Turbo Source Compression with Jointly Optimized Inner Irregular and Outer Irregular Codes

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***Abstract*—In this paper, we present a near-lossless compression scheme for scalar-quantized source codec parameters based on iterative source-channel decoding (ISCD). The scheme is comparable to a Turbo source encoder and can inherently incorporate protection against transmission errors. In order to realize a close-to-capacity transmission/compression scheme, we employ irregular redundant bit mappings and irregular inner channel codes. The irregular inner code is composed of (pseudo) randomly punctured convolutional codes of rate > 1, i.e., the compression is performed by strong puncturing of the inner code. The optimization goal is a minimum number of transmitted bits given certain source and channel conditions. We show how the inner and outer component can be jointly optimized resulting in a constrained nonlinear programming problem. An additional successive approximation based on linear programming is also presented. Simulation examples show the advantage over systems employing only a single irregular component.**

I. Introduction

In order to exploit the residual redundancy in source codec parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, *iterative source-channel decoding* (ISCD) has been proposed in [1], [2] as a means to further improve the quality of *soft decision source decoding* (SDSD) [3]. This residual redundancy occurs due to non-ideal source encoding resulting from, e.g., delay or complexity constraints.

It has been shown in, e.g., [4] and [5], that Turbo codes can also be used as source encoders, realizing a near-lossless compression scheme, meaning that a perfect reconstruction cannot be guaranteed. Conventional entropy source codes like Huffman codes are able to achieve high compression ratios with moderate computational complexity, however, in the presence of channel noise, severe error propagation and synchronization losses are possible. It has been shown by an example in [6] that in the presence of channel noise, a system utilizing fixed-length codes and channel codes of rates > 1 can achieve comparable results (in terms of reconstruction quality and symbol error rate) to a system with variable-length codes, with reduced computational complexity at the receiver.

Irregular codes for error protection have been introduced in [7], [8] as optimized outer component of serially concatenated convolutional codes. Later, this concept was successfully adapted to the area of joint source-channel coding with iterative decoding for variable- and fixed-length codes [9], [10], [11]. In [11], irregular bit mappings have been used together with a regular inner code and an optimization approach suitable for realizing error-resilient compression. In [12], [13], the (regular) bit mapping was fixed to a single parity check while optimizing an irregular inner convolutional code. The combination of irregular inner and outer codes has been proposed in [14], [15] by successively optimizing the inner and outer components in an iterative matching algorithm.

In this contribution, we formulate a single joint optimization of the inner and outer component, leading to a minimum number of bits after encoding while guaranteeing (near-lossless) reconstruction at the receiver. This leads to a single nonlinear optimization problem jointly matching the inner and outer component. Additionally, we also modify the iterative, successive matching algorithm of [15] for the application in a (near-lossless) source-compression environment. The advantage of using irregular inner and outer codes compared to the approaches with a single irregular component is that a higher degree of freedom is provided, resulting in better matching EXIT characteristics and thus better compression rate. The performance of the system employing two irregular components is evaluated by means of simulation examples.

II. System Model

In Fig. 1 the baseband model of the considered ISCD system is depicted. A frame $\underline{u}_t$ consists of K_S unquantized codec parameters $u_{t,\kappa}$ that are individually quantized using a Q-level scalar quantizer. The mapping function $\mathcal{M}_\kappa$ assigns a unique bit pattern $\mathbf{x}_{t,\kappa}$ of $M_\kappa^\star > \log_2 Q$ bits to each quantizer level according to the *irregular redundant bit mapping*. Irregular signifies that each parameter may be encoded using a different amount of bits $M_\kappa^\star$. The bit mapping is redundant as more bits than actually necessary are spent to represent a quantizer reproduction level. This means that a total number of $N_X = \sum_{\kappa=1}^{K_S} M_\kappa^\star$ bits result after bit mapping. We furthermore define $M \doteq \log_2 Q$ to be the required amount of non-redundant bits for each parameter. We furthermore assume that at the irregular redundant bit mapping stage, one out of L_{BM} distinct bit mappings can be selected from a set $\mathbb{M} = \{\mathcal{M}^{(1)}, \ldots, \mathcal{M}^{(L_{\text{BM}})}\}$. The rate of each of the L_{BM} bits mappings is defined by $r_\ell^{\text{BM}} = M/M_\ell^\star, \ell \in \{1, \ldots, L_{\text{BM}}\}$ with $M_\ell^\star$ being the number of output bits of the ℓ'th bit mapping. The number of source coded parameters to be encoded by the

