HYBRID DIGITAL ANALOG TRANSFORM CODING

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ABSTRACT

One problem of conventional digital transmission systems is that the transmission fidelity always saturates at the design channel quality (worst-case channel condition); this saturation is due to the irreversible quantization error introduced by the source coder. Hybrid digital analog (HDA) codes address this problem by additionally transmitting the inherent quantization error by pseudo-analog methods. However, the design and the decoding of these HDA codes is complex or even impossible for long block lengths (look-up table decoding). In this study we propose an HDA transmission system for sources with correlation supporting long block lengths. This is achieved by combining transform coding with HDA transmission using well-known digital channel codes. For the evaluation Monte Carlo simulations are used with a normalized discrete cosine transform (DCTN) and Turbo codes as the digital part and LMMSE estimation in the pseudo-analog part of the HDA system. The proposed HDA system with transform coding outperforms the purely digital transmission system at all channel qualities and exploits the expected gain due to source correlation.

Index Terms— Hybrid Digital Analog (HDA), transform coding, Turbo code, LMMSE estimator

1. INTRODUCTION

Consider the problem of transmitting real-valued discrete-time source symbols or source parameters over a Gaussian channel. The source-channel separation principle [1] leads to the common solution: separate digital source coding with binary representation of the source symbols and digital channel coding. These transmission systems are excellent at the design channel quality which is usually the assumed worst-case channel condition, e.g., at the cell edge of a radio system.

One drawback of this solution, however, is often tolerated: If the receiver experiences a better channel quality (cSNR) than the transmission system is designed for, the transmission fidelity, i.e., the signal-to-noise ratio (pSNR) does not exceed the design transmission fidelity and, hence, it saturates. This saturation is caused by the inherent quantization error introduced by the digital source coder which, even in the case of error-free transmission of the coded bits, can never be compensated for. In contrast, discrete-time analog transmission systems working with continuous-amplitude processing do not show this adverse characteristic. For increased channel qualities, the transmission fidelity improves, which is referred to as *graceful improvement*.

Such systems showing *graceful improvement* can be applied where real-valued source symbols or source parameters are transmitted over channels having a higher channel quality at particular times or locations than dictated by the given (worst case) design. Examples are systems with different channel qualities for different receivers and systems with fast changing channel quality which prevents revealing the current channel quality to the transmitter. Applications include systems with inexpensive transmitters not designed to adapt to changing channels, e.g., transmitters used in wireless microphones or headsets and wireless sensors.

Several analog discrete-time transmission systems working with different approaches, e.g., chaotic dynamical systems, Archimedes spirals, orthogonal polynomials or shift maps [2, 3, 4, 5] have been proposed. They all show *graceful improvement* of the transmission fidelity. Usually these systems work well for very small block sizes. For larger block sizes, however, the design is too complex or shows poor performance. The idea of combining digital and analog coding, the so-called hybrid digital analog (HDA) coding, has been analyzed in several papers, e.g., [6, 7, 8].

Relation To Prior Work

In the context of a combined digital and analog transmission, there are, in general, several methods which could be employed to exploit correlation in source signals. In [9, 10] a system optimized for speech signals uses predictive source coding while the spectral envelope is transmitted digitally, and the prediction error is transmitted using continuous-amplitude processing.

Skoglund et al. [11] published an HDA system for sources with and without correlation by using vector quantization and a redundant mapping of quantization indices to channel vectors as the digital part and uncoded transmission of the quantization errors as the analog part. The digital and analog channel symbols are multiplexed and transmitted over an additive white Gaussian noise (AWGN) channel. The vector quantizer, the decoder codebook at the transmitter side (to calculate the quantization error), the decoder codebook at the receiver side, the redundant mapping and normalization factors for the analog part are optimized jointly using numerical methods. For an 8-dimensional Gaussian vector source whose symbols are transmitted with 16 channel symbols, Monte Carlo simulations show [11] the superiority of the HDA system over a

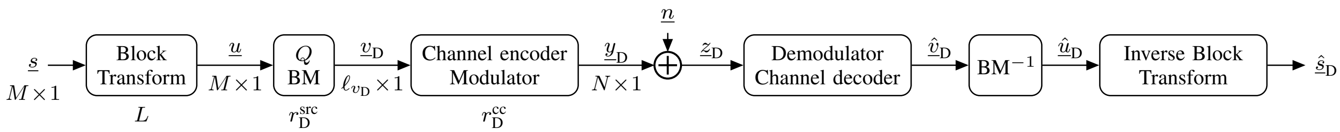

Fig. 1. Purely digital transmission system with transform coding.

purely digital system which uses the same numerical optimization strategy.

