

# ADAPTIVE EXPLOITATION OF RESIDUAL REDUNDANCY IN ITERATIVE SOURCE-CHANNEL DECODING

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## ABSTRACT

Iterative source-channel decoding (ISCD) aims at the exploitation of the time-variant residual redundancy of the source samples, e.g., source codec parameters, for error concealment and quality improvements. In most previous publications the receiver had perfect knowledge of the amount of residual redundancy. This assumption would require a reliable, i.e. highly redundant, transmission of side information. In contrast, in this paper we present a relatively simple scheme, yet efficient and robust, by which the residual redundancy at the receiver can be estimated accurately without any side information, and then can be exploited adaptively. We present the achievable performance gains in an ISCD system including the estimation of the residual source redundancy at the receiver for various scenarios. Several methods of different performance and computational complexity are proposed, with some of them even outperforming a system with perfect side information. The latter, not quite intuitive fact, is explained in the paper.

## I. INTRODUCTION

In recent years the application of the Turbo principle, i.e., the exchange of extrinsic information between at least two receiver components, has been expanded from channel coding to the whole receiver chain. This extension results in a steady approach of the Shannon limit with only a moderate rise of complexity.

In iterative source-channel decoding (ISCD)<sup>1</sup> [1, 2, 3] the residual redundancy of source codec parameters such as predictor coefficients or scale factors of speech, audio, and video signals is exploited in a Turbo process. Due to imperfect source coding, which results from complexity and delay constraints, the source coded signal still contains a non-negligible amount of residual redundancy. The a priori information of this redundancy, e.g., a non-uniform probability distribution or an autocorrelation, is used in a soft decision source decoder (SDSD) [4, 5]. The SDSD, which possesses error concealing capabilities rather than error correcting capabilities, iteratively exchanges extrinsic information with a channel decoder.

<sup>1</sup>Note that the term iterative source channel decoding is also used in a different context, i.e., for the iterative evaluation of variable-length source codes and channel codes. Here, ISCD is used to reconstruct the most probably sent source parameter from the channel decoded soft bit stream of fixed length.

For the reconstruction of the transmitted source samples in the SDSD it makes sense to exploit the current source correlation, though, in general the source correlation is not known to the receiver, since we neither assume any transmission of side information nor perfect knowledge at the receiver of the amount of residual source redundancy as in previous publications, e.g., [6]. Another approach that deals with the estimation of the source correlation at the receiver should also be mentioned here and may be found in [7]. The authors apply ISCD to spatially separated correlated sources, which is a different scenario than the one we deal with in this paper as we have only a single source that emits a stream of correlated source parameters. For this scenario we propose some (algorithmically simple, yet efficient) methods of different performance and computational complexity wherewith it is feasible to estimate the source correlation and exploit it in the SDSD.

Using a general Gauss-Markov source and an arbitrary time-varying source correlation function as a model for the correlation of the source parameters we obtain simulation results that demonstrate the capabilities of the proposed methods. As channel coding scheme we apply a flexible low-density parity-check (LDPC) [8, 9] coding scheme as introduced in [10] to ISCD.

## II. ISCD TRANSMISSION SCHEME

The baseband model of the utilized ISCD transmission scheme is depicted in Fig. 1. Source codec parameters  $u$  are generated, e.g., by a Gauss-Markov source, with an inherent autocorrelation  $\rho$  in order to obtain comparable and reproducible results. At time instant  $\tau$ ,  $K$  source codec parameters  $u_{k,\tau}$  are assigned to one frame  $\underline{u}_\tau$  with  $k=0, 1, \dots, K-1$  denoting the position in the frame. In this paper the autocorrelation  $\rho$  is time-variant framewise, i.e.,  $\rho = \rho_\tau$  is constant during one frame. The autocorrelation takes on values from a finite set, e.g.,  $\rho \in \{0.0, 0.1, \dots, 0.9\}$ . The value-continuous and time-discrete source samples  $u_{k,\tau}$  are each quantized to a quantizer reproduction level  $\bar{u}_{k,\tau}$ . To each  $\bar{u}_{k,\tau}$  a unique bit pattern  $\mathbf{x}_{k,\tau}$  of  $M$  bits is assigned according to the utilized index assignment. The single bits of a bit pattern  $\mathbf{x}_{k,\tau}$  are indicated by  $x_{k,\tau}^{(m)}$  with  $m=0, 1, \dots, M-1$ , and the frame of bit patterns is denoted as  $\underline{\mathbf{x}}_\tau$ .