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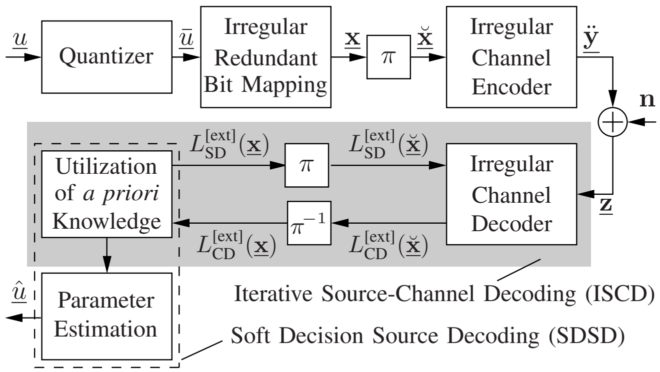

Fig. 1. Baseband model of the utilized ISCD system

bit mapping $\mathcal{M}^{(\ell)} \in \mathbb{M}$ is denoted by $K_{S,\ell}$. The total number of parameters K_S is thus given by $K_S = \sum_{\ell=1}^{L_{\mathrm{BM}}} K_{S,\ell}$.

After interleaving, the frame $\underline{\breve{\mathbf{x}}}_t$ is encoded by an irregular inner channel encoder. We use irregular inner convolutional codes of rate $r^{\mathrm{CC}} > 1$ as proposed in [12]. As pointed out in [16], the overall rate of the inner code in a serially concatenated system should be of rate ≥ 1 in order to allow the construction of a capacity-achieving system.

The bit mappings utilized throughout this paper are based on block codes with generator matrix $\mathbf{G}_4$ for $M = 4$ ($Q = 16$) which is able to generate bit mappings of rates $r_1^{\mathrm{BM}} = 4/5$ up to $r_{L_{\mathrm{BM}}}^{\mathrm{BM}} = 4/15$ (generally $r_\ell^{\mathrm{BM}} = 4/M_\ell^\star$ by utilizing only the first $M_\ell^\star$ columns of $\mathbf{G}_4$. The matrix $\mathbf{G}_4$ utilized in this paper is

$$\mathbf{G}_4 = \begin{pmatrix} 1\,0\,0\,0\,1\,1\,1\,1\,0\,1\,1\,1\,0\,0\,0 \\ 0\,1\,0\,0\,1\,1\,1\,0\,1\,0\,0\,1\,1\,1\,0 \\ 0\,0\,1\,0\,1\,1\,0\,1\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,0\,1\,1\,0\,1\,1\,1\,1\,0\,0\,0\,1\,1 \end{pmatrix}. \tag{1}$$

as already utilized in [9].

The input bit frame $\underline{\breve{\mathbf{x}}}_t$ is partitioned into L_{CC} different sub-frames $\underline{\breve{\mathbf{x}}}_\zeta^{[\mathrm{irr}]}$ which are individually encoded by one of the L_{CC} dedicated convolutional encoders $\mathcal{C}^{(\zeta)} \in \mathbb{C} = \{\mathcal{C}^{(1)}, \ldots, \mathcal{C}^{(L_{\mathrm{CC}})}\}$. Each of these convolutional encoders is a (pseudo) randomly punctured recursive convolutional code of (constant) memory J and of rate $r_\zeta^{\mathrm{CC}} \geq 1$ according to [6]. A frame consisting of N_X bits is encoded into a frame of N_E bits. The randomly punctured codes mainly consist of a rate $1/2$ recursive systematic convolutional (RSC) code punctured with a random puncturing matrix to a rate of r_ζ^{CC}. Besides their final rate, they are also characterized by the fraction $P_{\mathrm{sys},\zeta}$ of punctured systematic bits. Using $P_{\mathrm{sys},\zeta}$ and r_ζ^{CC}, the fraction of punctured non-systematic bits can be computed and the puncturing can be performed using a Bernoulli random number generator. For more details, we refer the reader to [6]. The parameters of the codes utilized in the remainder of the paper are summarized in Table I. The parameters given in Table I completely describe the codes. Note that synchronized random number generators are required at the transmitted and the receiver. All the codes are based on a recursive systematic mother code of rate $\frac{1}{2}$ with octal generator polynomials $(1, \frac{11}{17})$ and $J_\zeta = 3$, $\forall \zeta \in \{1, \ldots, L_{\mathrm{CC}}\}$.

