Capturing in Wireless BICM-ID Systems

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***Abstract*— In this contribution we analyze the residual bit error rate for bit-interleaved coded modulation with iterative decoding after packet collisions that may occur in wireless transmission systems. Different signal-to-noise ratios, signal-to-interferer ratios, and time offsets between the useful transmitted and the interfering packet at the receiver are considered as well as two different interference characteristics. Simulation results of an OFDM system and a single carrier system show significant performance differences which are analyzed using the technique of extrinsic information transfer (EXIT) charts.**

I. Introduction

In wireless transmission systems such as IEEE 802.11 (W-LAN) collisions of independently transmitted packets occur, e.g., due to uncoordinated medium access and/or the so-called hidden station problem. Different evaluation methods exist to determine if a packet suffering from interference caused by collisions is captured, i.e., correctly decoded or has to be declared lost [1], [2]. These methods often rely on statistical models mainly based on transmitter positions and resulting received powers of the useful and the interfering packet. However, these methods may not fully take into account the underlying physical layer structure of the receiver or the time offset between reception of useful and interfering packet.

In this contribution we analyze the residual bit error rate (BER) for bit-interleaved coded modulation with iterative decoding (BICM-ID), a system which has proven to allow for data transmission near the Shannon limit [3]. We demonstrate the influence of receiver signal-to-noise ratio (SNR), signal-to-interferer ratio (SIR), and offset between useful and interfering packet on successfully capturing interfered packets by means of simulation. Two different models for the interference are used. For the considered scenarios the comparison of single carrier (SC) and orthogonal frequency multiplexing (OFDM) modulation reveals advantages of the SC system over OFDM which are analyzed using extrinsic information transfer (EXIT) charts. This analysis allows for a better understanding of the capturing effect in wireless networks and may serve as basis for a refinement of existing and new capturing models.

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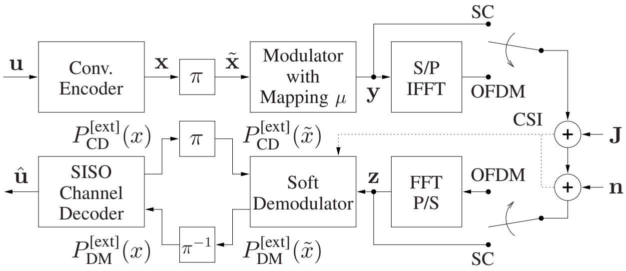

Fig. 1. Considered BICM-ID transmission system optionally using OFDM.

After this introduction the considered system model is described in Sec. II. Simulation parameters and results are given in Sec. III followed by an analysis and discussion in Sec. IV. The final Sec. V concludes this contribution.

II. System Model

For our investigations we consider the transmission system depicted in Fig. 1. A block of binary information bits $\mathbf{u}$ is encoded by a convolutional encoder to become the encoded data stream $\mathbf{x}$ which is then interleaved by a bit-interleaver π resulting in the interleaved encoded data stream $\tilde{\mathbf{x}}$. The bit-interleaved data bits $\tilde{\mathbf{x}}$ are grouped into sets of I bits and mapped to modulation symbols $y \in \mathcal{Y} \subset \mathbb{C}$ of a constellation $\mathcal{Y}$ using a certain mapping rule μ. That creates the symbol stream $\mathbf{y}$ of length L. If the OFDM system is considered, the complex modulation symbols are grouped to form blocks of size M with M indicating the FFT/IFFT size and therefore the number of subchannels. Each block is then processed with an IFFT. As for simplicity we do not consider multipath propagation here, any cyclic prefix is omitted.

Interference is simulated by a vector $\mathbf{J}$:

$$\mathbf{J} = \left[\mathbf{O}_{1\times\kappa}\,\hat{\mathbf{J}}_{1\times L-\kappa}\right] \qquad (1)$$

with all-zero vector $\mathbf{O}$ of size $\kappa \in [0, L]$ denoting an integer sample offset between useful packet and interfering packet at the receiver which we consider to be sufficiently well estimated. Interference can be realized in two different ways:

a) CWGN: The samples of $\hat{\mathbf{J}}$ consist of complex white Gaussian noise (CWGN) with power σ_j^2.
b) Modulated: The samples of $\hat{\mathbf{J}}$ are generated using the same modulation (SC or OFDM) and coding as the useful packet before being scaled to a power of σ_j^2.

