ON THE DESIGN OF HIERARCHICALLY MODULATED BICM-ID RECEIVERS WITH LOW INTER LAYER INTERFERENCES

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ABSTRACT

In this paper, we present a novel methodology to optimize hierarchically modulated Bit Interleaved Coded Modulation with Iterative Decoding (HM-BICM-ID). This methodology allows designing a receiver which supports several configurations. Each configuration is able to decode the same transmitted signal over the air with different fidelity. This concept permits using radios with varying processing capabilities, e.g. handhelds, vehicular based etc. However, earlier simulation results have shown that HM-BICM-ID loses, if compared to non-hierarchical schemes, in BER performance due to *Inter-Layer Interferences* (ILI). Our proposed iterative tunable algorithm optimizes hierarchical modulation schemes considering several criteria by moving critical constellation points towards the optimal direction. A novel modulation scheme has been found with minimized ILI and simulation results show an improved asymptotic BER performance in a wide range of channel conditions and for a two layered HM-BICM-ID.

1. INTRODUCTION

A transceiver system based on *Bit Interleaved Coded Modulation* with *Iterative Decoding* (BICM-ID) has been introduced the first time by X. Li and J. A. Ritcey in [1][2]. In their work they have shown that a BICM-ID system significantly outperforms *Trellis-Coded Modulation* (TCM) which has been introduced much earlier by G. Ungerboeck in [3]. A BICM system is given by a serial concatenation of channel coder, bit-interleaver, and modulator. At the receiver side, the arrangement of a demodulator, de-interleaver and channel decoder is used to reconstruct the originally transmitted signal. Compared to G. Ungerboeck's TCM scheme, BICM improves in terms of the *Bit Error Rate* (BER) especially under fading channels. The asymptotic performance of BICM depends mainly on the modulation scheme. For example, a Gray symbol labeling is often used in BICM because neighbored symbols differ at only one bit position which yields a small number of bit errors. The bit-interleaver as another key element in BICM helps to remove long burst errors and correlations between neighbored bits in such a way that the decoder is able to improve decoding. The channel decoder further uses the redundant information introduced at the transmitter side for protection and reconstructs the net bits with high reliability. With the introduction of the well-known turbo principle of digital signal processing [4] it was possible to improve BICM by extending it to BICM-ID. In BICM-ID an additional feedback line is used to exchange information between decoder and demodulator. In [1], a hard-decision feedback version of BICM-ID has been introduced. Additional improvements can be made when extending BICM-ID with soft-decision [5]. Then, reliability information in terms of so-called extrinsic information or log-likelihood ratios (LLR) is exchanged between channel decoder and demodulator. In a BICM-ID system, the BER curves get better with the number of iterations and converge until the *bit error floor* is reached. The *bit error floor* is the part of the BER curve that can be interpreted as the lowest achievable bound for the BICM-ID system and can be simulated according to an *Error Free Feedback* (EFF) scheme. To measure the EFF, *a priori* knowledge is relayed as error free and reliable LLR values to the demodulator. Another typical observation in performance measurements for BICM-ID is the *waterfall region* where the BER is reduced considerably in a small range of E_S/N_0. For a good performance of BICM-ID the channel code and symbol labeling must be optimized together. But significant improvements in BICM-ID, e.g., reaching a very low *bit error floor*, can only be realized when a symbol labeling with a high *Harmonic Mean* d_h^2 [2] is used. In other words, the *Euclidean Distance* of labels that differ only in one bit position shall be maximized. Such an optimization is contrary to the optimization of the modulation scheme in BICM. However, the improved performance for good channel conditions comes with the cost of complexity at the receiver side.

In the literature, *Hierarchical Modulation* [6][7] or *Layered Modulation* has been introduced to give a transceiver system the possibility to receive data under different circumstances. *Hierarchical Modulation* allows the operator to send multiple data streams modulated to a single symbol stream. The different data streams are called *base layer* (BL) or *enhancement layers* (EL). Depending on the channel condition and computational capability of the receiver the modulated symbols can be demodulated in such a way that all or a subset of data streams are recovered. Therefore, the BL provides the most important information being transmitted in a robust way to all radio devices. In addition, each EL carries optional information used to provide further valuable information. This information can be used at the receiver to improve the communication in several ways, e.g., a higher data rate, reliability, or range. One major challenge of hierarchical systems is the design of the different layers which is equivalent to the minimization of *Inter-Layer Interference* (ILI). However, hierarchical modulation is very attractive because of its possibility to switch between different receiver configurations. Therefore, it is used in broadcast systems like *Digital Video Broadcasting over Satellite* (DVB-S2) [8] or over *Terrestrial Antennas* (DVB-T2) [9].

In the literature, digital communication systems exploiting both the benefits of BICM-ID and the advantages of *Hierarchical Modulation* have not been extensively discussed so far. In [10][11], first solutions to a hierarchically modulated BICM-ID (HM-BICM-ID) have been introduced. In our previous work [12], we proposed a HM-BICM-ID system based on a hierarchical 8x8-PSK where each constellation point is composed by a group of eight labels with *Hamming Distance* $d_{ham} = 2$. This was a systematic way to reduce the influence of ILI. Now, we propose a novel algorithm to give the system designer the possibility to develop novel hierarchical modulation schemes with reduced ILI and improved BER performances.

This paper is structured as follows: Chapter 2 introduces the two layered HM-BICM-ID system. The effect of ILI is described in details and reasons for typical performance degradations in hierarchical systems are given. In Chapter 4 we propose a novel algorithm which moves the constellation points of a certain modulation scheme to a proper direction considering two main criteria, the *Harmonic Mean* and the *Bit Error Probability*. Then, the novel algorithm is used in Chapter 4 to develop a novel hierarchical modulation scheme. This scheme is used in a HM-BICM-ID system to perform a BER simulation and the results are presented and discussed. The paper concludes with Chapter 5.

2. HIERARCHICALLY MODULATED BICM-ID

The block diagram of a two configuration transmitter and receiver based on HM-BICM-ID as proposed in [12] is depicted in Fig. 1. It shows two different transmitters, an additive white Gaussian noise (AWGN) channel, and two different receivers. Assuming that two radios of different capabilities can be chosen at the transmitter and receiver side independently, four different configurations for a communication link can be considered for analysis.

