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \frac{\sigma_n^2}{|C_i|^2}\right) . \qquad (10)$$

It is obvious that this method can easily be extended to ISI channels where the CIR $\mathbf{c}$ varies during the transmission of one data block $\mathbf{u}$ simply by taking into account different CIRs $\mathbf{c}^{(t_\tau)}$ of the channel at Θ different time instances t_τ with $\tau = 0, \dots, \Theta - 1$ but still assuming that $\mathbf{c}^{(t_\tau)}$ is constant during the transmission of one OFDM symbol $\mathbf{y}$ of duration $(M + L)\cdot T$. As different realizations $\mathbf{c}^{(t_\tau)}$ will only result in different noise powers $\sigma_{n,m,\tau}^2$ on the subcarriers m at different times t_τ, (10) can be rewritten as

$$\mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \mathbf{c}^{(t_\tau)}) = \frac{1}{M\cdot\Theta}\sum_{\tau=0}^{\Theta-1}\sum_{m=1}^{M}\mathcal{T}_{\text{DM},n,m,\tau}$$

$$= \frac{1}{M\cdot\Theta}\sum_{\tau=0}^{\Theta-1}\sum_{m=1}^{M}\mathcal{T}\left(\mathcal{I}_{\text{DM}}^{[\text{apri}]}, \frac{\sigma_n^2}{|C_m^{(t_\tau)}|^2}\right) . \qquad (11)$$

The size of the set of SDM trajectories recorded from AWGN simulations depends on the minimum and maximum attenuation that may occur during transmission taking into account all subcarriers and all channel realizations. Let us define:

$$C_{\min} := \min_{m,\tau}(C_m^{(t_\tau)}) \text{ and } C_{\max} := \max_{m,\tau}(C_m^{(t_\tau)})$$

$$\Rightarrow \sigma_{\min}^2 := \frac{\sigma_n^2}{|C_{\max}|^2} \text{ and } \sigma_{\max}^2 := \frac{\sigma_n^2}{|C_{\min}|^2}\,.$$

Then the set size can be determined by choosing an arbitrary number of AWGN SDM characteristics simulated for noise powers σ_n^2 with $\sigma_{\min}^2 \leq \sigma_n^2 \leq \sigma_{\max}^2$ at different resolutions $\Delta\sigma_n^2$.

So far we have only considered one fixed modulation with a certain mapping μ. For single carrier modulation it is known [7] that different mappings μ and non-uniform symbol constellations $\mathcal{Y}$ [9], [10] can improve the performance of BICM-ID systems. In the case of OFDM carrier-wise allocation of different types of modulation and/or transmit powers is used to optimize the throughput, e.g., [15]. The extension of our method to systems using different modulations and transmit powers per subcarrier is straight forward: Instead of using one set of AWGN SDM trajectories $\mathcal{T}_{\text{DM},n}$ for different noise powers and a constant modulation, for each kind of modulation a different set of SDM characteristics has to be recorded under AWGN conditions. In that way a convergence prediction at the receiver is easily at hand. Furthermore, if the CIR is changing slowly enough in time and therefore can be fed back to the transmitter sufficiently accurate, it is also possible to predict the convergence behavior of the transmission of one data block for a certain allocation of different modulation types and transmit powers to single subcarriers. In this case, optimization of the allocation with respect to the ISI channel becomes possible.

4 Simulation Results

In order to evaluate the performance of the employed system and to show the applicability of the suggested method of weighted summation of AWGN SDM characteristics computer simulations have been executed with the following parameters: The block size of $\mathbf{u}$ is set to 10000, and a rate $1/2$ feed-forward convolutional encoder with zero termination, constraint length 4, and octal generator polynomials $G_0 = \{17\}_8$ and $G_0 = \{15\}_8$ is chosen for channel coding. For modulation we use 8PSK with $I = 3$ bits and Gray mapping, set partitioning (SP), or semi set partitioning (SSP) [11] as mapping rule μ. The number of subcarriers (FFT size) is set to $M = 256$. For better reproducibility of the results, the ISI channel in all simulations is arbitrarily chosen as real FIR with CIR $\mathbf{c} \sim [1.0, 0.0, 0.5, 0.2, 0.1]$. Consequently the length of the cyclic prefix is set to $L = 4$.

4.1 BER Performance of OFDM BICM-ID

The bit error rate (BER) performance of the system is depicted in Fig. 2 for SP and SSP as mapping rules μ. Perfect CSI is assumed at the receiver for calculation of the extrinsic information. However, this prerequisite can be relaxed for the approximation of ISI SDM characteristics (c.f. results in Sec. 4.2).