After being (partially) interfered the signal is subject to additive white Gaussian noise (AWGN) $\mathbf{n}$ of power σ_n^2 comprising the effects of receiver noise. In case of OFDM transmission the signal is processed block-wise with an FFT. The resulting signal $\mathbf{z}$ is then passed on to the soft demodulator (SDM). The soft demodulator and the soft input soft output (SISO) channel decoder (CD) exchange extrinsic information in a Turbo process [4]. The SDM computes extrinsic probabilities $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ for each bit $\tilde{x}^{(i)}$ [3]:

$$P_{\text{DM}}^{[\text{ext}]}(\tilde{x}^{(i)} = b) \sim \sum_{\hat{y} \in \mathcal{Y}_b^i} P(z|\hat{y}) \prod_{\substack{j=1 \\ j \neq i}}^{I} P_{\text{CD}}^{[\text{ext}]}\Big(\tilde{x}^{(j)} = \mu^{-1}(\hat{y})^{(j)}\Big) \quad (2)$$

with $b \in \{0,1\}$. Each $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ consists of the sum over all possible channel symbols $\hat{y}$ for which the ith bit of the corresponding bit pattern $\underline{\tilde{x}} = \mu^{-1}(\hat{y})$ is b. These channel symbols form the subset $\mathcal{Y}_b^i$ with $\mathcal{Y}_b^i = \{\mu([\tilde{x}^{(1)}, \ldots, \tilde{x}^{(I)}]) | \tilde{x}^{(i)} = b\}$. In the first iteration the feedback probabilities $P_{\text{CD}}^{[\text{ext}]}(\tilde{x})$ from the channel decoder (CD) are initialized as equiprobable, i.e., $P_{\text{CD}}^{[\text{ext}]}(\tilde{x}) = 0.5$. The conditional probability density $P(z|\hat{y}) = (1/\pi\sigma_{n,j}^2) \exp(-d_{z\hat{y}}^2/\sigma_{n,j}^2)$ with $d_{z\hat{y}}^2 = \|z - \hat{y}\|^2$ describes the complex channel and interference for each received sample. The resulting interferer and noise power $\sigma_{n,j}^2$ is estimated at the receiver, where we consider the influence of the estimation error negligible.

After appropriately deinterleaving the $P_{\text{DM}}^{[\text{ext}]}(\tilde{x})$ to $P_{\text{DM}}^{[\text{ext}]}(x)$, the $P_{\text{DM}}^{[\text{ext}]}(x)$ are fed into a Soft-Input Soft-Output (SISO) channel decoder, which computes extrinsic probabilities $P_{\text{CD}}^{[\text{ext}]}(x^{(i)})$ for the encoded bits $x^{(i)} = \{0, 1\}$ in addition to the preliminary estimated decoded data bits $\hat{\mathbf{u}}$. For the next iteration the $P_{\text{CD}}^{[\text{ext}]}(x)$ are interleaved again to $P_{\text{CD}}^{[\text{ext}]}(\tilde{x})$ in order to be fed into the demodulator.

III. Simulation Parameters and Results

As an example scenario we consider packets consisting of 1149 bits which will result in an integer number of complete OFDM symbols as shown in the following. A rate $\frac{1}{2}$ feed-forward convolutional encoder with zero termination, constraint length 4, and octal generator polynomials $G_0 = \{17\}_8$ and $G_0 = \{15\}_8$ is chosen for channel coding. For modulation we use 8PSK with $I = 3$ bits and semi set partitioning (SSP) as mapping rule μ which has proven to obtain optimum results in BICM-ID systems [3]. As a result we obtain $L = 768$ complex base band symbols per packet. In the case of OFDM transmission, packets are additionally mapped to 3 OFDM symbols with $M = 256$ subcarriers. The receiver is set to perform 10 soft demodulation and decoding iterations which represents a trade-off between energy consumption and BER performance.