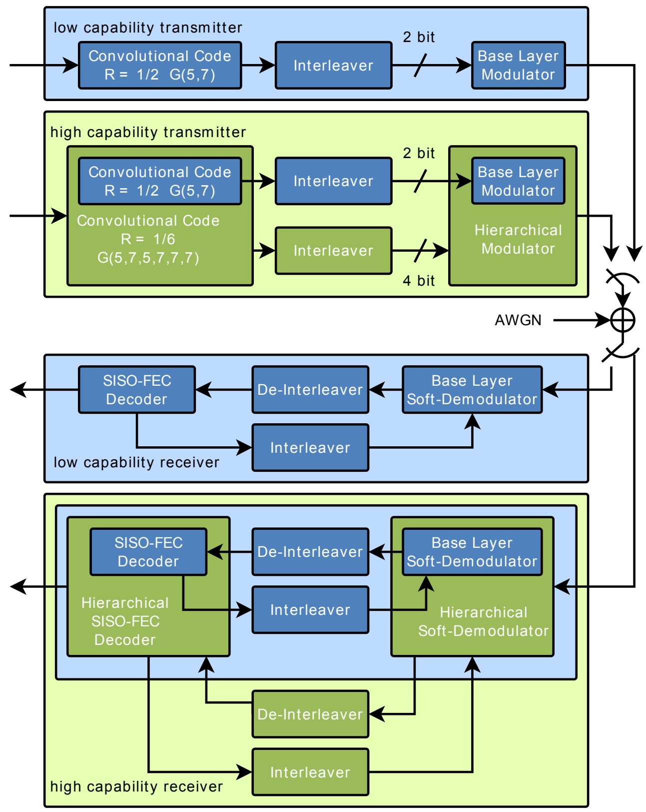
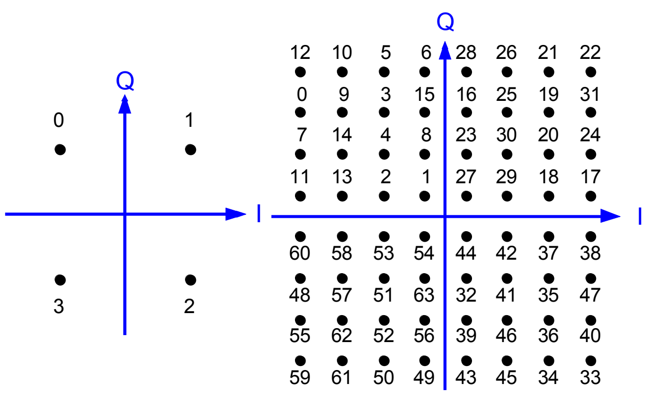


Figure 1: Block Diagram of a BICM-ID (BL) and HM-BICM-ID (EL) considering different configurations for transmitter and receiver.

The BL signal processing boxes used by a low capability transmitter and receiver are shown in Fig. 1 within the blue boxes. The situation is equivalent to signal processing in BICM-ID. Due to the higher computational power available for a high capability radio a HM-BICM-ID transmitter and/or receiver can be used as EL. They are shown in Fig. 1 by green boxes. Please note that the EL implies the blue signal processing blocks from the BL. However, only three of the configurations in Fig. 1 are relevant. Receiving signals, sent by a low capability radio, with the high capability device does not yield any improvements and is therefore irrelevant. Just as well, using a low capability radio at transmitter and receiver side corresponds to a state of the art BICM-ID scheme and thus is not considered in our work. Anyways, to support reliable communication while being flexible in the choice between specific configurations, a fully hierarchical system is used to guarantee interoperability over the air for the other three cases. Therefore, HM-BICM-ID is a suitable solution to manage such communication scenarios. In the following, we consider a high capability transmitter based on HM-BICM-ID. A fully hierarchical transceiver as depicted in Fig. 1 uses the concept of *incremental redundancy* [13][14] in *forward error correction*, e.g., a setup of different convolutional codes as mentioned in [12]. For the BL the generator polynomial $G(5,7)_8$ with a *free distance* of $d_f = 5$ and a coding rate of $R_c = 1/2$ is used. The EL uses the $R_c = 1/6$ convolutional code $G(5,7,5,7,7,7)_8$ with $d_f = 16$. Both convolutional codes with *constraint length* $K = 3$ are proposed by [15][16] due to the maximum *free distance* and *optimal distance spectra*. Because the first code is a subset of the latter one, a separation of the encoded data stream into BL and EL information is possible. Then, each data stream is independently bit-interleaved and merged together to groups of bits. The hierarchical modulator is designed in such a way that the BL modulator is a subset of the EL modulator. A typical hierarchical modulation scheme with 2 bits for the BL and 4 extra bits for the EL is shown in Fig. 2 (a) and (b).



(a) 4-QAM (b) 64-QAM

Figure 2: Labelling of signal constellation points for the BL (a) and the EL (b).

Thus, groups of 6 bits are mapped to one single symbol by the hierarchical modulator. This keeps the overall rate of the system constant. Such a HM-BICM-ID system guarantees the use of high capability radios at receiver side while also keeping signal interoperability over the air with a low capability receiver. In the case where a high capability receiver is used, HM-BICM-ID helps to demodulate and decode the additional information carried by the EL. In our work HM-BICM-ID is designed to provide additional information to improve robustness and range. This is novel compared to broadcast systems where the throughput is increased under good channel conditions [8][9].

2.1 System Design

A big challenge in HM-BICM-ID is the design of the modulation scheme and channel coder in the EL. The BL is often state of the art and somehow fixed. Therefore, especially in the case where a high capability transmitter and low capability receiver are used, interoperability must be kept in mind during the design process. This can be solved when both the BL modulator and BL decoder are subsets of the corresponding EL modulator and EL decoder. With this side constraint so-called ILI is introduced which reduces the overall performance of the communication link. To prevent such negative effects, ILI has to be minimized. ILI can be characterized by two main effects. The first effect is caused by a design constraint of the labels in the EL modulator. Due to the hierarchical nature, the choice for the labels and positions of the constellation points are restricted to a given range. This gives the operator with the low capability radio the possibility to receive the BL information even when the high capability transmitter has been used. Unfortunately, restrictions in the design of the modulation scheme lead to a reduced *Harmonic Mean* d_h^2. A high d_h^2 causes BICM-ID systems to improve the overall BER performance under good channel conditions. Therefore, we expect a degradation of the BER performance for HM-BICM-ID systems. The second effect of ILI affects the BL itself. The use of an EL modulator at the transmitter side and a BL demodulator at the receiver side results in a mismatch between the symbols and introduces additional noise at the receiver. For example, to transmit the EL modulator symbol 49_{10} / [MSB]110001_2 as depicted in Fig. 2 (b) causes the BL demodulator to decide for the correct symbol 3_{10} / [MSB]11_2 when AWGN is absent. However, the probability for a wrong decision increases because some EL symbols are close to the BL decision bounds. This causes the BL demodulator to decide wrong when AWGN is present. This is illustrated in Fig. 3 where EL symbol 49_{10} is noisy received (green arrow) and demodulated by the BL to 2_{10}.