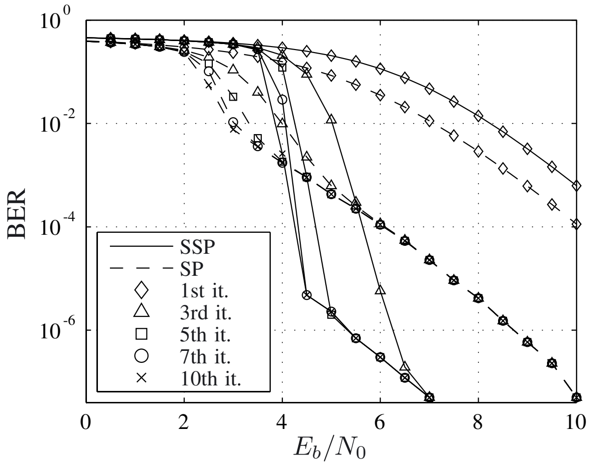

Fig. 2. BER performance of OFDM BICM-ID transmission over an ISI channel using 8PSK SP and SSP.

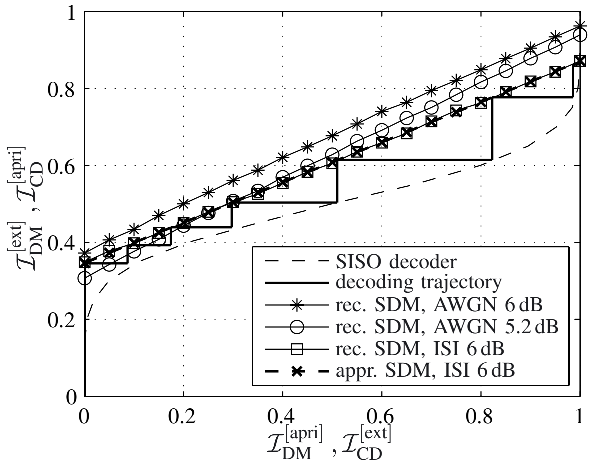

Fig. 3. Comparison of different SDM characteristics using 8PSK with SSP.

As expected for SP the BER waterfall region (or Turbo cliff) starts at low values of the ratio of bit energy to noise power spectral density (E_b/N_0 with $N_0 = \sigma_n^2$) ending in an error-floor. The waterfall region for SSP on the other hand starts at slightly higher E_b/N_0 values but leads to an lower error-floor, indicating the known superior convergence behavior of SSP in better channel conditions [11]. In an EXIT chart this fact is visualized by the amount of extrinsic information generated by the SDM $\mathcal{I}_{\text{DM}}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]} \approx 1)$, which is less for SP than for SSP assuming constant noise powers N_0.

4.2 SDM Characteristics for ISI Channels

Figure 3 depicts the convergence analysis for the BICM-ID OFDM system using SSP mapping at a symbol energy to noise power spectral density ratio (E_S/N_0) of 6 dB (which corresponds to $E_b/N_0 \approx 4.24$ dB in Fig. 2). The dashed line illustrates the characteristic of the SISO decoder, the solid line is the decoding trajectory of the iterative decoding process, which in accordance with Fig. 2 approaches the point $\mathcal{I}_{\text{DM}}^{[\text{apri}]} = 1$ very closely. Regarding the BER performance, that means an error-floor is reached as $\mathcal{I}_{\text{DM}}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}_{\text{DM}}^{[\text{apri}]} \approx 1) < 1$: no error-free decoding will be possible.

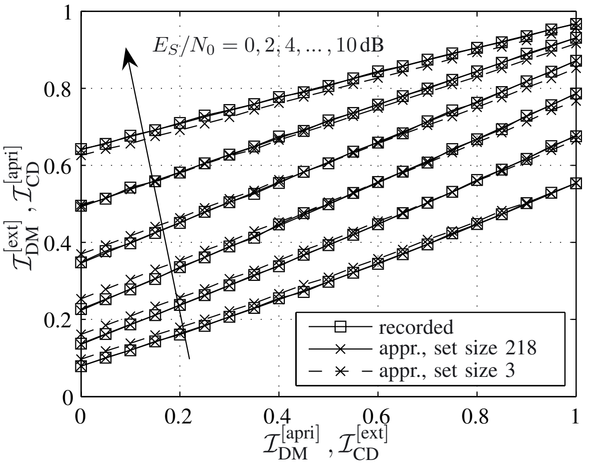

Fig. 4. Comparison of weighted and measured SDM characteristics: 8PSK, SSP.