Prior to transmission over the channel, the encoded bits are mapped to bipolar values $\underline{\ddot{\mathbf{y}}}_t$ (symbol energy $E_{\mathrm{s}} = 1$)

TABLE I
UTILIZED RANDOMLY PUNCTURED CONVOLUTIONAL CODES, MOTHER CODE WITH GENERATOR POLYNOMIAL $(1, \frac{11}{17})$

ζ	r_ζ^{CC}	$P_{\mathrm{sys},\zeta}$	ζ	r_ζ^{CC}	$P_{\mathrm{sys},\zeta}$
1	1	0.995	8	3.5	0.99
2	1.25	0.995	9	4.0	0.99
3	1.5	0.995	10	5.0	0.99
4	1.75	0.995	11	6.0	0.99
5	2.0	0.995	12	8.0	0.99
6	2.5	0.99	13	10.0	0.99
7	3.0	0.99			

which may be subject to AWGN with power spectral density $\sigma_n^2 = N_0/2$. The received symbols z_k are transformed to L-values [17] prior to being evaluated in a Turbo process which exchanges *extrinsic* reliabilities between channel decoder (CD) and soft decision source decoder (SDSD). The irregular channel decoder demultiplexes the received symbols into L_{CC} different sub-streams according to the partitioning performed inside the irregular encoder. Each of these L_{CC} sub-streams is individually decoded using a MAP algorithm [18] and the resulting L-values are multiplexed again to a single stream prior to deinterleaving. For the equations of the SDSD, we refer to the literature, e.g., [1], [2], [19]. After a given number of iterations, the block entitled *Parameter Estimation* performs a MAP estimation and selects an estimated parameter $\hat{u}_{t,\kappa} \in \mathbb{U}$.

III. Near-Lossless Source Compression Using Irregular Inner and Outer Codes

First, we show how the minimization of the number of bits N_E prior to transmission over the channel can be achieved with a system employing inner and outer irregular components. The resulting non-linear joint optimization problem is derived in Section III-A. In Section III-B on the other hand, an iterative optimization which successively optimizes outer and inner component is presented. This latter approach resembles the successive optimization given in [15].

A. Joint Optimization of Inner and Outer Code

The system introduced in Section II is used in this paper for realizing a near-lossless source compression system. The task of the optimization is to find a redundant bit mapping as well as an irregular channel encoder which jointly minimize the number of transmitted bits and allow near-lossless decoding of the parameters at the receiver/decoder. This combines the approaches of [11] and [13] which have fixed one of the components while optimizing the other component.

In order to perform source compression, the optimization goal is a different one as in [7] and [9], where N_E was fixed and the goal was to realize a capacity-achieving coding scheme with given N_E and K_S. The optimization goal is to find an EXIT characteristic which results in the smallest number of transmitted bits with the constraint that an open decoding tunnel exists. We suppose that a total number of L_{BM} different bit mappings are used. The weighting factors $\mathbf{w}_{\mathrm{BM}} = (w_{\mathrm{BM},1}, \ldots, w_{\mathrm{BM},L_{\mathrm{BM}}})^T$ of the L_{BM} different utilized bit mappings determine the fraction of bits $w_{\mathrm{BM},\ell} N_X$ after bit mapping that are encoded by the ℓ'th bit mapping (with

$\sum_{\ell=1}^{L_{\text{BM}}} w_{\text{BM},\ell} = 1$). Furthermore, the weighting factors $\mathbf{w}_{\text{CC}} = (w_{\text{CC},1}, \ldots, w_{\text{CC},L_{\text{CC}}})^T$ (with $\sum_{\zeta=1}^{L_{\text{CC}}} w_{\text{CC},\zeta} = 1$) of the irregular channel code determine the size $w_{\text{CC},\zeta} N_X$ of the sub-frame $\underline{\check{\mathbf{x}}}_\zeta^{[\text{irr}]}$ to be encoded by the ζ'th convolutional code.