Although there exist several index assignments [11, 3, 12] designed for ISCD, we choose the natural binary (NB) index assignment. Since the enhanced index assignments are de-

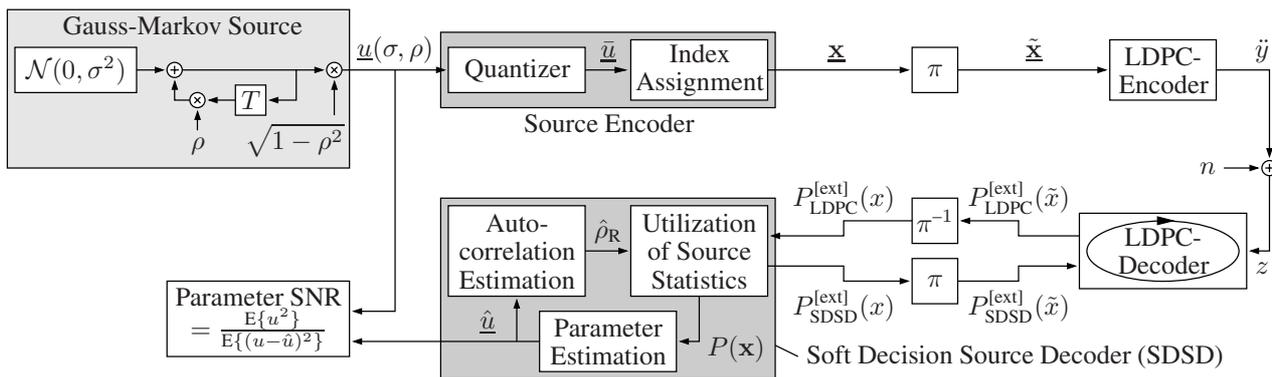


Figure 1: Baseband model of ISCD with LDPC codes.

signed for a certain amount of residual redundancy, they work well only if the signal bears that specific amount of residual redundancy, else the performance decreases. One way is to choose the index assignment corresponding to the current amount of residual redundancy of the signal, which would require a transmission of side information on the utilized index assignment. Another way is to use an unadapted index assignment, which is independent of the amount of residual redundancy such as the natural binary index assignment, which we utilize in this paper,

The bit interleaver  $\pi$  scrambles the incoming frame  $\underline{x}_\tau$  of bit patterns to  $\tilde{\underline{x}}_\tau$  in a deterministic manner. To simplify the notation, we restrict the interleaving to a single time frame with index  $\tau$  and omit the time frame index  $\tau$  in the following where appropriate.

For the channel encoding of a frame  $\tilde{\underline{x}}$  of interleaved bits  $x$  we utilize LDPC codes, which were first proposed by Gallager [8] and rediscovered by MacKay [9]. LDPC codes have a very high error correction capability with iterative decoding that is very close to the Shannon limit. Their performance is comparable or even superior to that of convolutional Turbo codes. In this paper we use a modification of short LDPC codes as presented in [10]. Identical instances of a short LDPC code are combined to a long LDPC code, whose frame size is flexible in multiples of a subframe size, i.e., the frame size of the short LDPC code. By serially concatenating the subframes with a bit-interleaver and a second component that provides extrinsic information according to the Turbo principle (e.g., a soft decision source decoder (SDSD) as in this paper), extrinsic information can also be exchanged between subframes. Such concatenated LDPC codes approach very well the performance of long monolithic LDPC codes of the same frame size [10]. The performance of the concatenated LDPC code strongly depends on the performance of the short code. Therefore, the short code has to be chosen carefully. As short LDPC code a (21,11) difference set cyclic (DSC) code [13] is used. DSC codes feature a high minimum Hamming distance, and especially at short block lengths they can outperform comparable pseudo-random LDPC codes [14].

The resulting codeword is denoted as  $\underline{y}$  with bits  $y$ , which are mapped to bipolar bits  $\tilde{y} \in \{\pm 1\}$  for BPSK transmission with

symbol energy  $E_S = 1$ . We choose the simple BPSK modulation scheme, since modulation is no design issue in this paper.