Assuming normalized signal power on the channel the signal-to-noise ratio (SNR) is defined as

$$\text{SNR} \doteq -10 \log_{10} \left(\sigma_n^2\right) \text{ dB} \quad (3)$$

and the signal-to-interferer ratio (SIR) as

$$\text{SIR} \doteq -10 \log_{10} \left(\sigma_j^2\right) \text{ dB.} \quad (4)$$

An effective signal-to-interferer-and-noise ratio (SINR) for the complete packet can then be defined:

$$\text{SINR} \doteq -10 \log_{10} \left(\sigma_n^2 + \left(1 - \frac{\kappa}{L}\right) \sigma_j^2\right) \text{ dB.} \quad (5)$$

Figure 2 depicts the obtained simulation results. The residual BER is plotted over the offset κ between the reception of the useful packet and an interfering packet, i.e. $\kappa = 0$ marks complete overlapping while an symbol offset of $\kappa = 768$ (packet length on channel) results in reception without interference. The corresponding SINR is given on the top abscissa. BERs achieved with OFDM BICM-ID transmission are depicted as solid lines (—) while the results of SC BICM-ID system are depicted as dashed lines (- -). All simulations have been performed assuming CWGN and modulated interference.

In the top plot of Fig. 2 the receiver noise power is set to SNR = 10 dB resembling average channel conditions. The interferer powers are set to equal the signal power, i.e., SIR = 0 dB. As expected, the capability to capture interfered packets with low BER increases with the offset κ. By using iterative demodulation and decoding significant gains can be achieved. For instance, for a desired residual BER of 10^{-3} no symbol offset can be tolerated in the non-iterative case[1], while in the iterative case an offset of $\kappa \approx 380$ samples (approx. 50 % of useful packet disturbed by interference) can be tolerated in case of SC transmission. In case of OFDM transmission an offset of $\kappa \approx 480$ samples (approx. 38 % disturbed by interference) results in the desired BER.

[1] In this contribution "non-iterative" refers to the first decoding and demodulation step of the given BICM-ID system. An optimized BICM system (without iterative decoding) uses Gray mapping instead of SSP, which obtains a BER performance in-between those given in Fig. 2, but exhibits the same significant differences between SC and OFDM.