Nonetheless, the aforementioned system has drawbacks: the design of the HDA codes is complex or even impossible for long block lengths. Additionally, exhaustive search in the decoding algorithms is required for the general vector quantization and the redundant mapping (look-up table decoding).

In this contribution, we propose the use of transform coding to exploit correlation of the source in the context of HDA transmission. The ideas of our recent study [12] are taken on for a new hybrid digital analog transform coding system. It is based on a block transform to exploit source correlation and a simplified HDA design employing well-known digital codes and potentially long block lengths with analog transmission of the quantization error.

2. SYSTEM MODEL

2.1. Purely Digital Transmission System

Figure 1 shows a conventional digital transmission system with transform coding. The source emits continuous-amplitude and discrete-time source symbols $\underline{s}$ with dimension $M \times 1$ following the probability density function (pdf) $p_{\underline{\mathbf{s}}}$ and correlation of successive symbols ρ. Then, a block transform is applied which breaks up the correlation of the source symbols. The entries of the vector $\underline{u}$ are quantized (Q) and a bitmapper (BM) generates $\ell_{v_{\mathrm{D}}}$ source bits, yielding the vector $\underline{v}_{\mathrm{D}}$. The rate $r_{\mathrm{D}}^{\mathrm{src}} = \frac{M}{\ell_{v_{\mathrm{D}}}}$ describes the ratio between the source dimension and the number of bits. Subsequently, a digital channel code followed by digital modulation transforms the source bits into N real-valued symbols forming the vector $\underline{y}_{\mathrm{D}}$ with $\mathrm{E}\{y_{\mathrm{D}}^2\} = 1$. Modulation schemes using complex-valued symbols (e.g., QPSK, 8PSK) are also considered by noting the equivalence between one complex symbol and two real symbols. The ratio between the number of bits $\ell_{v_{\mathrm{D}}}$ and the number of real symbols N is denoted by $r_{\mathrm{D}}^{\mathrm{cc}} = \frac{\ell_{v_{\mathrm{D}}}}{N}$. Additive white Gaussian noise $\underline{n}$ with variance σ_n^2 per dimension disturbs the channel symbols, thereby yielding the received symbols $\underline{z}_{\mathrm{D}}$. The channel quality is denoted by

$$\mathrm{cSNR} = \frac{\mathrm{E}\{y^2\}}{\sigma_n^2}. \tag{1}$$

After demodulation, channel decoding, reconstruction of quantization levels, and finally the inverse block transform, $\hat{\underline{s}}_{\mathrm{D}}$ gives an estimate of the initial source symbols $\underline{s}$.

2.2. Block Transform and Bit Allocation

The block transform can be described by a matrix multipliation:

$$(u_{j \cdot L}, ..., u_{j \cdot L+L-1})^{\mathrm{T}} = \mathbf{T} \cdot (s_{j \cdot L}, ..., s_{j \cdot L+L-1})^{\mathrm{T}} \quad \forall j \in \mathbb{N}$$

The matrix $\mathbf{T}$ has the dimension $L \times L$. If $M > L$, the vector $\underline{s}$ is partitioned to an integer number of blocks of the length L, $\mathbf{T}$ is applied to each block and the result is again concatenated to yield $\underline{u}$ with length M. If the transform matrix $\mathbf{T}$ is orthogonal (i.e. unitary, but real valued), then the subsequent bit allocation works best and the matrix used for the inverse block transform is $\mathbf{T}^{\mathrm{T}}$. A Karhunen-Loève transform (KLT) is an orthogonal transform, but, e.g., the discrete cosine transform (DCT) is not. It can be noted that also for a source without correlation the first coefficient of the DCT transform has double the variance than the other coefficients while a normalization by $1/\sqrt{2}$ of the first coefficient again ensures orthogonality. In this paper this *normalized* DCT (DCTN) will be employed to use a orthogonal transform but to avoid sending the source specific KLT matrix as side information.