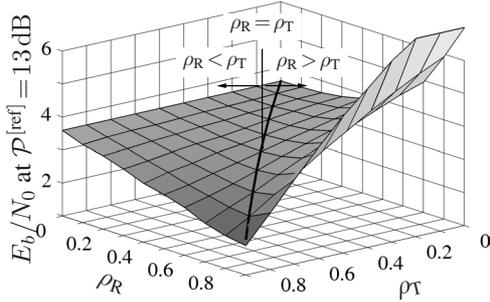
On the channel, the signal  $\tilde{y}$  is superposed with additive white Gaussian noise (AWGN)  $n$  with the known power spectral density  $\sigma_n^2 = N_0/2$ , i.e.,  $z = \tilde{y} + n$ .

The received symbols  $z$  are evaluated in a Turbo process, in which extrinsic reliabilities between the LDPC decoder and the SDSD are exchanged. Utilizing LDPC codes results in an additional iterative loop in the LDPC decoder, in which extrinsic information is exchanged between the variable nodes and the check nodes. These iterations are denoted as LDPC-iterations.

Details about the ISCD receiver can be found in [1, 2, 3]. The LDPC decoder uses the belief propagation algorithm [15, 9] to generate extrinsic information. The SDSD determines the extrinsic information mainly from the natural residual source redundancy, which generally remains in the bit patterns  $\mathbf{x}_k$  after source encoding. Such residual redundancy appears on parameter-level, e.g., as a non-uniform distribution  $P(\tilde{u}_k)$ , in terms of a correlation, or as any other possible time-dependencies. The latter terms of residual redundancy are generally approximated by a first order Markov chain, i.e., by exploiting the conditional probabilities  $P(\mathbf{x}_k | \mathbf{x}_{k-1})$ . These conditional probabilities heavily depend on the source correlation. For specific source correlations, e.g.,  $\rho \in \{0.0, 0.1, \dots, 0.9\}$ , they can be calculated once in advance. The technique how to combine this a priori information  $P(\mathbf{x}_k | \mathbf{x}_{k-1})$  on parameter-level with the soft input values  $P_{\text{LDPC}}^{\text{extl}}(x)$  on bit-level is also well known in the literature. The algorithm how to compute the extrinsic  $P_{\text{SDSD}}^{\text{extl}}(x)$  has been detailed, e.g., in [1, 2, 3].

### III. ESTIMATION OF TIME-VARIANT SOURCE CORRELATION

In Fig. 2 the required  $E_b/N_0$  to achieve the arbitrary reference parameter SNR  $\mathcal{P}^{\text{refl}} = 13\text{dB}$  is depicted for all combinations of source correlation  $\rho_T$  and assumed correlation at the receiver  $\rho_R$  with  $(\rho_T, \rho_R) \in \{0.0, 0.1, \dots, 0.9\}$ . The simulation parameters correspond to the ones detailed in Sec. IV.. The solid line (matching line) on the surface connects the points  $\rho_R = \rho_T$ . Left to this line, i.e.,  $\rho_R < \rho_T$ , a shallow degradation can be observed



**Figure 2:**  $E_b/N_0$  at  $\mathcal{P}^{[\text{ref}]} = 13\text{dB}$  of ISCD with source autocorrelation  $\rho_T$  and assumed autocorrelation  $\rho_R$  at the receiver.

by decreasing  $\rho_R$  until reaching a constant level for  $\rho_R \equiv 0 \forall \rho_T$ , which corresponds to the case of unexploited source correlation. To the right of the matching line, i.e.,  $\rho_R > \rho_T$ , severe degradations occur by increasing  $\rho_R$ , especially for low  $\rho_T$ . This shows that the system reacts highly sensitive to an over-estimation of the correlation, i.e.,  $\rho_R > \rho_T$ . Thus, the receiver must obviously know the residual redundancy quite accurately, either by transmitted side information or by estimation. In the following, we present efficient methods for the latter task.