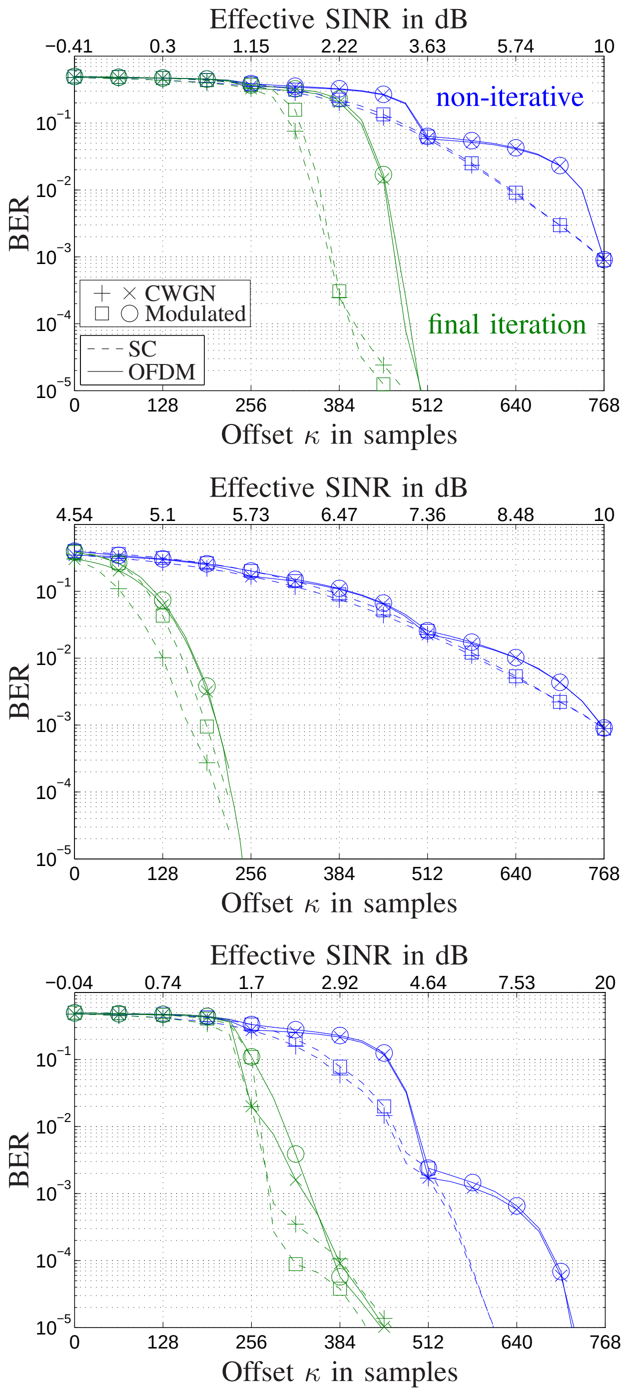

Fig. 2. BER vs. sample offset and effective SINR per packet with correlated and uncorrelated interference.
Top – SNR = 10 dB, SIR = 0 dB
Middle – SNR = 10 dB, SIR = 6 dB
Bottom – SNR = 20 dB, SIR = 0 dB

The middle and the bottom plot of Fig. 2 show the obtained residual BERs for variations of the scenario used in the top plot. For the middle plot the receiver noise power is left unchanged at SNR = 10 dB while the interferer power is reduced resulting in SIR = 6 dB. The BER performance for the non-iterative case is similar compared to the case with SIR = 0 dB: a residual BER of e.g. 10^{-3} can only be achieved without interference which is no surprise considering the receiver noise as main source of disturbance for high symbol offsets κ. For the iterative case, however, large gains can be achieved: Already with an offset of $\kappa \approx 190$ samples between useful and interfering packet (approx. 75 % disturbed by interference) a BER of 10^{-3} can be achieved. The BER performance advantage of the SC system that appeared for SIR = 0 dB is less significant for SIR = 6 dB.

The influence of SNR variation is shown in the bottom plot of Fig. 2: The interferer power is set to SIR = 0 dB while the receiver noise power is decreased (SNR = 20 dB) resembling good channel conditions. As expected, the BER performance increases for all depicted cases as compared to the case of average channel conditions (SNR = 10 dB, Fig. 2, top). To achieve an exemplary residual BER of 10^{-3}, a minimum offset of $\kappa = 520$ samples has to be guaranteed in case of the non-iterative SC, in case of OFDM the offset should exceed $\kappa = 600$. Similarly in the iterative case: A minimum offset of $\kappa = 290$ samples between useful and interfering packet is required for an BER of 10^{-3} with the SC system, while it takes an offset of at least $\kappa = 330$ for OFDM. This (partial) advantage that the SC system exhibits over OFDM in both the iterative and the non-iterative case is a rather surprising observation and is analyzed in detail in the following section.

The character of the interference seems to play a minor role in all shown BER figures. For the OFDM system no significant difference in BER performance can be noticed in Fig. 2. It is only for the results of the SC system in the top and bottom plot of Fig. 2 and for a small range of offsets κ that the results after iterative demodulation and decoding for modulated interference slightly outperform those for CWGN interference. In the middle plot of Fig. 2 a slight advantage of the SC transmission disturbed by CWGN interference can be observed as the so-called waterfall or Turbo cliff of the iterative system appears slightly earlier compared to the transmission disturbed by modulated interference.

IV. Analysis and Discussion

A powerful tool to analyze the convergence behavior of iterative systems utilizing the Turbo principle are extrinsic information transfer (EXIT) charts [5]. These depict the amount of extrinsic information $\mathcal{I}^{[\text{ext}]}$ generated by one component from a given amount of a-priori information $\mathcal{I}^{[\text{apri}]}$ in form of an EXIT characteristic $\mathcal{I}^{[\text{ext}]} = \mathcal{T}(\mathcal{I}^{[\text{apri}]})$. Gains by iterative exchange of extrinsic information are achieved if a "tunnel" exists between the characteristics of the two involved components. The number of necessary demodulation and decoding iterations can be estimated from the width and length of that decoding tunnel.