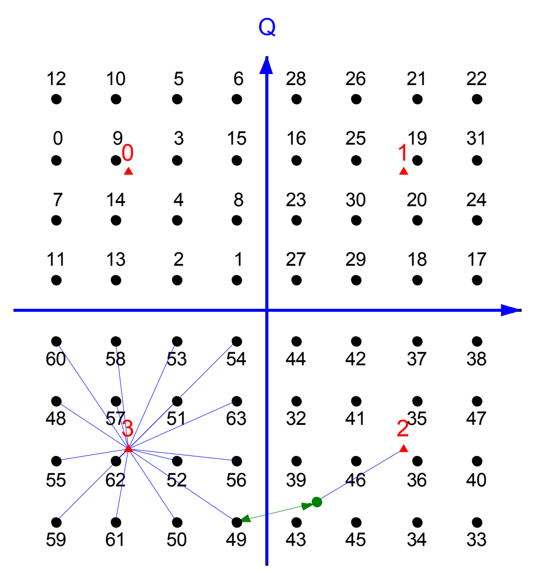


Figure 3: Constellation points mismatch between EL and BL modulation scheme causes additional interferences.

3. NOVEL ALGORITHM FOR JOINT MULTI-LAYER OPTIMIZATION OF HIERARCHICAL MODULATION SCHEMES

Due to the fact that ILI causes performance degradations in hierarchical receivers, our main idea is to minimize ILI by minimizing the underlying effects that cause ILI. Therefore, we propose a novel algorithm that starts from a hierarchical modulation scheme whose labels have already been optimized for BICM-ID. The algorithm moves the constellation points towards the direction of interest in a serial iterative manner. We propose to optimize two main criteria for each layer. On the one hand, we want to increase the *Harmonic Mean* $d^2_{h,layer}$ of each layer. On the other hand, the algorithm decreases the *Bit Error Probability* $P_{b,layer}$ in each layer. To prevent an uncontrolled growth of the energy per symbol E_S during the movement of the constellation points a normalization is done after each algorithm step. Thus, the energy per symbol is kept constant during the process of the algorithm and a convergence of the algorithm is guaranteed. A description for a comprehensive optimization of a hierarchical modulation scheme is given by Algorithm 1.

Algorithm 1: Comprehensive optimization

```
Initialize constellation points and labels
normalize
measure d²_h,layer & P_b,layer for each layer
set optional stop criteria for lower (LB) and upper
  bound (UB), e.g. d²_h,layer,LB; d²_h,layer,UB; P_b,layer,LB; P_b,layer,UB
set flag as true
set maximum number of iterations
loop until stop criteria fulfilled or flag equals false
      or number of iterations exceeded
  set for each layer: d²_h,layer,old as d²_h,layer
  set for each layer: P_b,layer,old as P_b,layer
  optimize d²_h,layer with Algorithm 2 for each layer
  optimize P_b,layer with Algorithm 3 for each layer
  measure d²_h,layer & P_b,layer for each layer
  if d²_h,layer < d²_h,layer,old then
    set flag as false
  end if
  if P_b,layer > P_b,layer,old then
    set flag as false
  end if
end loop
```

First, the initial modulation scheme, i.e., constellation points and labels, is normalized and the *Harmonic Mean* and the *Bit Error Probability* of each layer are calculated. Additionally, optional stopping criteria for the algorithm are defined, e.g., an upper and a lower bound for the main criteria. For example, a very small *Harmonic Mean* for the EL might be a good choice to optimize BICM-ID performance for the EL. The algorithm optimizes for each layer two criteria. The parameters are measured once again and compared with the old values. This is important because the optimization of the first parameter, e.g., the *Harmonic Mean*, might degrade the *Bit Error Probability* and vice versa. A comparison between the two constellations is done to recognize an improvement during the iteration process. If one parameter degrades, the algorithm stops immediately and outputs the final constellation. To maximize the *Harmonic Mean*, we propose to use an algorithm as described in Algorithm 2.

Algorithm 2: Maximize $d^2_{h,layer}$ of a specific layer

```
Initialize normalized modulation scheme
for k = 1 to number of constellation points do
  compute direction vector r_k,layer using Eq. (3)
end for
for k = 1 to number of constellation points do
  move constellation point to direction of r_k,layer
end for
normalize
```

The algorithm initializes the normalized modulation scheme which has been committed by algorithm 1. Then the direction vector $\vec{r}_{k,layer}$ for a specific layer is computed. A more detailed explanation of the derivation and definition of $\vec{r}_{k,layer}$ including Eq. (3) are given in sub-section 3.1. The computation is performed for each symbol of the modulation scheme. Finally, the constellation points are moved towards the computed direction and normalized. The concept of Algorithm 2 can easily be adapted to the *Bit Error Probability* in the same way and is described in Algorithm 3. The direction vector is here identified as $\vec{s}_{k,layer}$ which is derived and explained in more details in sub-section 3.3.