#### A. Source Correlation Model Functions

The source correlation  $\rho_T$  in this section is time-variant frame-wise. In order to determine the correlation  $\rho_R$  at the receiver, four estimation methods of different performance and computational complexity are proposed in the following. The performance of the estimation methods is exemplarily presented by means of two discrete sinusoidal source correlation functions, visualized in Fig. 3, differing in frequency:

$$\rho_{T1}(\tau) = 0.4 + \frac{1}{10} \cdot [2.9 \cdot (1 + \sin(\omega_1 \tau - \frac{\pi}{2}))], \quad \text{with } \omega_1 = \frac{8\pi}{25} \quad (1)$$

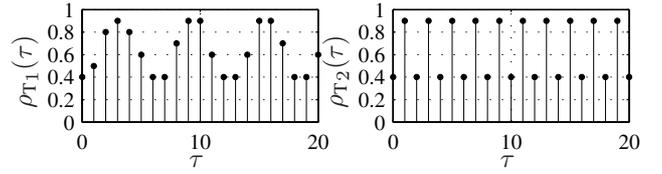
and

$$\rho_{T2}(\tau) = 0.4 + \frac{1}{10} \cdot [2.5 \cdot (1 + \sin(\omega_2 \tau - \frac{\pi}{2}))], \quad \text{with } \omega_2 = \pi. \quad (2)$$

These source correlation functions have been defined to generate reproducible, time-varying source correlations. The source correlations vary from frame to frame. The first function has a moderate variation, i.e., the source correlation difference from one frame to the following is rather small, whereas the second function has the maximum variation possible. Thus, the difference between the correlations of two adjacent frames is maximal. We assume a minimum source correlation of  $\rho_T = 0.4$  and a maximum of  $\rho_T = 0.9$  as a larger difference seems unlikely.

#### B. Estimation Algorithms

Since the estimation of the autocorrelation is based on estimated parameters, these have to be determined before each autocorrelation estimation as depicted in Fig. 1. The autocorrelation estimate  $\hat{\rho}_R(\tau, K)$  is then determined from  $K$  successive



**Figure 3:** Time-variant source correlation model functions.

estimated parameters  $\hat{u}(\tau)$  from the frame with time index  $\tau$  as follows

$$\hat{\rho}_R(\tau, K) = \frac{\sum_{k=1}^{K-1} \hat{u}_{k-1}(\tau) \cdot \hat{u}_k(\tau)}{\sum_{k=0}^{K-1} \hat{u}_k^2(\tau)}. \quad (3)$$

The estimation methods are now briefly described, starting with the least complex one:

**1<sup>st</sup> method:** The autocorrelation  $\hat{\rho}_R(\tau, K_{\max})$  is determined after the parameter estimation block from the parameter estimates of the complete present frame, i.e.,  $K_{\max}$  corresponds to the frame size. Then,  $\hat{\rho}_R(\tau, K_{\max})$  is used as the true value to choose the right a priori information in the SDSD for the decoding of the following frame. This method is very simple to implement and has an extremely low computational complexity, since the autocorrelation estimation is performed only once per frame and no additional parameter estimation is needed. However, this method is only justified for slowly varying source correlations.

**2<sup>nd</sup> method:** The second method is a variation of the first one. In the parameter estimation of each frame the autocorrelation  $\hat{\rho}_R(\tau, K)$  with  $K_{\min} \leq K \leq K_{\max}$  is updated after each parameter, as soon as  $K_{\min}$  parameters of the current frame have been estimated. For the parameter estimation of these first  $K_{\min}$  parameters the autocorrelation of the previous frame is utilized. The updating of  $\hat{\rho}_R(\tau, K)$  in one frame is stopped as soon as  $K = K_{\max}$ . In the simulations  $K_{\min} = 7$  turned out to be a good choice. The current value  $\hat{\rho}_R(\tau, K)$  is utilized for estimating the following parameter with the index  $K + 1$ . This estimation method has a slightly increased complexity and a better performance compared to method one. But since the autocorrelation is estimated only in the last execution of the SDSD, the previous iterations between LDPC decoder and the SDSD (LS-iterations) are based on the autocorrelation from the previous frame and might be incorrect. Since no iterative adaption of the correlation estimation is performed in this method, it is still only appropriate for moderately varying source correlations.