For the bit allocation, the well known water filling technique is used which repetitively assigns single bits to the quantizer of the vector entry with the highest input variance and then lowers the corresponding variance by $6\mathrm{dB}$ until all bits are spent [13]. The maximum gain in dB which can be expected exploiting source correlation is given by $10 \cdot \log_{10}\left((1-\rho^2)^{-1}\right)$ [13].

2.3. Hybrid Digital Analog Transmission System

Figure 2 illustrates the HDA transmission system [12] which is extended here by transform coding. The general idea of

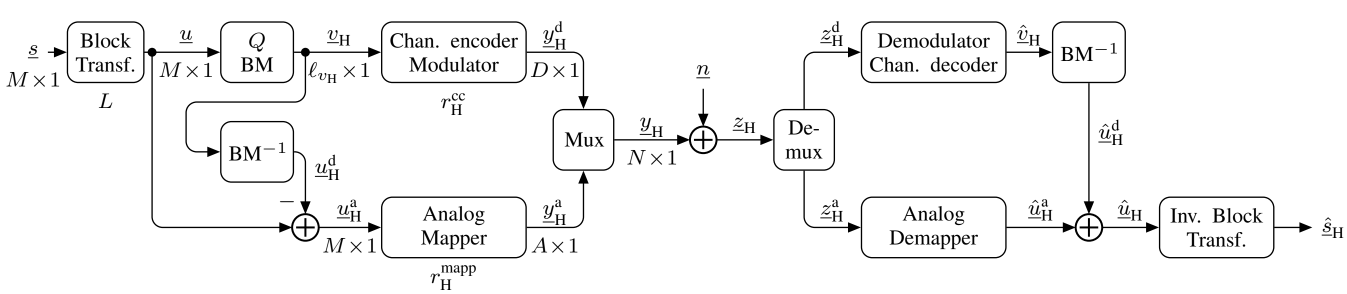

Fig. 2. HDA transmission system with transform coding. All operations are carried out by digital signal processing.

this HDA system is to use a conventional digital transmission system for the transformation coefficients $\underline{u}$ and additionally transmit the quantization error $\underline{u}_{\mathrm{H}}^{\mathrm{a}}$ in the transform domain by using continuous-amplitude (pseudo-analog) processing. The upper branch of the hybrid encoder and decoder is referred to as the *digital* part and the lower branch as the *analog* part. All operations, also in the analog part, are conducted using digital signal processing. The analog symbols are floating point variables with a precision depending on the digital architecture. In a real-world system, only the transmission channel and the signals at the interfaces of the system are "truely" analog.

The digital part is a purely digital transmission system; the number of real channel dimensions used by the digital part is denoted by D. The analog part utilizes $A > 0$ channel uses. Thus, the total number of channel uses in the HDA system is:

$$N = D + A. \tag{2}$$

In order to compare both systems, the respective numbers of channel uses (N) in the digital system and in the HDA system are always equal.

In the hybrid encoder, first the block transform is applied to the source vector $\underline{s}$ to calculate $\underline{u}$. The entries in the vector $\underline{u}$ are quantized (Q) and a bitmapper (BM) generates the source bits $\underline{v}_{\mathrm{H}}$. Then, the bitmapping is again inverted yielding a quantized source representation $\underline{u}_{\mathrm{H}}^{\mathrm{d}}$. The distortion in the transform domain introduced by the quantizer $\underline{u}_{\mathrm{H}}^{\mathrm{a}} = \underline{u} - \underline{u}_{\mathrm{H}}^{\mathrm{d}}$ is processed in the analog branch. The analog mapper uses continuous-amplitude processing to map the vector $\underline{u}_{\mathrm{H}}^{\mathrm{a}}$ to the vector $\underline{y}_{\mathrm{H}}^{\mathrm{a}}$ with length A and average energy $\mathrm{E}\{(y_{\mathrm{H}}^{\mathrm{a}})^2\} = 1$. Due to the bit allocation and the resulting different quantizers for different entries of $\underline{u}$, the variance of the quantization error is not the same for all entries. Therefore, for each entry, a different normalization has to be applied:

$$y_{\mathrm{H}i}^{\mathrm{a}} = \sqrt{\frac{1}{\mathrm{E}\{(f(u_{\mathrm{H}i}^{\mathrm{a}}))^2\}}} \cdot f(u_{\mathrm{H}i}^{\mathrm{a}}) \qquad \forall\, 0 \leq i < M. \tag{3}$$

The bit allocation may lead to entries which are not transmitted digitally at all (allocation of 0 bits). The quantization error, which in this case is the original entry, is still transmitted, but using only the analog branch. The ratio between the input and the output dimensions of the analog mapper is $r_{\mathrm{H}}^{\mathrm{mapp}} = \frac{M}{A}$. This mapping $f(\cdot)$ could, e.g., be a linear amplification or a nonlinear function with a rate of $r_{\mathrm{H}}^{\mathrm{mapp}} = 1$ or an Archimedes Spiral [3, 14] which maps one symbol onto two symbols ($r_{\mathrm{H}}^{\mathrm{mapp}} = 1/2$).

After multiplexing the symbols from the digital and analog branch and transmitting over the AWGN channel, the symbols are demultiplexed and conveyed to the digital and analog decoding branches. The analog demapper then estimates ($\hat{\underline{u}}_{\mathrm{H}}^{\mathrm{a}}$) the original quantization error which can be facilitated using several methods such as maximum likelihood (ML), maximum a posteriori (MAP), linear minimum mean square error (LMMSE) and minimum mean square error (MMSE) estimators. The outputs of the analog and digital branches are added, and after inverse block transformation, $\hat{\underline{s}}_{\mathrm{H}}$ gives an estimate of the initial source symbols. The end-to-end parameter SNR for both systems is described by

$$\mathrm{pSNR} = \frac{\sigma_{\underline{s}}^2}{\mathrm{E}\{(\underline{s} - \hat{\underline{s}})^2\}}. \tag{4}$$

2.4. HDA System Parametrization and Performance

To ensure a fair comparison between a purely digital and an HDA transmission system, the source dimension (M), the number of channel uses (N) and the energy of the channel symbols is kept constant.

Since the HDA system needs A channel uses for the analog part, fewer digital dimensions remain for the digital part compared to the purely digital system. To keep the rate of the channel coding and the modulation constant for both systems, or even achieve a better rate for the HDA system, the number of source bits $\ell_{v_{\mathrm{H}}}$ needs to be reduced. This is achieved by lowering the fidelity of the quantizer of the HDA system.

As the analog demapper, a LMMSE estimator is employed. Thus, it holds $\mathrm{E}\{(\underline{u}_{\mathrm{H}}^{\mathrm{a}} - \hat{\underline{u}}_{\mathrm{H}}^{\mathrm{a}})^2\} \leq \mathrm{E}\{(\underline{u}_{\mathrm{H}}^{\mathrm{a}})^2\}$ [15] and therefore the distortion variance introduced by the analog branch never exceeds the original quantization noise, even for very bad channel conditions. Thus, for bad channel conditions, the overall distortion is dominated by transmission errors in the digital branch. Due to the equal (or stronger) channel coding and modulation for the HDA system than in the purely digital system, the bit error rate is equal (or lower) and therefore the distortion of the HDA system is equal (or lower) than in the purely digital case. Increased distortion due to fewer quantization levels in case of bit errors are compensated for by the lowered probability of wrong quantization levels due to fewer bits for each quantized symbol [16].