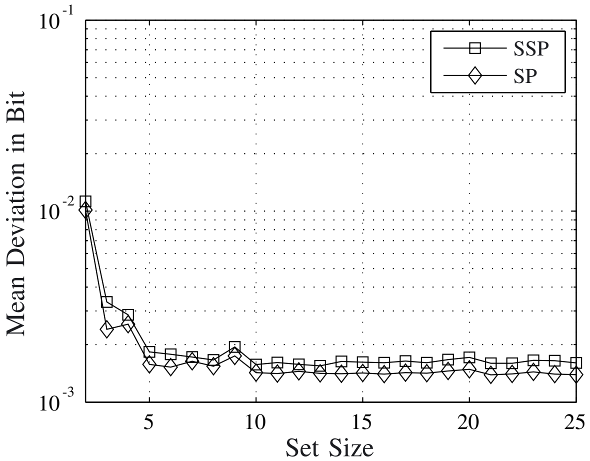

Fig. 5. Mean deviation between weighted and measured SDM characteristics: 8PSK, SP and SSP.

The marked lines represent different approximations ($*$, $\circ$, $\times$) and the actual recording ($\square$) of the SDM characteristic $\mathcal{T}_{\mathrm{DM},n}$ for the corresponding channel noise power. As predicted in Sec. 3 neglecting of the influence of the ISI channel ($*$) results in an overestimation of the capability of the SDM: the decoding trajectory falls short of the approximated SDM characteristic recorded with 6 dB AWGN.

The second approximation using the SDM characteristic recorded under AWGN conditions with $1/\overline{\sigma}_n^2 = 5.2\,\mathrm{dB}$ ($\circ$) first underestimates the capability by suggesting a narrow tunnel for $\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} < 0.3$: The decoding trajectory overshoots the characteristic in this region. For $\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} > 0.3$, however, the approximated SDM characteristic overestimates the capability of the SDM suggesting a lower error-floor than the one reached in the simulation.

The most exact approximation of the SDM characteristic is delivered by the suggested weighted summation of AWGN SDM characteristics (dashed, bold $\times$) which completely coincides with the characteristic recorded in an ISI simulation ($\square$). The decoding trajectory is exactly upper bounded by this characteristic. It becomes possible by counting the number of "stair steps" to predict the convergence of the system after seven demodulation and decoding iterations which is in exact accordance with the findings in Fig. 2.

Figure 4 depicts SDM characteristics using 8PSK SSP mappings for different noise powers recorded directly from simulations under ISI conditions ($\square$) and approximated by weighted summation ($\times$). Two different sizes of AWGN SDM characteristic sets were used. For the given CIR $\mathbf{c}$ the noise rise (difference between AWGN noise power σ_n^2 and resulting noise power on the subcarriers $\sigma_{n,m}^2$) is between -7.7 dB and 4 dB. These figures in combination with the considered E_S/N_0 values in the range of 0 dB to 10 dB demand for AWGN SDM characteristics recorded in simulations with noise powers between -7.7 dB and 14 dB. For the first approximation (solid lines in Fig. 4) the resolution for the AWGN simulations was set to $\Delta\sigma_n^2 = 0.1\,\mathrm{dB}$ leading to a set of $(14 + 7.7) \cdot 10 + 1 = 218$ AWGN SDM characteristics each with 21 positions ($\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} = 0, 0.05, 0.1, \ldots, 1$). This set size leads to a high degree of congruence of the recorded and approximated SDM characteristics for the same E_S/N_0. The mean deviation between the recorded and the approximated SDM characteristics averaged over all points of the characteristics for all E_S/N_0 values was observed to be around 0.0015 Bit which is expected to be in the range of the statistical error of directly recording an SDM characteristic.

The dashed lines in Fig. 4 show the approximated SDM characteristics using a set of only three AWGN SDM characteristics per E_S/N_0 value resulting for the six given channel qualities in a set size of $3 \cdot 6 = 18$. For each channel E_S/N_0 the set contains the AWGN SDM characteristics for the corresponding $\sigma_{\min}^2$, $\sigma_{\max}^2$, and their mean. Comparing these SDM characteristics to the recorded ones reveals higher deviations. The average deviation of the SDM characteristic was observed to be slightly above 0.01 Bit.

Figure 5 shows the mean deviations observed for different set sizes optimized for $E_S/N_0 = 4\,\mathrm{dB}$ and the given CIR, i.e., sets containing AWGN SDM characteristics for the corresponding $\sigma_{\min}^2$, $\sigma_{\max}^2$ and uniformly distributed values in-between. The deviations for other channel noises were observed to yield similar figures and progressions. Surprisingly, a set of only five AWGN SDM characteristics per expected E_S/N_0 seems to be sufficient for a very accurate approximation of the ISI SDM characteristics for the given CIR. It can be seen that the mean deviation using a set of five AWGN SDM characteristics is approximately the same as for the earlier mentioned set of 218 AWGN SDM characteristics with $\Delta\sigma_n^2 = 0.1\,\mathrm{dB}$. These findings can also be interpreted as a relaxation of the previously demanded prerequisite of perfect CSI: Also a coarsely quantized estimate of the expected noise power $\sigma_{n,m}^2$ per subcarrier seems to be sufficient for a satisfactorily accurate estimate of the corresponding ISI SDM characteristic.