The optimization goal is in fact to minimize the number of bits N_E after channel encoding. N_E can be written as

$$\begin{aligned} N_E &\approx \sum_{\zeta=1}^{L_{\text{CC}}} \frac{w_{\text{CC},\zeta} N_X + J_\zeta}{r_\zeta^{\text{CC}}} = \sum_{\zeta=1}^{L_{\text{CC}}} \frac{w_{\text{CC},\zeta} N_X}{r_\zeta^{\text{CC}}} + \sum_{\zeta=1}^{L_{\text{CC}}} \frac{J_\zeta}{r_\zeta^{\text{CC}}} \\ &= N_X \cdot \tilde{\mathbf{r}}_{\text{CC}}^T \mathbf{w}_{\text{CC}} + C \end{aligned} \tag{2}$$

with $\tilde{\mathbf{r}}_{\text{CC}} = \left(\frac{1}{r_1^{\text{CC}}}, \ldots, \frac{1}{r_{L_{\text{CC}}}^{\text{CC}}} \right)^T$ containing the inverse of the rates of the individual sub convolutional codes. The factor $C = \tilde{\mathbf{r}}_{\text{CC}}^T \mathbf{j}$, with $\mathbf{j} = (J_1, \ldots, J_{L_{\text{CC}}})$ is a constant offset factor which is due to the termination of the different sub-codes. In the case of large block lengths, $N_X \cdot \tilde{\mathbf{r}}^T \mathbf{w} \gg C$ and the effect of the termination can be neglected. However, as soon as N_X becomes smaller, the length of the compressed bit stream considerably increases with the number of utilized codes, as the constant additive term $C = \tilde{\mathbf{r}}_{\text{CC}}^T \mathbf{j}$ grows with L_{CC}. Approaches for reducing the influence of the termination include the use of tailbiting codes or a modified optimization producedure which leads to a sparse weight vector $\mathbf{w}_{\text{BM}}$ [20].

The number of bits after bit mapping N_X can be expressed by $N_X = K_S \overline{M}^\star$ with $\overline{M}^\star$ being the average number of bits per parameter. As the weights $\mathbf{w}_{\text{BM}}$ indicate the proportions of bits *after* bit mapping, the different $K_{S,\ell}$, i.e., the number of parameters encoded with the ℓ'th bit mapping, can be determined from the weights. The number of resulting output bits after encoding a portion of $K_{S,\ell}$ parameters with a bit mapping of rate r_ℓ^{BM} thus amounts to

$$K_{S,\ell} \frac{M}{r_\ell^{\text{BM}}} = w_\ell N_X \tag{3}$$

and it holds $\sum_\ell K_{S,\ell} \stackrel{!}{=} K_S$. Rewriting (3) to $N_X w_\ell r_\ell^{\text{BM}} = M K_{S,\ell}$ and summing up over all L_{BM} different bit mappings leads to

$$N_X \sum_{\ell=1}^{L_{\text{BM}}} r_\ell^{\text{BM}} w_\ell = M \sum_{\ell=1}^{L_{\text{BM}}} K_{S,\ell}$$

which can be rewritten as

$$N_X \mathbf{r}_{\text{BM}}^T \mathbf{w}_{\text{BM}} = M K_S \Rightarrow N_X = \frac{M K_S}{\mathbf{r}_{\text{BM}}^T \mathbf{w}_{\text{BM}}}\,, \tag{4}$$

with $\mathbf{r}_{\text{BM}} = (r_1^{\text{BM}}, \ldots, r_{L_{\text{BM}}}^{\text{BM}})^T$. Inserting (4) into (2) leads to

$$N_E \approx M K_S \frac{\tilde{\mathbf{r}}_{\text{CC}}^T \mathbf{w}_{\text{CC}}}{\mathbf{r}_{\text{BM}}^T \mathbf{w}_{\text{BM}}} + C\,. \tag{5}$$