In case of good channel conditions, the digital branch transmits error freely and the performance of the digital branch as well as the performance of the purely digital system saturates. In contrast, the performance of the whole HDA system still improves due to the influence of the analog branch according to [12]

$$\mathrm{pSNR}_{\mathrm{H,\,sat.}} = \frac{\sigma_{\underline{u}}^2}{\mathrm{E}\{(\underline{u}_{\mathrm{H}}^{\mathrm{a}} - \hat{\underline{u}}_{\mathrm{H}}^{\mathrm{a}})^2\}} = \frac{\sigma_{\underline{u}}^2}{\sigma_{\underline{u}_{\mathrm{H}}^{\mathrm{a}}}^2} \cdot \mathrm{pSNR}_{\mathrm{H}}^{\mathrm{a}}, \tag{5}$$

while $\mathrm{pSNR}_{\mathrm{H}}^{\mathrm{a}}$ is defined as

$$\mathrm{pSNR}_{\mathrm{H}}^{\mathrm{a}} = \frac{\sigma_{\underline{u}_{\mathrm{H}}^{\mathrm{a}}}^2}{\mathrm{E}\{(\underline{u}_{\mathrm{H}}^{\mathrm{a}} - \hat{\underline{u}}_{\mathrm{H}}^{\mathrm{a}})^2\}}. \tag{6}$$

The fraction $(\sigma_{\underline{u}}^2/\sigma_{\underline{u}_{\mathrm{H}}^{\mathrm{a}}}^2)$ in (5) refers to the pSNR of the quantizer and $\mathrm{pSNR}_{\mathrm{H}}^{\mathrm{a}}$ refers to the pSNR of just the analog part. The performance for the LMMSE estimator in the analog part is dependent on the channel quality and can be expressed as $\mathrm{pSNR}_{\mathrm{H}}^{\mathrm{a}} = \mathrm{cSNR} + 1$ [15].

In [12] we show that even for channel codes reaching the Shannon capacity, the loss in quantization fidelity is compensated for by the analog branch at all channel qualities and thus for every purely digital system a corresponding superior HDA system can be designed. These results which hold for sources without correlation apply to sources with correlation, too. For

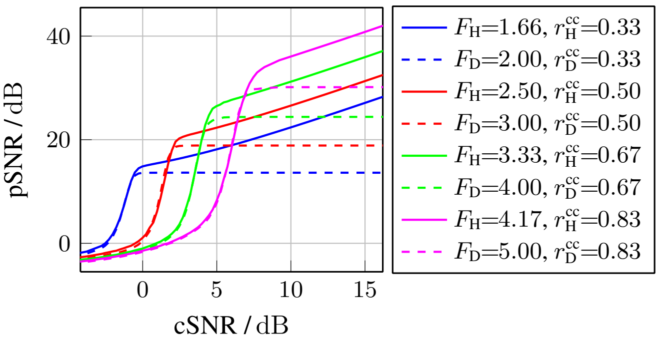

Fig. 3. HDA and digital transmission for a Gaussian source with $\rho = 0.8$, DCTN transform and Turbo coding. $M = 128$, $L = 128$, $N = 768$. F_{H} and F_{D} are the number of quantization bits per source symbol for the HDA and purely digital system, respectively.

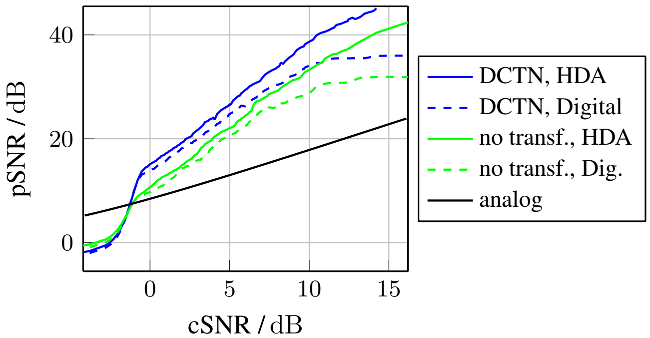

Fig. 4. Envelopes for HDA and digital transmission for a Gaussian source with $\rho = 0.8$. Simulations with Turbo coding and with and without DCTN transform. $2 \leq F_{\mathrm{D}} \leq 6$, $1.66 \leq F_{\mathrm{H}} \leq 5$, $M = 128$, $L = 128$, $N = 768$. Purely analog transmission system for comparison.

orthogonal transforms, the SNR with regard to s and u is invariant to the block transform and Parseval's theorem applies, e.g., [17]. Thus, the aforementioned properties of HDA transmission systems for sources without correlation also hold for sources with correlation using orthogonal transforms such as the DCTN or the KLT.