Algorithm 3: Maximize $P_{b,layer}$ of a specific layer

```
Initialize normalized modulation scheme
for k = 1 to number of constellation points do
  compute direction vector s_k,layer using Eq. (9)
end for
for k = 1 to number of constellation points do
  move constellation point to direction of s_k,layer
end for
normalize
```

3.1 Maximization of the *Harmonic Mean* $d^2_{h,layer}$

To maximize the *Harmonic Mean* d^2_h of both layers in a HM-BICM-ID we first need to define it for hierarchical modulation schemes. From [2] the definition of d^2_h can be modified and rewritten in such a way that $d^2_{h,layer}$ is given by the invers of the sum of the *Harmonic Mean Costs* H_k^{cost} for each Symbol x_k with $k \in [1..M]$ of an M-ary modulation scheme with predefined labels and constellation points:

$$d^2_{h,layer} = \left(\frac{1}{M} \sum_{k=1}^{M} H_k^{cost} \right)^{-1} \quad (1)$$

To improve a HM-BICM-ID system $d^2_{h,layer}$ needs to be maximized which corresponds to the minimization of all *Harmonic Mean Costs* H_k^{cost}:

$$H_k^{cost} = \frac{1}{m_{layer}} \sum_{l=1}^{m_{layer}} \left(\frac{2^{m_{layer}}}{M} \sum_{z \in X^l_{\bar{b}(x_k)}} \frac{1}{\|x_k - z\|^2} \right) \quad (2)$$

H_k^{cost} is defined as the sum of the inverse *Euclidean Distances* between the symbol x_k and all neighbors z defined by the subset $X^l_{\bar{b}(x_k)}$. The subset includes all symbols z with an inversed bit at the l^{th} bit position and equal bit patterns for the remaining BL. For example, the EL symbol 32_{10} / [MSB]100000_2 is related to the BL symbol 2_{10} / [MSB]10_2. Considering the 2nd bit position $l = 2$ all EL symbols with binary labelling [MSB]$11xxxx_2$ are neighbors because they are all related to the BL symbol 3_{10} / [MSB]11_2. This is shown in Fig. 4 by the green lines. The second group with inversed bit at bit position $l = 1$ is defined by the BL symbol 0_{10} / [MSB]00_2 and related to all EL symbols with bit pattern equal to [MSB]$00xxxx_2$. This is shown in Fig. 4 by blue lines. As a consequence, the number of neighbors for a hierarchical modulation scheme for a specific layer increases compared to the definition of the *Harmonic Mean* in non-hierarchical modulation schemes.

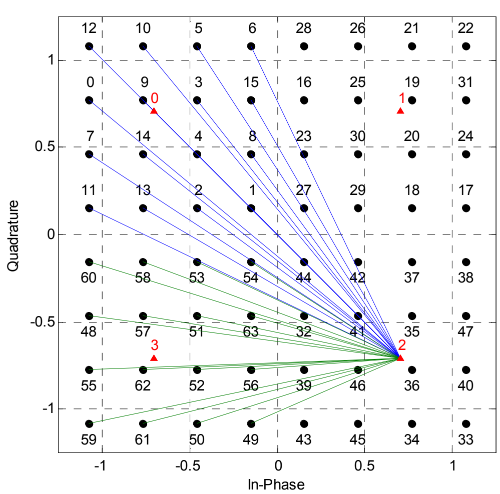


Figure 4: Constellation diagram of a QPSK (BL) and 64-QAM (EL) with corresponding pairs of neighbors in terms of the optimization of the *Harmonic Mean* of the BL.

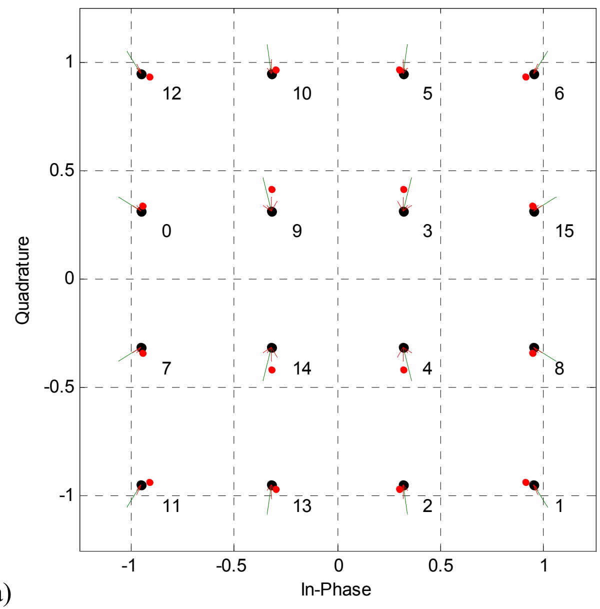
With the definition form Eq. (1) and Eq. (2), we propose the maximization of the *Harmonic Mean* by moving the constellation points far away from all neighbors with high influence. As a consequence, the vector for the movement considering a specific layer can be identified as follows:

$$\vec{r}_{k,layer} = \frac{\xi_{layer}}{m_{layer}} \sum_{l=1}^{m_{layer}} \left(\frac{2^{m_{layer}}}{M} \sum_{z \in X^l_{\bar{b}(x_k)}} \frac{e^{j \cdot \measuredangle(x_k - z)}}{\|x_k - z\|^2} \right) \quad (3)$$

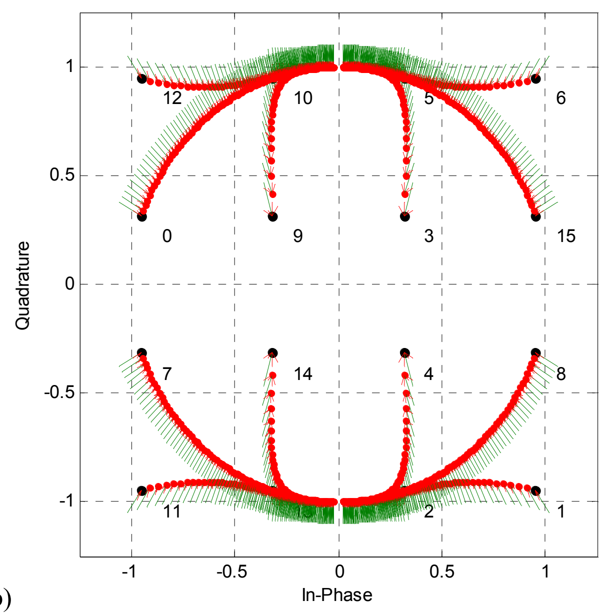
The vector $\vec{r}_{k,layer}$ describes the vector to move the k^{th} symbol of an M-ary modulation scheme in a proper direction. The definition is similar to Eq. (2) and introduces an additional term for the direction $e^{j \cdot \measuredangle(x_k - z)}$. The inverse of the *Euclidean Distance* is used to weight the direction vector. A lower distance between neighbors in the terms of the *Harmonic Mean* causes an increased value and influence of the movement. The direction component in x_k for the movement away from z_l is defined by the vector subtraction. The final direction vector is the superposition of the direction vectors of each neighbor. For hierarchical modulation schemes the number of neighbors differs significantly. The scaling factor ξ_{layer} is introduced to further control the convergence behavior of Algorithm 1 and shall be a positive real value $\xi_{layer} \in \mathbb{R}^+$. Considering a two layered modulation scheme with a BL of 2 bits per symbol and an EL with 6 bits per symbol values for m_{layer} are given by $m_{BL} = 2$ and $m_{EL} = 6$. Please note, in case of the EL the definition of the *Harmonic Mean* from Eq. (1) and (2) falls back to the definition of [2] because the number of neighbors reduces to m_{EL}.