As K_S and M are constant, minimizing the fraction in (5) minimizes N_E. Thus, the following constrained nonlinear programming optimization

$$\mathbf{w}_{\text{opt}} = (\mathbf{w}_{\text{BM,opt}}^T\ \mathbf{w}_{\text{CC,opt}}^T)^T = \underset{\mathbf{w}_{\text{BM}},\mathbf{w}_{\text{CC}}}{\arg\min} \frac{\tilde{\mathbf{r}}_{\text{CC}}^T \cdot \mathbf{w}_{\text{CC}}}{\mathbf{r}_{\text{BM}}^T \cdot \mathbf{w}_{\text{BM}}} \tag{6}$$

subject to

$$\mathbf{C}_{\text{CD}} \cdot \mathbf{w}_{\text{CC}} > \underline{f}_{\text{inv}}(\mathbf{C}_{\text{SD}} \cdot \mathbf{w}_{\text{BM}}) + \mathbf{o} \tag{7}$$

$$0 \le w_\ell \le 1, \qquad \forall \ell \in \{1, \ldots, L_{\text{BM}} + L_{\text{CC}}\} \tag{8}$$

$$\sum_{\ell=1}^{L_{\text{BM}}} w_{\text{BM},\ell} = 1 \quad \text{and} \quad \sum_{\ell=1}^{L_{\text{CC}}} w_{\text{CC},\ell} = 1\,, \tag{9}$$

can be formulated. The matrices $\mathbf{C}_{\text{SD}}$ and $\mathbf{C}_{\text{CD}}$ contain P sample points of the constituent EXIT characteristics of SDSD and channel decoding and $\underline{f}_{\text{inv}}(\cdot)$ interpolates sample points of the inverse EXIT function. The constraint (7) guarantees an open EXIT chart decoding tunnel while constraints (8) and (9) guarantee the validness of the weights. The decoding tunnel width and thus the decoding complexity can be adapted by the offset vector $\mathbf{o}$. However, widening the decoding tunnel leads to a penalty in the compression performance as the optimization tends to select codes with lower rates in order to fulfill the constraints. Numerical algorithms to solve the optimization can be found in, e.g., [21]. For further information about how to set up the matrices $\mathbf{C}_{\text{SD}}$ and $\mathbf{C}_{\text{CD}}$, we refer the reader to [11], [13].

Using the factors $\mathbf{w}_{\text{CC}}$, the irregular convolutional code can be set up by just encoding a portion of $w_{\text{CC},\zeta} N_X$ bits by the ℓ'th code. After the optimization, N_X can be found by inserting $\mathbf{w}_{\text{BM}}$ into (4). The number of parameters $K_{S,\ell}$ to be encoded by the ℓ'th bit mapping still have to be determined. Using the vector $\mathbf{w}_{\text{BM,opt}}$, the different $K_{S,\ell}$, $\ell = 1, \ldots, L_{\text{BM}}$ can easily be found by combining (3) and (4)

$$\begin{aligned} K_{S,\ell} &= w_{\text{BM,opt},\ell} \cdot r_\ell^{\text{BM}} \cdot \frac{N_X}{M} \\ &= K_S \cdot \frac{w_{\text{BM,opt},\ell} \cdot r_\ell^{\text{BM}}}{\mathbf{r}_{\text{BM}}^T \mathbf{w}_{\text{BM,opt}}}\,. \end{aligned} \tag{10}$$

The drawback of the presented approach is that the optimization problem (6) is in general not convex, however, due to the small dimensionality of the problem (usually, less than $10 - 20$ code configurations are considered), it can be solved with reasonable computational complexity.

Finally note that the proposed approach also allows the joint optimization of the inner and outer components in a non-compression scenario, i.e., if the overall coding rate (or the number of bits N_E) is fixed. In this case, an additional constraint fixing N_E using (5) has to be added to the optimization problem (6).