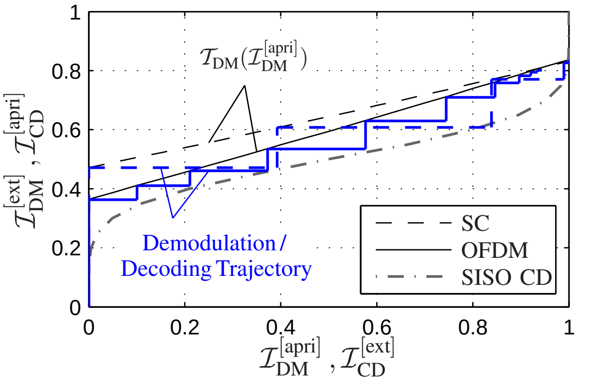

Fig. 3. EXIT chart of iterative decoding/demodulation, offset κ = 448 samples, SNR = 10 dB, SIR = 0 dB.

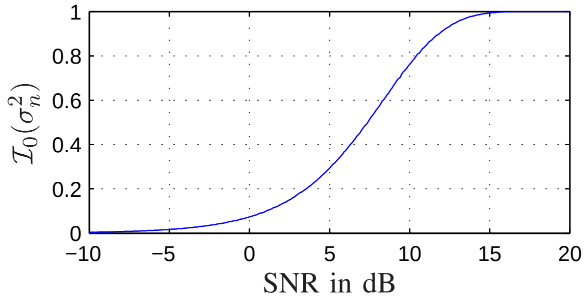

Fig. 4. Extrinsic information from SDM for $\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} = 0$ bit.

An example is given in Fig. 3 which shows the EXIT characteristic of the SISO channel decoder (dash-dotted line) and the characteristics of the soft demodulators $\mathcal{T}_{\mathrm{DM}}(\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]})$ for the SC (- -) and OFDM (—) system for the reference scenario from the top plot in Fig. 2: SNR = 10 dB, SIR = 0 dB. Additionally, the resulting demodulation and decoding trajectories are depicted for the considered number of 10 Turbo iterations. We will constrain ourselves to the case of CWGN interference. As SDM characteristics heavily depend on the channel quality [5], the offset κ between the useful and interfering packet plays a crucial role. To illustrate the cause for the difference in BER performance of SC and OFDM systems, we choose $\kappa = 448$ samples here.

Obviously, the decoding and demodulation tunnel of the SC system is significantly wider than the tunnel for the OFDM system in the given scenario. While the trajectory of the SC system reaches the top right corner of its tunnel already after 4 iterations (dashed stair steps), the trajectory of the OFDM system (solid stair steps) does not reach this point after 10 iteration. An hindering factor is that after 5 iterations the OFDM trajectory does not reach the SISO characteristic anymore thus creating a smaller amount of extrinsic information that serves as input to the SDM in the next step. This is due to the limited block size of the considered system, which prevents the decoder to generate independent extrinsic information. As a result, the amount of extrinsic information produced by the SDM of the OFDM system stays below that of the SC system, which finally leads to the higher residual BER depicted in the top plot in Fig. 2 for $\kappa = 448$ samples.

It is easy to observe and well known that the BER performance of the non-iterative system relies on the starting point of the SDM characteristic $\mathcal{I}_0(\sigma_n^2) \doteq \mathcal{T}_{\mathrm{DM}}(0)$ which can be derived analytically as a function of the noise power σ_n^2 for any modulation constellation $\mathcal{Y}$ and mapping μ [6]. As the SDM characteristics for 8PSK modulation have been observed to resemble straight lines, the width of the demodulation and decoding tunnel and therefore the number of necessary iterations to reach convergence also depends on $\mathcal{I}_0$. Figure 4 depicts this function which is monotonically increasing but non-linear over the SNR for 8PSK with SSP mapping and AWGN channels.