3.2 Convergence Behavior during Optimization of the *Harmonic Mean*

In the previous subsection we described the algorithm for optimizing the *Harmonic Mean* d_h^2 for each layer. A high *Harmonic Mean* guarantees a convergence to an asymptotic low BER during the iterative process in BICM-ID and HM-BICM-ID systems. This is mainly because the *a priori* knowledge is going to improve during the iterations and thus the demodulator is able to distinguish much better between two symbols of similar labels (one bit position differs). However, in the first iteration in BICM-ID another challenge has to be mastered because no *a priori* information is available at the beginning. During the development of Algorithm 1, it has been observed for a given constellation, e.g., 16-QAM as shown in Fig. 5 (a), and several algorithmic iterations that the novel constellation may converge to a multi-labeled *Binary Phase-Shift Keying* (BPSK). Two groups of eight constellation points with even and odd parity bits in the labels appear as depicted in Fig. 5 (b). This is caused by the fact that BPSK has the highest *Harmonic Mean* of $d^2_{h,BPSK} = 4$.



(a)



(b)

Figure 5: 16-QAM-Ray after 1st iteration (a) and 100th iteration (b) with Algorithm 2 and $\xi = 0.1$.

However, this causes a new challenge. Without any *a priori* knowledge at the first step in a BICM-ID system, the soft-demodulator, based on a multi-labeled BPSK, can only distinguish between the even and odd parity group but not between different labels within the same group. This is because the labels within one group have the same constellation points. Therefore the demodulator cannot produce any valuable extrinsic information and the BER performance will degrade.

In a second simulation we analyzed the behavior of the *Harmonic Mean* under the influence of the number of iterations and the scaling factor ξ. The results are shown in Table 1.

Table 1: Influence of the scaling factor ξ and the number of iterations on the convergence behavior of Algorithm 1

Iteration number	*Harmonic Mean* d_h^2		
	$\xi = 0.01$	$\xi = 0.1$	$\xi = 1.0$
0th (initial)	2.7190	2.7190	2.7190
1st iteration	2.7411	2.9102	3.4454
2nd iteration	2.7625	3.0555	3.6244
5th iteration	2.8232	3.3369	3.8483
10th iteration	2.9136	3.5669	3.9619
100th iteration	3.5856	3.9946	4.0000

For this, the algorithm has been executed with an initial 16-QAM-Ray, as given in [17][18], and without any restrictions, i.e., no partitioning into different layers and no further stopping criteria. 16-QAM-Ray labelling has been chosen because of its *Harmonic Mean* of $d^2_{h,\text{16-QAM-Ray}} = 2.719$ which is the highest compared to any other 16-QAM labelling. Within Table 1, a general convergence behavior towards a multi-labeled BPSK scheme can be observed for all parametrizations. But with a higher value for ξ a reduced number of iterations is needed to reach the multi-labeled BPSK and therefore a faster convergence behavior is observed. But this might also result in the instability of the algorithm when used with a hierarchically modulated scheme. Thus, the movement must be controlled by a carefully chosen ξ to ensure that no constellation point will cross a decision bound of another layer as this would violate the premise of one layer being a subset of another layer.

3.3 Minimization of the *Bit Error Probability* $P_{b,layer}$

As a consequence of the multi-labeling problem, we propose not only to use the *Harmonic Mean* d_h^2 as an optimization criterion, but also the *Bit Error Probability* P_b. The main idea is to prevent the collapse of several constellation points to groups and thus achieve a minimization of the BER for the first iteration in BICM-ID.

In Algorithm 3, we propose the calculation of the direction vector $\vec{s}_{k,layer}$ for a given constellation point x_k based on the *Probability Density Functions* (PDF) of each symbol in a modulation scheme as well as the labels and the decision bounds. The main challenge to calculate the influence is the fact that the symbols are moving and changing during the iteration process of the algorithm and therefore the decision bounds and PDF are changing. This renders the determination of the *Bit Error Probability* in an analytical way, e.g., by integration, very complex or even impossible. Therefore, we propose a numerical way of calculating the *Bit Error Probability* considering a division of the I/Q-plane into sufficiently segments. Assuming an additive white Gaussian noise (AWGN) channel, the PDF $p(\boldsymbol{w}, \boldsymbol{x}_k)$ in $\boldsymbol{w}$ with the mean equal to the k^{th} symbol $\boldsymbol{x}_k$ and a variance of $\sigma^2 = N_0/E_S$ can be described by:

$$p(\boldsymbol{w}, \boldsymbol{x}_k) = Pr(\boldsymbol{x}_k) \cdot \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{|\boldsymbol{w}-\boldsymbol{x}_k|^2}{2\sigma^2}} \quad k \in [1..M] \tag{4}$$

To fulfill the side constraint that the segment under the sum of the PDF $p(\boldsymbol{w}, \boldsymbol{x}_k)$ in $\boldsymbol{x}_k$ over all symbols must be equal to one, the $p(\boldsymbol{w}, \boldsymbol{x}_k)$ must be normalized by the symbol probability $Pr(\boldsymbol{x}_k)$. Considering that all symbols are equally distributed, we can define the symbol probability as $Pr(\boldsymbol{x}_k) = 1/M$. Assuming, that a noisy symbol has been received in the specified small segment with center $\boldsymbol{\delta}_{i,j}$ and, assuming that the symbol $\boldsymbol{x}_k$ has been sent, the *Bit Error Probability* $Pr(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k)$ is given by:

$$Pr(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k) = \int_{\delta_j-\frac{\Delta}{2}}^{\delta_j+\frac{\Delta}{2}} \int_{\delta_i-\frac{\Delta}{2}}^{\delta_i+\frac{\Delta}{2}} p(\boldsymbol{w}, \boldsymbol{x}_k)\, dw_x\, dw_y \tag{5}$$