**3<sup>rd</sup> method:** Before the first invocation of the SDSD an additional auxiliary parameter estimate  $\tilde{u}$  is generated. This is utilized for extracting a priori information by a successive autocorrelation estimation. Then the SDSD can use the a priori information of the corresponding estimated correlation  $\hat{\rho}_R$ , rather than the a priori information corresponding to the initial value  $\rho = 0.0$  or to the correlations of the previous frame as in methods one and two. In both, the auxiliary and the final parameter estimation, the autocorrelation is estimated according to

the second estimation method. This method is more robust and it has a good performance even with strongly varying source correlations.

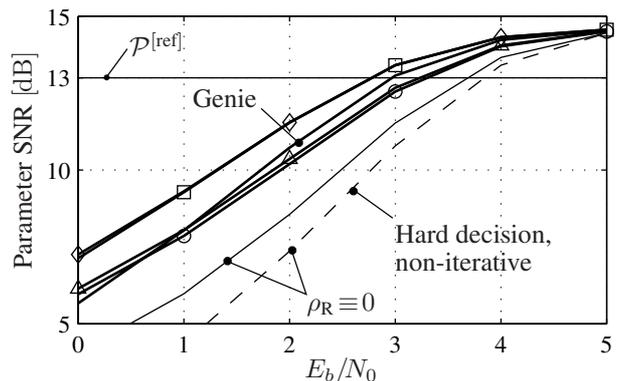
**4<sup>th</sup> method:** The fourth method performs an auxiliary parameter estimation with a successive autocorrelation estimation before every SDDS execution, additionally to the final parameter and autocorrelation estimation. This combination iteratively updates the autocorrelation estimate, so that the SDDS always can utilize the most recent  $\hat{\rho}_R$ . Again, the autocorrelation estimation is performed according to method two. The computational complexity of this method is a multiple of the third one, though the performance in the tested scenarios is very similar.

#### IV. SIMULATION RESULTS

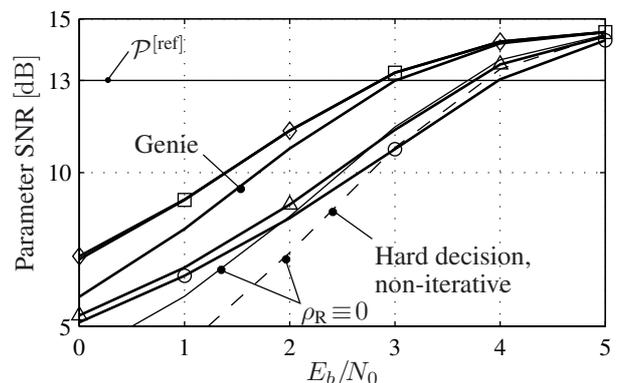
In Figs. 4(a) and 4(b) the parameter SNR performance is depicted for the four autocorrelation estimation methods for the source correlation functions  $\rho_{T_1}(\tau)$  and  $\rho_{T_2}(\tau)$  as well as the so-called Genie curves, which result, when the transmitter correlation is known to the receiver. Also depicted in both plots are the curves for  $\rho_R \equiv 0.0$ , i.e., when no autocorrelation estimation is performed, and additionally the curves for the non-iterative case with hard decision and  $\rho_R \equiv 0.0$ . In the iterative case six LS-iterations are carried out with three LDPC-iterations per LS-iteration (i.e.,  $(L^3S)^6$  iterations). In the non-iterative case only the three LDPC-iterations are carried out (i.e.,  $(L^3S)^1$  iterations).

For  $\rho_{T_1}(\tau)$  the performance differences between the estimation methods are quite small, but all show a better performance than the non-estimating case with  $\rho_R \equiv 0.0$  as visible in Fig. 4(a). Consequently, one of the simpler methods should be sufficient for this source correlation function.