Let in the following $\mathcal{I}_0^{[\mathrm{SC}]}(\kappa)$ and $\mathcal{I}_0^{[\mathrm{MC}]}(\kappa)$ denote the extrinsic SDM output as function over κ for the SC and the OFDM system respectively. Let us further restrict to CWGN interference with power σ_j^2 which – when present – adds up to the receiver noise power σ_n^2. Each modulation symbol at a certain position within a packet can then be considered as being transmitted over a separate AWGN channel. Using the principle of weighted summation of SDM characteristics described in [7], $\mathcal{I}_0^{[\mathrm{SC}]}(\kappa)$ can now be constructed as weighted sum of the starting points of two SDM characteristics simulated under AWGN conditions with two different noise powers:

$$\mathcal{I}_0^{[\mathrm{SC}]}(\kappa) = \frac{\kappa}{L}\mathcal{I}_0(\sigma_n^2) + \left(1 - \frac{\kappa}{L}\right)\mathcal{I}_0(\sigma_n^2 + \sigma_j^2). \quad (6)$$

For the derivation of $\mathcal{I}_0^{[\mathrm{MC}]}(\kappa)$ let us consider the useful packet of length $L = K_\Sigma M$ consisting of $K_\Sigma \in \mathbb{N}$ OFDM symbols. Depending on the offset κ between useful packet and interfering packet we will observe

$$K_{\mathrm{u}}(\kappa) = \left\lfloor \frac{\kappa}{M} \right\rfloor \quad (7)$$

undisturbed symbols, one partially disturbed symbol, and

$$K_{\mathrm{d}}(\kappa) = K_\Sigma - K_{\mathrm{u}}(\kappa) - 1 \quad (8)$$

fully disturbed symbols. We further define

$$\hat{\kappa} = \kappa - M K_{\mathrm{u}} \quad (9)$$

with $\hat{\kappa} \in [0, M-1]$ denoting the undisturbed samples of the partially disturbed OFDM symbol. Note that for the offset κ being an integer multiple of the OFDM symbol length, i.e., $\hat{\kappa} = 0$, the symbol which is considered to be partially disturbed is actually

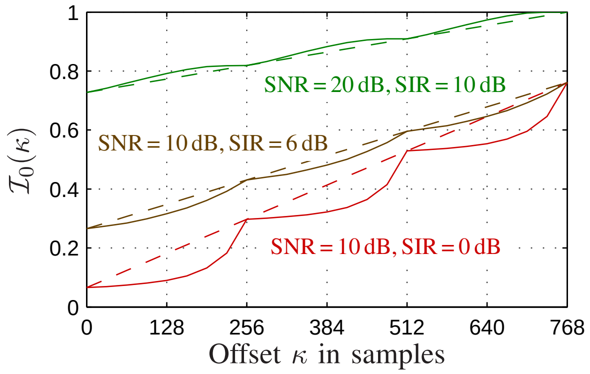

Fig. 5. Extrinsic information of SDM at $\mathcal{I}_{\mathrm{DM}}^{[\mathrm{apri}]} = 0\,\mathrm{bit}$ vs. sample offset for SC (- -) and OFDM (—) with different noise and interferer powers (CWGN).

fully disturbed. Due to the Fourier transform used for reception of the partially disturbed OFDM symbol the interferer power σ_j^2 will be equally spread over the complete symbol in frequency domain adding up to the receiver noise σ_n^2 resulting in a noise power σ_{p}^2 of the modulation symbols z within the partially disturbed OFDM symbol:

$$\sigma_{\mathrm{p}}^2(\hat{\kappa}) = \sigma_n^2 + \left(1 - \frac{\hat{\kappa}}{M}\right)\sigma_j^2. \tag{10}$$

Then $\mathcal{I}_0^{[\mathrm{MC}]}(\kappa)$ can finally be composed of the contributions of the undisturbed, the partially disturbed, and the fully disturbed OFDM symbols:

$$\mathcal{I}_0^{[\mathrm{MC}]}(\kappa) = \frac{1}{K_\Sigma}\left[K_{\mathrm{u}}\mathcal{I}_0(\sigma_n^2) + \mathcal{I}_0\left(\sigma_{\mathrm{p}}^2\right) + K_{\mathrm{d}}\mathcal{I}_0(\sigma_n^2 + \sigma_j^2)\right]. \tag{11}$$