3. SIMULATION RESULTS

Figure 3 shows the performance of several HDA and purely digital transmission systems. A Gaussian source with correlation $\rho = 0.8$ and dimension $M = 128$ is transformed by a DCTN with $L = 128$ and quantized using skalar Lloyd-Max quantizers whose fidelity is assigned by water filling. A parallel concatenated Turbo code using convolutional component codes with the generator polynomial $\{1, 15/13\}_8$ and random interleaving is used with 20 decoding iterations. The Turbo code is the same code which is used in UMTS-LTE, though the input block length is $\ell_{\mathbf{v}}$. The modulation is choosen to be BPSK. All simulations use $N = 768$ channel uses, while the HDA system uses a linear mapper ($f(\underline{u}_{\mathrm{H}}^{\mathrm{a}}) = \underline{u}_{\mathrm{H}}^{\mathrm{a}}$) with $A = M = 128$ for the analog branch and $D = N - A = 640$ dimensions for the digital branch. Puncturing of nonsystematic bits facilitates the adaptation of the channel code rate r^{cc}. The analog demapper uses a LMMSE estimator. For each shown purely digital system, a corresponding HDA system is designed using on average fewer quantization bits per source dimension $F_{\mathrm{H}} < F_{\mathrm{D}}$ to ensure the same channel coding rate $r_{\mathrm{H}}^{\mathrm{cc}} = r_{\mathrm{D}}^{\mathrm{cc}}$. In contrast to the simulations in [12], the water filling employed here facilitates the use of, in average, fractional bits per source dimension to ensure the same channel coding rate for HDA and purely digital systems. Most interestingly, all HDA systems surpass the corresponding purely digital system for all channel qualities and additionally improve the pSNR for increasing channel quality cSNR.

In Figure 4, the same simulation parameters are used as above, but here, only the upper envelope of the curve array for varying quantizer fidelities is shown. Two experiments use the same Turbo code as above but once with a DCTN and once without transform coding. For comparison, a purely analog transmission system is depicted in which the source vector is transmitted $N/M = 6$ times and at the receiver a LMMSE estimator combines the received symbols. Again all simulations use the same source- and channel dimensions.

Transform coding using the DCTN improves the pSNR of the transmission system. For both, the purely digital and the HDA transmission system, the performance enhances by around $4.2\,\mathrm{dB}$ which is quite close to the theoretical limit of $10 \cdot \log_{10}\left((1-\rho^2)^{-1}\right) = 4.4\,\mathrm{dB}$ [13]. All envelopes of HDA systems surpass the envelopes of their corresponding purely digital transmission systems for all channel qualities.

4. CONCLUSION

In this, we propose a new hybrid digital analog (HDA) transmission system for sources with correlation. A block transform is used to break up source correlation to exploit its potential to improve the overall end-to-end fidelity. Additionally to the digital representation of the transform coefficients, the HDA system also transmits the quantization error in the transform domain using pseudo-analog methods. Thus, the inherent error introduced by the quantizer is compensated for by additional analog transmission. With improving channel qualities, thereby, the saturation of the transmission fidelity is eliminated. In comparison to a purely digital system, the transmission fidelity of the HDA system is superior for all channel qualities while the overall number of channel uses is kept constant. Moreover, since well-known, excellent digital codes are employed, long block lengths can be supported.

Simulations comparing purely digital and HDA transmission systems employing Gaussian source pdfs with correlation, an orthogonal transform with water filling as the bit allocation technique and Turbo codes show the superiority of HDA codes over purely digital codes for all channel qualities. These simulations also indicate that the performance of HDA codes additionally rises for increasing channel qualities (*graceful improvement*). Ultimately, the proposed HDA transmission system surpasses the purely digital transmission system at all channel qualities.

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