The second source correlation function  $\rho_{T_2}(\tau)$  represents the worst case for the first method. The correlation at the receiver is estimated in one frame, but is exploited only in the following one, and since the source correlation varies strongly from one frame to the next the estimate utilized in the current frame widely differs from the true value. Additionally to the direct deteriorating effect which is based on utilizing a wrong correlation value, a second effect degrades the performance of method one. The received source samples with an originally rather low source correlation  $\rho_T(\tau)$  that are source decoded utilizing too high correlations  $\rho_R$  show a higher correlation  $\rho$  after source decoding than the original source correlation. By the resulting overestimation ( $\hat{\rho}_R > \rho_T$ ) the performance degrades even more, as already shown in Fig. 2. Note that before utilization, the estimated correlations are quantized in order to choose the corresponding a priori probabilities, which have been measured in advance for a finite set of correlations. The source correlations and the corresponding unquantized estimated and/or utilized correlations at the receiver are depicted in Fig. 5 for different estimation methods for the second source correlation model function. In Fig. 5(a) the utilized unquantized correlation estimate for method one is depicted, which is a right-shifted version by one frame of the estimated correlation. The two effects that are responsible for the performance degradation are clearly visible, i.e., first, the utilized correlation is high when



(a) Parameter SNR for source correlation function  $\rho_{T_1}(\tau)$ .



(b) Parameter SNR for source correlation function  $\rho_{T_2}(\tau)$ .

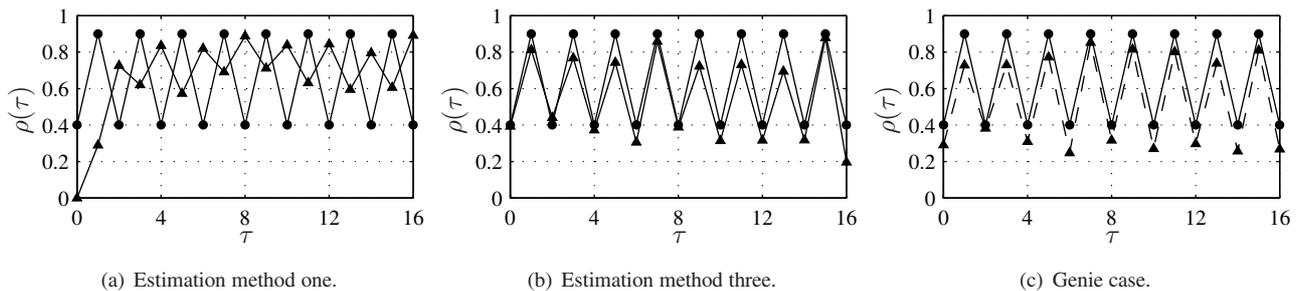
**Figure 4:** Parameter SNR performance of different autocorrelation estimation methods.

Frame size  $K = 330$ , 8-level LMQ, index assign. NB,  $(L^3S)^6$  iterations.

—○— 1<sup>st</sup> method, —△— 2<sup>nd</sup> method,  
—□— 3<sup>rd</sup> method, —◇— 4<sup>th</sup> method.

the source correlation (and consequently the correlation at the receiver, which is not depicted) is low and vice versa, and secondly, the estimate of the lower source correlation ( $\rho_T = 0.4$ ) is much higher than 0.4. With the variations of method two these effects can only be slightly attenuated, but the performance is still worse than the non-estimating case as visible in Fig. 4(b).

Methods three and four, however, show about the same performance as the Genie curve in good channels or even outperform the Genie curve in bad channels for both functions  $\rho_{T_1}(\tau)$  and  $\rho_{T_2}(\tau)$ . The reason for the good performance of estimation method three and four is that the correlation at the receiver is continuously estimated and is directly exploited in the same frame. Due to the channel noise, the resulting higher BER at the channel decoder output and the wrongly source decoded parameters the effective correlation at the receiver can be lower than the source correlation as it can be seen in Fig. 5(b). Nevertheless, this lower estimated correlation has to be chosen at the receiver, since we decode the noisy signal. Note, although method four has a higher computational complexity than method three which is proportional to the number of LS-



**Figure 5:** Comparison of the unquantized estimated and/or utilized correlation at the receiver  $\hat{\rho}_R(\tau)$  ( $\text{---}\blacktriangle\text{---}$ ) with the source correlation  $\rho_{T_2}(\tau)$  ( $\text{---}\bullet\text{---}$ ) at  $E_b/N_0 = 0$  dB (cf. Fig. 4(b)).

- $\hat{\rho}_R(\tau)$  is the utilized unquantized correlation estimate, it is estimated in the previous frame,
- continuous update of  $\hat{\rho}_R(\tau)$  and instant utilization in the same frame,
- $\hat{\rho}_R(\tau)$  is measured at the end of frame  $\hat{u}_\tau$  for comparison only,  $\rho_T(\tau)$  is utilized for the parameter estimation.

iterations, it has only an insignificantly better parameter SNR performance. Consequently, for strongly varying source correlations it is sufficient to choose estimation method three.