Δ describes a sufficiently small grid size for a specific segment of the I/Q-plane for both directions w_x and w_y. $\boldsymbol{\delta}_{i,j} = (\delta_i, \delta_j) \in \mathbb{R}^2$ is the center of the specific segment. For all segments which are part of a given border, a slightly modified bound of $\pm\infty$ for the integral shall be used. In a further step, only those segments where the labels between the nearest and the originally transmitted symbol differs in at least one bit position will have an influence on the *Bit Error Probability* from (5). Therefore, it must be multiplied by the label error rate Λ_{layer} which is related to the normalized *Hamming Distance*. We can derive:

$$P_{b,layer}(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k) = \Lambda_{layer}(\mu(\boldsymbol{\delta}_{i,j}), \mu(\boldsymbol{x}_k)) \cdot Pr(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k) \tag{6}$$

$\Lambda_{\text{layer}}(\mu(\boldsymbol{\delta}_{i,j}), \mu(\boldsymbol{x}_k))$ is defined by the *Hamming Distance* considering only a subset of m_{layer} bits of the labels divided by the maximum number of layer bits:

$$\Lambda_{layer}\left(\mu(\boldsymbol{\delta}_{i,j}), \mu(\boldsymbol{x}_k)\right) = \frac{d_{ham,m_{layer}}(\mu(\boldsymbol{\delta}_{i,j}), \mu(\boldsymbol{x}_k))}{m_{layer}} \tag{7}$$

$d_{ham}(\mu(\boldsymbol{\delta}_{i,j}), \mu(\boldsymbol{x}_k))$ is the *Hamming Distance* between the label of $\boldsymbol{x}_k$ and the label related to $\boldsymbol{\delta}_{i,j}$ where $\boldsymbol{\delta}_{i,j}$ is mapped to the nearest symbol $\boldsymbol{x}_l$ with:

$$l = \underset{n \in [1..M]}{\arg\min} \left|\boldsymbol{\delta}_{i,j} - \boldsymbol{x}_n\right|^2 \tag{8}$$

A colored projection of the mapping μ is shown in Fig. 6. The values in the color bar are arranged from 1 to 64 according to the mapping of the hierarchical 64-QAM modulation scheme from Fig. 2 (b).

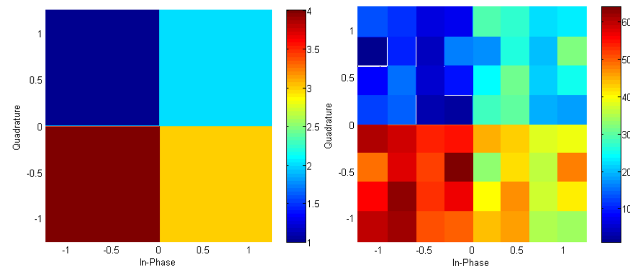


Figure 6: Colored projection of the mapping $\mu(\boldsymbol{\delta}_{I,Q})$ of a hierarchical 64-QAM modulation scheme from Fig. 2. BL (a) and EL (b).

Similarly, the hierarchical BL part of the 64-QAM modulation scheme which is given by the QPSK from Fig. 2 (a) is also shown in Fig. 6. The colors of the four quadrants are similar to the mean color of those quadrants in Fig. 6 which is typical for a hierarchical modulation scheme. The corresponding *Bit Error Probability* $P_b(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k)$ of all symbols $\boldsymbol{x}_k$ over the I/Q-plane is shown as a colored scatterplot in Fig. 7.

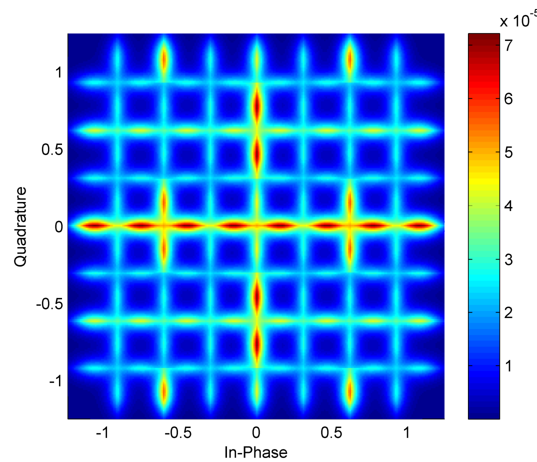


Figure 7: Colored projection of the *Bit Error Probability* for all symbols of a given two layered 64-QAM as depicted in Fig. 2 (b) for E_s/N_0 = 15 dB.

With the given *Bit Error Probability* $P_b(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k)$ for a small segment in the constellation diagram the direction of influence for x_k can be expressed as follows:

$$\vec{s}_{k,layer} = \beta_{layer} \sum_{\boldsymbol{\delta}_{i,j} \in \mathbb{R}^2} P_{b,layer}(\boldsymbol{\delta}_{i,j}, \boldsymbol{x}_k) \cdot e^{j \cdot \measuredangle(x_k - \delta_{i,j})} \tag{9}$$

$\vec{s}_{k,layer}$ is a vector describing the direction of movement as a result of the superposition of the movements expressed by each small segment. For all segments a direction based on the vector subtraction of $\boldsymbol{x}_k - \boldsymbol{\delta}_{i,j}$ can be determined. The weight for the direction is directly related to the bit error rate of each segment. Therefore, a low E_s/N_0 and a short distance to a neighbor will result in a higher weight of the segment. Due to the nature of the Gaussian distribution the segments with highest impact are arranged at the borders of the decision bounds. Consequently, the direction of movement is very often similar to the task of increasing the distance to the nearest neighbor. The convergence behavior can be controlled by the positive real value $\beta_{layer} \in \mathbb{R}^{+}$.

4. PERFORMANCE ANALYSIS OF HM-BICM-ID WITH OPTIMIZED MODULATION SCHEME

In the previous sections we introduced our proposed algorithm. Two parameters, the *Harmonic Mean* and the *Bit Error Probability*, have been used as optimization criteria for the algorithm. Now, we want to use the novel algorithm to develop a novel hierarchical modulation scheme with optimized performance in terms of BER for both the BL and EL of a two layer modulation scheme.