B. Successive Inner and Outer Matching

Maunder *et al.* propose to use an iterative matching approach [15] for successively optimizing the inner and outer component. First the outer component is fixed and the inner is optimized. This inner optimized component is then fixed while trying to match the outer component. This procedure is iteratively repeated several times until convergence is observed. The approach in [15] uses a similar approach as [7] by optimizing the L_2-norm of the EXIT chart tunnel width and fixed rates (unity rate for the inner code and a rate < 1 for the outer code). The successive matching approach can also be realized for source compression by successively optimizing the outer and inner component. The optimization starts at iteration $i = 1$ by first optimizing the number of bits N_X after bit mapping by solving the linear program (see [11] for details)

$$\mathbf{w}_{\text{BM,opt}}^{(i)} = \arg\max_{\mathbf{w}_{\text{BM}}} \mathbf{r}_{\text{BM}}^T \mathbf{w}_{\text{BM}} \tag{11}$$

subject to

$$\mathbf{C}_{\text{SD}} \cdot \mathbf{w}_{\text{BM}} > \mathbf{d}_{\text{CC,inv}}^{(i-1)} + \mathbf{o} \tag{12}$$

$$0 \leq w_{\text{BM},\ell} \leq 1 \quad \forall \ell \in \{1, \ldots, L_{\text{BM}}\} \tag{13}$$

$$\sum_{\ell=1}^{L_{\text{BM}}} w_{\text{BM},\ell} = 1\,, \tag{14}$$

with $\mathbf{d}_{\text{CC,inv}}^{(0)}$ containing P sample points of an initial (usually regular) inner convolutional code characteristic. Using the optimized weights $\mathbf{w}_{\text{BM,opt}}^{(i)}$, an optimized EXIT curve of the bit mapping can be computed according to $\mathbf{C}_{\text{SD}}\mathbf{w}_{\text{BM,opt}}^{(i)}$ and its inverse is interpolated and stored in the vector $\mathbf{d}_{\text{BM,inv}}^{(i)}$, with i denoting the iteration counter. Subsequently, the number N_E of channel coded bits is minimized using the linear program (see [13] for details)

$$\mathbf{w}_{\text{CC,opt}}^{(i)} = \arg\min_{\mathbf{w}_{\text{CC}}} \tilde{\mathbf{r}}_{\text{CC}}^T \mathbf{w}_{\text{CC}} \tag{15}$$

subject to

$$\mathbf{C}_{\text{CC}} \cdot \mathbf{w}_{\text{CC}} > \mathbf{d}_{\text{BM,inv}}^{(i)} + \mathbf{o}' \tag{16}$$

$$0 \leq w_{\text{CC},\zeta} \leq 1 \quad \forall \zeta \in \{1, \ldots, L_{\text{CC}}\} \tag{17}$$

$$\sum_{\zeta=1}^{L_{\text{CC}}} w_{\text{CC},\zeta} = 1\,. \tag{18}$$

Using the result of the optimization, the optimized EXIT curve of the inner convolutional code is given by $\mathbf{C}_{\text{CC}}\mathbf{w}_{\text{CC,opt}}^{(i)}$. After computing the inverse of this characteristic and interpolating P sample points to be stored in $\mathbf{d}_{\text{CC,inv}}^{(i)}$, the iteration counter can be increased ($i = i+1$) and a subsequent iteration can be carried out. The iterations can be stopped if the number of bits N_E, evaluated using (5) does not change from iteration $i-1$ to iteration i. The assignment of bit mappings to parameters can again be found using (10).

IV. Simulation Examples

The capabilities of the proposed EXIT chart matching techniques shall be illustrated by means of simulation examples. For reasons of reproducibility, we utilize a Gauss-Markov source modelling a first order autoregressive process with autocorrelation $\rho = 0.9$, a value which can be found e.g., in MP3 and CELP encoders. A frame consisting of K_S correlated parameters is quantized using a $Q = 16$ scalar quantizer ($M = 4$). In order to maximize the quality of the signal, *Lloyd-Max* quantization is employed (see, e.g., [22]).