Finally, Fig. 6 shows a possible application in the transmitter. In this example we consider the SDM characteristic for 8PSK with SSP ($\square$) and with Gray mapping ($\circ$) for a channel noise of $E_S/N_0 = 5\,\mathrm{dB}$. The difference between the mappings is obvious and well known: While for Gray mapping the information gain for $\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} = 0$ is relatively high as compared to SSP the

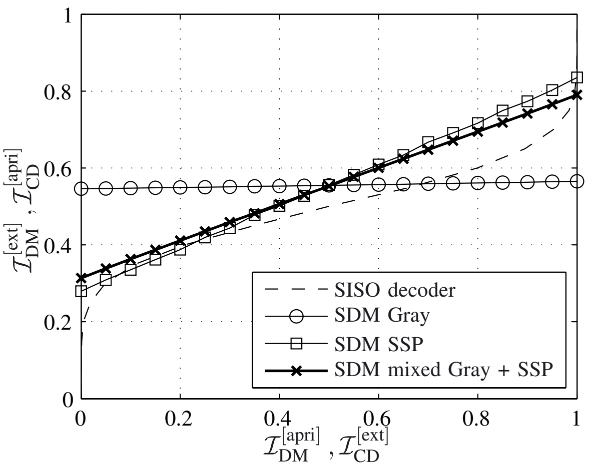

Fig. 6. EXIT optimized construction of SDM characteristic using Gray and SSP mappings.

possible information gain for good a priori knowledge ($\mathcal{I}_{\text{DM}}^{[\text{apri}]} \approx 1$) for Gray falls far behind the possibilities of SSP. Consequently, the SDM characteristic using SSP intersects very early with the SISO characteristic (dashed line) preventing BER gains after more then 3 iterations (c.f. Fig. 2, $E_b/N_0 = 3.24\,\text{dB}$). With Gray mapping on the other hand the SDM will not be able to generate extrinsic information $\mathcal{I}_{\text{DM}}^{[\text{ext}]} > 0.7\,\text{Bit}$ resulting in a high error-floor.

However, with the CIR and E_S/N_0 known at the transmitter, it is possible to design an SDM characteristic which yields a decoding tunnel and enables the SDM to generate a maximum amount of extrinsic information that allows for a good BER performance (low error-floor) after SISO decoding. Such an SDM characteristic can easily be constructed by weighted superposition of AWGN SDM characteristics of both kinds of mapping. In our example this is simply done by choosing an threshold σ^2_{thresh} and assigning Gray mapped 8PSK to subcarriers with $\sigma^2_{n,m} \leq \sigma^2_{\text{thresh}}$ while assigning 8PSK with SSP mapping to subcarriers with $\sigma^2_{n,m} > \sigma^2_{\text{thresh}}$. The resulting SDM characteristic (bold $\times$) obtained for $1/\sigma^2_{\text{thresh}} \hat{=} 0\,\text{dB}$ predicts a narrow decoding tunnel and the possibility to generate a large amount of extrinsic information which should enable the SISO decoder to decode at a BER on a comparably low error-floor after a limited number of demodulation and decoding iterations.

5 Conclusion and Outlook

EXIT charts are an important tool for predicting the convergence behavior of iterative processes. From the EXIT characteristics of decoder and demodulator it is possible to estimate the necessary number of decoding and demodulation iterations needed for receiving transmitted data in a BICM-ID system at the lowest possible BER. In this paper we addressed the construction of SDM characteristics for transmissions using an OFDM BICM-ID system in time-varying ISI conditions. As the CIR changes in time, one SDM characteristic for each CIR realization and channel noise power is necessary to predict the number of necessary iterations at the receiver based on EXIT charts. We used weighted summation of SDM characteristics prerecorded in a limited number of AWGN conditions to approximate the ISI SDM characteristic for a known CIR and channel noise power. This method does not only provide a means for easily estimating the number of necessary demodulation/decoding iterations at the receiver, it can also be used in the transmitter to allocate resources (modulation types, mappings, power) to the subcarriers with respect to the predicted convergence behavior of the receiver. The accuracy of the method has been illustrated by computer simulation. No noticeable deviations between measured and approximated SDM characteristics occurred using a set of 218 AWGN SDM trajectories ($\Delta\sigma_n^2 = 0.1\,\text{dB}$) for the given CIR. However, already small sets consisting of only five AWGN SDM characteristics achieve an comparably accurate approximation for one given CIR and channel noise power.

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