Figure 5 depicts $\mathcal{I}_0^{[\mathrm{MC}]}(\kappa)$ (- -) and $\mathcal{I}_0^{[\mathrm{MC}]}(\kappa)$ (—) for the same SNR and SIR examples used in the top and middle plot Fig. 2. The findings fit well to the observations concerning the BER performance. Only at offsets κ equaling integer multiples of the OFDM symbol length M the curves show the same values, otherwise the extrinsic information $\mathcal{I}_0$ generated after the first soft demodulation step in the OFDM system is smaller than that of the SC system. Whenever the $\mathcal{I}_0$ difference between SC and OFDM decreases, the BER performance decreases as well and vice versa.

A third example in Fig. 5 with SNR = 20 dB (good channel conditions) and SIR = 10 dB (low interfering power) shows that in some cases the OFDM system can still outperform the SC system in terms of higher values of $\mathcal{I}_0$. However, in the shown case the values of $\mathcal{I}_0$ are already large enough to ensure a wide decoding tunnel and therefore a fast convergence of both the SC and the OFDM system for the given channel decoder.

V. Conclusions

In this contribution we consider wireless transmissions disturbed by different kinds of interference and receiver noise. It is shown by computer simulation that packets can be captured, i.e., decoded at tolerable residual BERs, in presence of CWGN and modulated interference depending on the interference power (SIR), on the receiver noise (SNR), and on the time offset between useful and received packet at the receiver. Even for interference powers equaling the signal power of the useful packet (SIR = 0 dB) transmissions can be captured using OFDM BICM-ID receivers if the offset between useful and interfering packet does not drop below two thirds of the considered packet length. For the considered SC BICM-ID system this offset limit is even shifted to one half of the packet length. The relation between the residual BER for the iterative and non-iterative case and the extrinsic information generated by the SDM in the first demodulation step is illustrated by means of EXIT charts. Analytical functions $\mathcal{I}_0^{[\mathrm{SC}]}$ and $\mathcal{I}_0^{[\mathrm{MC}]}$ are derived for the SC and OFDM system that imply an advantage in BER performance of the SC system compared to OFDM for critical scenarios with low SNR and SIR. These findings can be easily extended to other state-of-the-art and future transmission systems to provide a better understanding of capturing effects in wireless communications.

References

[1] C. T. Lau and C. Leung, "Capture Models for Mobile Packet Radio Networks," *IEEE Transactions on Communications*, vol. 40, no. 5, pp. 917 – 925, May 1992.

[2] Z. Hadzi-Velkov and B. Spasenovski, "An Analysis of CSMA/CA Protocol with Capture in Wireless LANs," in *Wireless Communications and Networking, 2003. WCNC 2003. 2003 IEEE*, Mar. 2003, vol. 2, pp. 1303 – 1307.

[3] X. Li, A. Chindapol, and J. A. Ritcey, "Bit-Interleaved Coded Modulation with Iterative Decoding and 8PSK Signalling," *IEEE Transactions on Communications*, vol. 50, no. 8, pp. 1250 – 1257, Aug. 2002.

[4] J. Hagenauer, E. Offer, and L. Papke, "Iterative Decoding of Binary Block and Convolutional Codes," vol. 42, no. 2, pp. 429–445, Mar. 1996.

[5] S. ten Brink, "Designing Iterative Decoding Schemes with the Extrinsic Information Transfer Chart," *AEÜ Int. J. Electron. Commun.*, pp. 389 – 398, Nov. 2000.

[6] T. Clevorn, S. Godtmann, and P. Vary, "EXIT Chart Analysis of Non-Regular Signal Constellation Sets for BICM-ID," in *International Symposium on Information Theory and its Applications (ISITA)*, Oct. 2004.

[7] H. Lüders, L. Schmalen, and P. Vary, "Approximating EXIT Characteristics of Soft Demodulators in OFDM BICM-ID Systems for Time Variant Frequency Selective Fading Channels," in *Proceedings of International ITG Conference on Source and Channel Coding*, Ulm, Germany, Jan. 2008.