Figure 2 depicts an exemplary EXIT chart showing the characteristics of the $L_{\text{BM}} = 11$ different constituent bit mappings (rates ranging from $\frac{4}{5}$ to $\frac{4}{15}$) and $L_{\text{CC}} = 13$ different constituent inner convolutional codes given in Table I. The channel is assumed to be perfect, i.e., $E_s/N_0 \to \infty$, simulating a pure storage (no error) scenario. It can be seen that both EXIT curves match quite well and an extremely narrow decoding tunnel is present resulting in an almost perfect (ideal) compression. In fact, this example setup results in a total number of 2.6307 bit/parameter. The source conditional entropy amounts to $H(\bar{u}_t|\bar{u}_{t-1}) = 2.62$ bit. The system with irregular bit mapping and regular channel code (irrBM & regCC) [11] requires 2.6612 bit/parameter while the system with regular bit mapping and irregular channel code (regBM & irrCC) [13] requires 2.6481 bit/parameter.

TABLE II
BITS/PARAMETER ($Q = 16, \rho = 0.9$) REQUIRED FOR AN OPEN DECODING TUNNEL (DECODING THRESHOLD) AT DIFFERENT SYSTEM SETUPS

	irrBM & regCC	regBM & irrCC	jointly opt. irrBM & irrCC	successive irrBM & irrCC
$E_s/N_0 \to \infty$	2.6612	2.6481	**2.6307**	2.6358
$E_s/N_0 = 10$ dB	2.6673	2.6414	**2.6304**	2.6341
$E_s/N_0 = 5$ dB	2.7393	2.7046	**2.6910**	2.6991
$E_s/N_0 = 2$ dB	3.0737	3.0941	**3.0624**	3.0731
$E_s/N_0 = 1$ dB	3.3728	3.3524	**3.3171**	3.3251
$E_s/N_0 = 0$ dB	3.7891	3.7124	**3.6537**	3.6576

Table II shows the number of bits/parameter at the decoding threshold for the 3 different systems at different channel qualities. The system using a regular channel code utilizes the rate 2 randomly punctured code from Table I. The system employing a regular bit mapping utilizes the rate $4/5$ bit mapping corresponding to a simple parity check (5 first columns of the matrix $\mathbf{G}_4$ from (1) used).

If irregular inner and outer codes are employed, the estimated number of bits required for coding is always the lowest. The joint optimization of Sec. III-A also outperforms the successive optimization approach presented in Sec. III-B. During the generation of the results, we also have found that the successive optimization is quite sensitive to the starting point (i.e., the initial value $\mathbf{d}_{\text{CC,inv}}^{(0)}$). This first result confirms that employing irregular bit mappings and inner channel codes (irrBM & irrCC) leads to the best compression for the given

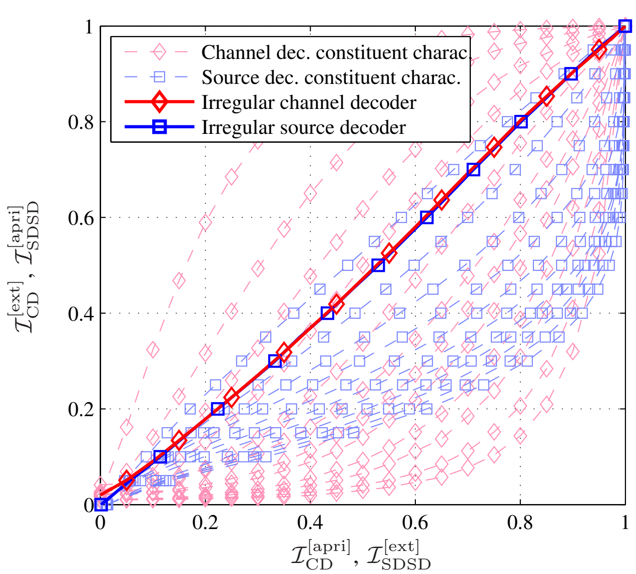

Fig. 2. EXIT chart example for irregular bit mappings and irregular inner convolutional codes ($\rho = 0.9$, $Q = 16$) for $E_s/N_0 \to \infty$