Utilizing the source correlation at the receiver, as in the Genie case, results in a performance degradation growing with the channel noise. Then the performance degradation corresponds to the case in which the correlation is overestimated at the receiver. This becomes obvious in Fig. 5(c) by measuring the correlation  $\hat{\rho}_R$  at the receiver for the Genie case, which is lower than the source correlation  $\rho_T$  that is utilized for the parameter estimation.

## V. CONCLUSION

In this paper we presented several estimation methods for the time-varying residual redundancy of transmitted source samples in order to adaptively exploit the redundancy for improving the parameter estimation at the receiver in an ISCD scheme. Their high performance has been demonstrated and verified by means of two source correlation model functions. The more advanced methods, which still exhibit only a moderate increase in complexity, even outperform the scenario with perfect knowledge of the source correlation at the receiver due to the instantaneous adaptation to the current effective correlation. Additionally, we integrated a sophisticated, flexible LDPC code as channel code into the ISCD system.

## REFERENCES

- [1] M. Adrat, P. Vary, and J. Spittka, "Iterative Source-Channel Decoder Using Extrinsic Information from Softbit-Source Decoding," in *IEEE ICASSP*, Salt Lake City, UT, USA, May 2001.
- [2] N. Görtz, "On the Iterative Approximation of Optimal Joint Source-Channel Decoding," *IEEE J. Select. Areas Commun.*, Sept. 2001.
- [3] M. Adrat and P. Vary, "Iterative Source-Channel Decoding: Improved System Design Using EXIT Charts," *EURASIP Journal on Applied Signal Processing (Special Issue: Turbo Processing)*, May 2005.
- [4] T. Fingscheidt and P. Vary, "Softbit Speech Decoding: A New Approach to Error Concealment," *IEEE Trans. Speech Audio Proc.*, pp. 240–251, Mar. 2001.
- [5] P. Vary and R. Martin, *Digital Speech Transmission: Enhancement, Coding and Error Concealment*, John Wiley & Sons Ltd, 2006.
- [6] T. Clevorn, P. Vary, and M. Adrat, "Iterative Source-Channel Decoding using Short Block Codes," in *IEEE ICASSP*, Toulouse, France, May 2006.
- [7] F. Daneshgaran, M. Laddomada, and M. Mondin, "LDPC-Based Channel Coding of Correlated Sources With Iterative Joint Decoding," *IEEE Trans. Comm.*, pp. 577–582, Apr. 2006.
- [8] R. G. Gallager, "Low-Density Parity-Check Codes," *IRE Trans. Inform. Theory*, pp. 21–28, Jan. 1962.
- [9] D. J. C. MacKay, "Good Error-Correcting Codes Based on Very Sparse Matrices," *IEEE Trans. Inform. Theory*, pp. 399–431, Mar. 1999.
- [10] T. Clevorn, F. Oldewurtel, and P. Vary, "Combined Iterative Demodulation and Decoding using very short LDPC Codes and Rate-1 Convolutional Codes," in *CISS*, Baltimore, MD, USA, Mar. 2005.
- [11] N. Görtz, "Optimization of Bit Mappings for Iterative Source-Channel Decoding," in *3rd International Symposium on Turbo Codes & Related Topics*, Brest, France, Sept. 2003.
- [12] T. Clevorn, P. Vary, and M. Adrat, "Parameter SNR Optimized Index Assignments and Quantizers based on First Order A Priori Knowledge for Iterative Source-Channel Decoding," in *CISS*, Princeton, NJ, USA, Mar. 2006.
- [13] E. J. Weldon, "Difference-set cyclic codes," *The Bell Syst. Tech. J.*, pp. 1045–1055, Sept. 1966.
- [14] R. Lucas, M. Fossorier, Y. Kou, and S. Lin, "Iterative Decoding of One-Step Majority Logic Decodable Codes Based on Belief Propagation," *IEEE Trans. Comm.*, pp. 931–937, June 2000.
- [15] J. Pearl, *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*, Morgan Kaufmann, 1988.