4.1 An Optimized Two Layer Modulation Scheme Based on a Hierarchical 64-QAM Modulation Scheme

In [12], HM-BICM-ID has been introduced for several configurations. Both three layer and two layer hierarchical modulation schemes have been investigated and it was shown that a reduced number of layers give the designer more freedom to optimize the modulation scheme. Therefore, we propose a HM-BICM-ID system with one BL and one EL to keep the freedom in the design of the hierarchical modulation as high as possible. Further, we propose the BL modulation scheme to be a QPSK as shown in Fig. 2 (a) with *Harmonic Mean* $d^2_{h,QPSK} = 2.6667$.

The initial EL is a hierarchical 64-QAM which can be constructed by superposition of QPSK mapping (first 2 bits fixed) and a 16-QAM-Ray labelling from [17][18] for each quadrant. The mapping is identical to Fig. 2 (b). The parameter set for the algorithm has been chosen to $\beta_{EL} = 25.0$, $\xi_{EL} = 0.01$, $\beta_{BL} = 0.2$, $\xi_{BL} = 0.01$. The maximum number of iterations is 10. There were no further restrictions, e.g., no optional stopping criteria. Executing the algorithm results in a constellation diagram as depicted in Fig. 8.

Due to the look of the scatter plot we proposed to call the constellation *64-QAM ButterFly* (64-QAM-BF). The normalized symbols of all labels (index) are given in details in Table 2.

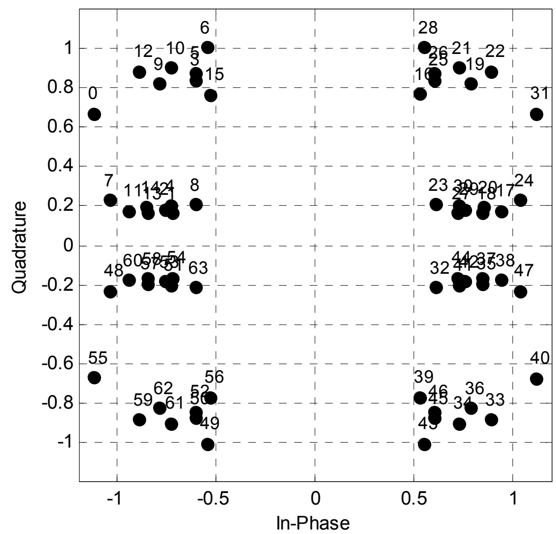


Figure 8: 64-QAM-BF modulation scheme

Table 2: Symbol mapping of the 64-QAM-BF

Map	Symbol	Map	Symbol
0	-0.8910 + 0.8805i	1	-0.7261 + 0.9005i
2	-0.6066 + 0.8749i	3	-0.5489 + 1.0047i
4	0.5500 + 1.0045i	5	0.6072 + 0.8750i
6	0.7262 + 0.9004i	7	0.8909 + 0.8806i
8	-1.1176 + 0.6701i	9	-0.7861 + 0.8201i
10	-0.6058 + 0.8386i	11	-0.5292 + 0.7662i
12	0.5303 + 0.7667i	13	0.6063 + 0.8386i
14	0.7863 + 0.8201i	15	1.1176 + 0.6703i
16	-1.0410 + 0.2303i	17	-0.8521 + 0.1918i
18	-0.7322 + 0.2010i	19	-0.6073 + 0.2088i
20	0.6079 + 0.2086i	21	0.7323 + 0.2009i
22	0.8521 + 0.1917i	23	1.0413 + 0.2301i
24	-0.9419 + 0.1735i	25	-0.8464 + 0.1646i
26	-0.7566 + 0.1831i	27	-0.7242 + 0.1625i
28	0.7243 + 0.1625i	29	0.7567 + 0.1830i
30	0.8463 + 0.1646i	31	0.9420 + 0.1734i
32	-0.9413 - 0.1755i	33	-0.8475 - 0.1665i
34	-0.7569 - 0.1841i	35	-0.7245 - 0.1648i
36	0.7246 - 0.1648i	37	0.7569 - 0.1840i
38	0.8475 - 0.1664i	39	0.9412 - 0.1754i
40	-1.0408 - 0.2326i	41	-0.8509 - 0.1922i
42	-0.7329 - 0.2021i	43	-0.6078 - 0.2108i
44	0.6084 - 0.2106i	45	0.7330 - 0.2020i
46	0.8510 - 0.1921i	47	1.0410 - 0.2324i
48	-1.1191 - 0.6706i	49	-0.7863 - 0.8226i
50	-0.6054 - 0.8405i	51	-0.5290 - 0.7672i
52	0.5301 - 0.7678i	53	0.6060 - 0.8405i
54	0.7864 - 0.8226i	55	1.1191 - 0.6707i
56	-0.8917 - 0.8813i	57	-0.7263 - 0.9021i
58	-0.6061 - 0.8763i	59	-0.5489 - 1.0052i
60	0.5499 - 1.0050i	61	0.6066 - 0.8763i
62	0.7264 - 0.9021i	63	0.8916 - 0.8815i

4.2 Performance Analysis for 64-QAM-BF

In a first step, we measured the *Harmonic Mean* for different modulation schemes as depicted in Table 3. The upper schemes are measured for a single layered typical BICM-ID system. The values are given from literature. It can be observed that hierarchical schemes used for BICM-ID systems have poor performance due to the significantly reduced *Harmonic Mean*. Therefore in [12], performance measurements in terms of BER have been proposed. The best performance has been reached by an 8x8-PSK modulation scheme because the *Harmonic Mean* is increased. The reason for this is a relaxed design of the constellation points which is no more fixed to the 64-QAM scheme. Compared to the latter one, the novel 64-QAM-BF modulation scheme has a slightly reduced *Harmonic Mean*. But, considering the *Harmonic Mean* of the BL in a HM-BICM-ID system we find the *Harmonic Mean* of the BL to be increased dramatically to $d_{h,BL}^2 = 2.847$. Therefore, we expect the 64-QAM-BF to outperform all other two and three layered hierarchical modulation schemes from [12].