TABLE III
BITS/PARAMETER ($Q = 32, \rho = 0.0$) REQUIRED FOR AN OPEN DECODING TUNNEL (DECODING THRESHOLD) AT DIFFERENT SYSTEM SETUPS

	irrBM & regCC	regBM & irrCC	jointly opt. irrBM & irrCC	successive irrBM & irrCC
$E_s/N_0 \to \infty$	*6.0508*	4.7098	4.6999	**4.6957**
$E_s/N_0 = 10\,\text{dB}$	*5.9471*	4.7339	**4.7203**	4.7831
$E_s/N_0 = 5\,\text{dB}$	*6.6950*	4.8404	**4.8297**	4.8606
$E_s/N_0 = 2\,\text{dB}$	*6.1637*	5.5140	**5.4916**	5.5195
$E_s/N_0 = 1\,\text{dB}$	*7.0012*	6.5110	**6.1267**	6.1835
$E_s/N_0 = 0\,\text{dB}$	–	–	**7.2186**	7.2335

setup. Note that the optimization above has been performed with $\mathbf{o} = (0, \ldots, 0)^T$, signifying that an infinitesimally small EXIT tunnel results. In this case, convergence can only be achieved for $K_S \to \infty$. For practical systems, either an offset vector has to be chosen or a penalty offset in terms of E_s/N_0 leading to a wider tunnel has to be tolerated.

In order to show the performance of the proposed approach, a second example utilizing $Q = 32$ ($M = 5$) level Lloyd-Max quantization together with a non-correlated source ($\rho = 0$) is presented. $L'_{\text{BM}} = 10$ bit mappings with rates ranging from $5/6$ to $5/15$ are generated using $\mathbf{G}_5$ given by

$$\mathbf{G}_5 = \begin{pmatrix} 1\,0\,0\,0\,0\,1\,1\,1\,1\,1\,0\,1\,1\,1\,0 \\ 0\,1\,0\,0\,0\,1\,1\,1\,1\,0\,1\,1\,1\,0\,0 \\ 0\,0\,1\,0\,0\,1\,1\,1\,0\,1\,1\,1\,0\,0\,1 \\ 0\,0\,0\,1\,0\,1\,1\,0\,1\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,0\,1\,1\,0\,1\,1\,1\,1\,0\,1\,1\,1 \end{pmatrix}. \quad (19)$$

The utilized convolutional codes are the ones given in Table I with an additional code of rate $r^{\text{CC}}_{L_{\text{CC}}+1} = 1.05$ and $P_{\text{sys},L_{\text{CC}}+1} = 0.99$. This means that $L'_{\text{CC}} = L_{\text{CC}} + 1$ codes are used in this second example.

Table III summarizes the results for this second setup. It can be seen that in this case the approaches optimizing only a single irregular component have more difficulties than a system design which optimizes two irregular components. Again the joint optimization leads to a superior performance in most cases. Only for $E_s/N_0 \to \infty$, it is outperformed by the successive optimization. Note that the utilized nonlinear optimizer does not necessarily lead to the global minimum.

Finally, it has to be noticed that comparable performance can also be achieved with systems having only a single irregular component, however, great care has to be taken for the selection of the regular component, such that the matching algorithms finds a reasonable result. In the irregular approach, finding good matching characteristics is easier as the search space is increased by the larger set of possible codes. Note that for finite E_s/N_0, the proposed approach does not only perform compression, but minimizes the number of bits necessary for successful transmission.

V. CONCLUSIONS

In this contribution we have shown that a joint source-channel coding approach with irregular bit mappings, irregular inner channel codes, and iterative decoding can be effectively utilized for near-lossless Turbo source compression. In a source compression setup, the number of transmitted bits shall be minimized. Minimizing the number of bits leads to a non-linear programming optimization problem which can be solved using numerical methods. Additionally, a second, iterative approximation scheme has been introduced. The advantage of the proposed schemes is that they can easily cope with channel noise and can be flexibly adapted to varying source and channel conditions. The capabilities and the flexibility of the proposed source compression scheme have been demonstrated in two simulation examples.

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