Table 3: *Harmonic Mean* d_h^2 for several modulation schemes

System	Modulation	Layer 1st	Layer 2nd	Layer 3rd
BICM-ID	BPSK (upper bound)	4.000		
	φ-4PSK; φ=90˚ [19]	2.667		
	16-QAM-Ray [17][18]	2.719		
	64-QAM Non Hierarch. [12]	2.874		
	16-QAM Straightforward [12]	0.853		
	64-QAM Straightforward [12]	0.290		
	8x8-PSK [12]	0.792		
	64-QAM-BF	0.689		
HM-BICM-ID	BL φ-4PSK; φ=90˚ [19] EL1 16-QAM Straightfor. [12] EL2 64-QAM Straightforward	1.407	0.665	0.290
	BL φ-4PSK; φ=90˚ [19] EL 8x8-PSK [12]	1.958	0.792	
	BL φ-4PSK; φ=90˚ [19] EL 64-QAM-BF	2.847	0.689	

In a second step, we performed a BER simulation for a two layer HM-BICM-ID system with 64-QAM-BF. BER curves and the corresponding EFF curves have been measured for both the BL and EL. The setup for the simulation environment has been chosen equivalent to [12] and is depicted in Table 4. In Fig. 9 the BER performance depending on the energy per symbol to noise ratio E_S/N_0 is depicted for three systems. The reference system is a non-iterative BICM and uses a QPSK modulation scheme with Gray labeling. The BER is described by the black solid curve. The second system is the HM-BICM-ID from [12] with a hierarchical modulation with relaxed design constraints. The BER performance of the BL and EL referred in [12] are depicted by the blue solid (square marker) and green solid (diamond marker) BER curves. The performance of the EL compared to the reference system considering a BER of 10^{-6} is a gain of 2.98 dB. But, comparing the reference system with the BL approach from [12] we find a loss of 2.02 dB.

Table 4: Setup of simulation parameters for HM-BICM-ID

Parameter	Value
Frame size	1000 net bits
BL FEC	Feed Forward Convolutional Code $G(5,7)_8$; R = 1/2; K = 3; L = 2 tail bits
BL encoded bits	2004 encoded bits
BL Interleaver	Random Interl.: size of 2004 bits
BL modulation	QPSK: 2 bits / symbol
EL FEC	FFCC: $G(5,7,5,5,5,7)_8$; R = 1/6; K = 3; L = 2 tail bits
EL encoded bits	6012 encoded bits
EL Interleaver	Hierarchical Random configuration Interl. 1: 2004 bits; Interl. 2: 4008 bits
EL modulation	hierarchical 64-QAM: 6 bits / symbol 2 bits for BL; 4 bits for EL
total symbols	1002 symbols / frame
total rate	≈ 1 net bit per symbol
Iterations	10

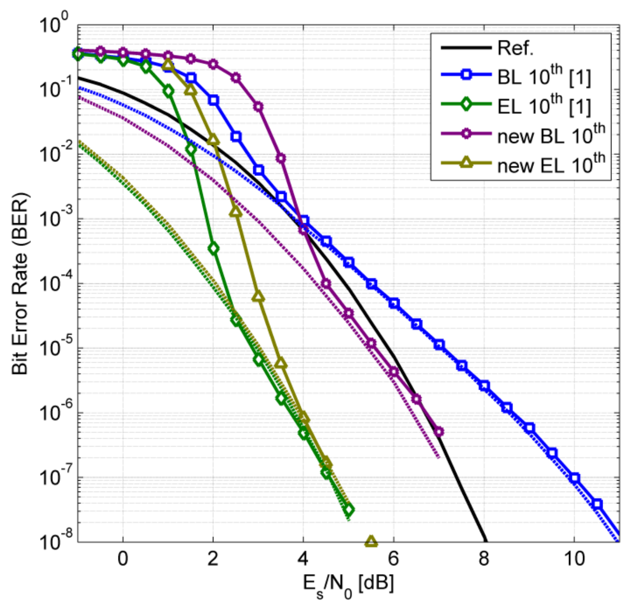


Figure 9: Comparison between HM-BICM-ID with novel modulation scheme and with relaxed design constraint [12]

However, the third system corresponds to our novel approach for a HM-BICM-ID with a modulation scheme as depicted in Fig. 9. The BER performance is given by the violet solid line (star marker) and yellow ocher solid line (triangle marker). Each dashed colored line describes the EFF curve of the associated HM-BICM-ID system. The EFF describes the lowest possible BER performance that can be reached with the corresponding system. As it can be seen, the novel EL approach has a similar EFF curve compared to the EL from [12]. For a BER of 10^{-6} dB a gain of approximately 2.71 dB can be reached. Considering now the BL and BER of 10^{-6} dB the novel approach has a very small Loss of 0.02 dB. Therefore, the BL performance of the novel approach outperforms [12]. This remains also valid for BER < 10^{-6}. Of course, the better performance in the BL comes with the cost of a shifted *waterfall region* in both the BL and EL. For the BL, the novel HM-BICM-ID already improves for E_S/N_0 > 4 dB and BER < 10^{-3}. Other regions of the BER and E_S/N_0 are irrelevant. Therefore, only in the EL the novel approach performs slightly worse because of the shifted *waterfall region*. For higher E_S/N_0 the novel approach performs similar to [12]. Therefore, we can conclude that the novel algorithm gives the designer more freedom to improve specific layers without degrading the performance of another layer in the same way. Thus, the overall BER of HM-BICM-ID is improved. Further, we can additionally deduct that ILI has been greatly reduced. Finally, the novel HM-BICM-ID system outperforms the BICM reference system in the EL for values of E_S/N_0 > 2 dB and in the BL between 4 dB < E_S/N_0 < 7 dB. Only in the relevant range E_S/N_0 > 7 dB a small loss of the BER performance in the BL can be observed although the EFF curve is below the BER curve of the reference system.

5. CONCLUSIONS

In our paper we proposed the concept of HM-BICM-ID. To further improve hierarchical modulation schemes, we developed a novel algorithm to move constellation points of a certain modulation scheme to a direction where critical parameters for a specific layer, i.e., the *Harmonic Mean*, and the *Bit Error Probability* are maximized. For a given parametrization, e.g., the number of iterations and the convergence behavior of each optimization step, we derived a novel modulation scheme termed 64-QAM-BF. To demonstrate the performance of the scheme we designed a HM-BICM-ID with 64-QAM-BF and compared it with those already known from literature. It has been observed that the novel algorithm provides more design freedom to improve BER performance. Finally, it has been shown that the novel HM-BICM-ID outperforms the reference system for a wide range of E_S/N_0 in both the BL and EL. In our future research work, we plan to modify the algorithm to improve the convergence behavior and the balance between the different optimization criteria. This shall help to fine tune modulation schemes. HM-BICM-ID uses BICM-ID in each layer. In future, we plan a novel receiver using more powerful codes, e.g., turbo codes. Thus, an unequal FEC for each layer shall help to balance performance between layers.

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